Introduction to the theory of confinement

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CEA Cadarache
Confinement is a crucial issue for fusion

- Lawson criterion for ignition
  \[ n_D T_D \tau_E = 3 \times 10^{21} \text{m}^{-3} \text{keV.s} \]

- Confinement
  \[ \tau_E = \frac{\text{Energy content}}{\text{Power losses}} \]
  \[ \sim 3.7 \text{ s in ITER} \]
  \[ \rightarrow \text{Transport} \]
Some orders of magnitude

• Heat flux equation
  \[ \frac{3}{2} n \partial_t T + \nabla \cdot \phi_T = S \]
  \[ \phi_T = -n \chi_T \nabla T \]

• Transport in a tokamak is diffusive
  \[ \tau_E \approx \frac{a^2}{\chi_T} \]
  \[ \text{If } \tau_E \approx 1 \text{s and } a \approx 1 \text{m, then} \]
  \[ \chi_T \approx 1 \text{m}^2\text{s}^{-1} \]
Orders of magnitude (cont.)

- Collisional transport:
  
  random displacement $\approx \rho_c$
  
  every collisional time $1/\nu_c$
  
  $\rightarrow \chi_{T,\text{coll}} \approx \nu_c \rho_c^2$

- Neoclassical theory: enhanced collisional transport due to magnetic pumping
  
  $\rightarrow \chi_{T,\text{neo}} \approx q^2 / (r/R) ^ {3/2} \nu_c \rho_c^2$

  Ions $\chi_{T,\text{coll},i} \approx 0.1 \text{m}^2\text{s}^{-1}$
  
  Electrons $\chi_{T,\text{coll},e} \approx 0.001 \text{m}^2\text{s}^{-1}$

- Usually smaller than experimental value.
• Losses are mainly conductive

\[ \tau_E \approx \frac{a^2}{\chi_{turb}} \]

→ Turbulent diffusion \( \chi_{turb} \) determines the confinement.

• However:
  - parallel transport is nearly collisional,
  - collisional transport can be dominant in transport barriers.

Turbulent transport is dominant

Turbulent flux
Outline

2) A powerful approach: dimensionless analysis.
3) Status of our understanding of turbulent transport: heat, particle, momentum.
4) Building a transport model: mixing-length estimate, quasi-linear theory.
6) Improved confinement, physics of transport barriers.
Part I - Basics

• A few reminders.

• Basics of turbulent transport: random walk, main instabilities.

• Some key ingredients of theory and modelling.
Geometry

• Field lines generate magnetic surfaces.
• Safety factor: \( q(r) = \frac{d\varphi}{d\theta} \)
• Density and temperature are constant on magnetic surfaces.

Magnetic surfaces

Safety factor

Field lines generate magnetic surfaces.

\[ q(r) = \frac{d\varphi}{d\theta} \]

Density and temperature are constant on magnetic surfaces.
Fluctuations of ExB drift velocity produce turbulent transport

ExB drift velocity

$$v_E = \frac{B \times \nabla \phi}{B^2}$$

$$D_{\text{turb.}} \approx \left| v_E \right|^2 \tau_c$$

$$q = \frac{d\phi}{d\theta}$$
Random walk process

- ExB drift

\[ \mathbf{v}_E = \frac{\mathbf{B} \times \nabla \phi}{B^2} \]

- Turbulent diffusion

\[ D_{\text{turb}} \propto \left| \mathbf{v}_E \right|^2 \tau_c \]
\[ \propto \frac{L_c^2}{\tau_c} \]

- Turbulent flux

\[ \phi_E = \frac{3}{2} \langle p v_E \rangle \]

Contour lines of electric potential $\phi$. 

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Main instabilities are interchange modes

- Exchange of two flux tubes is energetically favourable if
  \[(v_E \cdot \nabla B)(v_E \cdot \nabla p) > 0\]
- Stable and unstable regions are connected by field lines.
Several branches are potentially unstable

- Ion Temperature Gradient modes: driven by passing ions, interchange + “slab”
- Trapped Electron Modes: driven by trapped electrons, interchange type.
- Electron Temperature Gradient modes: driven by passing electrons
- Ballooning modes at high $\beta$
Electron and/or ion modes are unstable above a threshold

- Instabilities → turbulent transport
- Appear above a threshold $\kappa_c$.
- Underlie particle, electron and ion heat transport: interplay between all channels.

![Stability diagram - Weiland model](image)
A Self-Consistent Problem

Plasma response: kinetic or fluid equations

Maxwell Equations
\[ \rho = 0 \]
\[ \text{rot} (\mathbf{B}) = \mu_0 \mathbf{j} \]

e.m. field \( E, B \)

charge and current densities \( \rho, j \)
Calculating the plasma response: fluid equations

Continuity equation
\[ d_t n = -n \nabla \cdot \mathbf{V} \]

Force balance equation
\[ n m d_t \mathbf{V} = -\nabla p - \nabla \cdot \mathbf{\pi} + n e (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \]

Heat equation
\[ d_t p = -5/3 \ p \nabla \cdot \mathbf{V} - 2/3 \nabla \cdot \mathbf{q} - 2/3 \mathbf{\pi} : \nabla \nabla \]

Lagrangian derivative \[ d_t = \partial_t + \mathbf{V} \cdot \nabla \]

No wave particle resonant interaction, nor orbit effects: partly cured with closure schemes and gyroaverage operators.
Gyrokinetic theory

- Kinetic equation
  \[
  \partial_t F + \dot{x} \partial_x F + \dot{p} \partial_p F = 0
  \]
  \[
  = -[H, F]
  \]
- In principle a 6D calculation!
- However \( \omega_{turb} << \Omega_c \)
  \[
  \rightarrow \quad \mu = \frac{m_i v_{\perp}^2}{2B(x_G)} \quad \text{is an invariant}
  \]
Gyrokinetic theory
Brizard and Hahm 08

- Compute the distribution of gyrocenters $\overline{F}$

$$\partial_t \overline{F} - [\overline{H}, \overline{F}] = 0$$

$$F = B^{-1} \partial_\mu F_{eq} (H - \overline{H}) + \overline{F}$$

- $\overline{H}$ is the hamiltonian averaged over the fast motion (gyroaverage).
Coherence in gyrokinetics

- Maxwell equations: local charge and current densities
- Must be related to gyrocenter charge and current densities: an other gyroaverage!
- Difference between $F$ and $\bar{F}$ is the polarization term.

$$n(x, t) = \int \frac{d\varphi_c}{2\pi} \int d^3p \bar{F}(x - x_c, p, t)$$
After a lot of work to develop a gyrokinetic code (see lecture by C.S. Chang) …
Numerical simulations reproduce the main expected features of turbulence

- Structures aligned with the magnetic field.
- Fluctuations are ballooned on the low field side.
Part II - Dimensionless scaling laws

- Similarity principle.
- Numerical and experimental tests.
- Extrapolation using dimensionless scaling laws.
Dimensionless numbers

Kadomtsev ‘75

- Counting the dimensionless parameters for a given set of plasma parameters
- 8 numbers for a pure e-i plasma
  
  I. \( n^* = \frac{qR}{\lambda_{mfp}} \)  
  \( \rho^* = \frac{\rho_c}{a} \)  
  \( \beta = 2\mu_0p / B^2 \)
  
  II. \( A = \frac{R}{a} \)  
  \( \tau = \frac{T_e}{T_i} \)  
  \( q \)
  
  III. \( \mu = \frac{m_e}{m_i} \)  
  \( N = n_e\lambda_d^3 \)

- Implications on confinement time, II and III given

\[ \omega_c \tau_E = F(\rho^*, \beta, n^*) \]
Scale invariance

Connor&Taylor ‘77

• Analysis of scale invariance of Fokker-Planck equation coupled to Maxwell equations → local relations.

• If geometry, profiles, and boundary conditions are fixed, plasma is neutral, then

\[ \chi = \frac{T}{eB} G(\rho^*, \beta, v^*) \]

Bohm diffusion coefficient
Dimensionless scaling is a powerful tool to predict transport in a next step device

Similarity principle

\[ \omega_c \tau_E = F(\rho_*, \beta, \nu_*) \]

Normalised gyroradius:

\[ \rho_* = \frac{\rho_c}{a} \]

beta:

\[ \beta = \frac{\rho}{B^2 / 2\mu_0} \]

collisionality:

\[ \nu_* = \frac{\nu_{\text{coll}}}{c_s / R} \]

Measured \( \tau_E \) vs fit, ITPA

Scaling law \( \tau_E (s) \)
ρ* and ν* will be smaller in ITER.

\[
\frac{\rho^*}{\rho^\text{ITER}} \quad \frac{\nu^*}{\nu^\text{ITER}}
\]
What is gyroBohm scaling law?

• At fixed $\beta$ and $\nu^*$,

$$\frac{L_c}{a} \equiv \rho^*_\alpha^{\alpha+1} \frac{\alpha+1}{2} \quad \gamma \equiv \frac{c_s}{a} \rightarrow \chi \equiv \frac{T}{eB} \rho^{\alpha}_*$$

• Two main cases: $\alpha=1$ (gyroBohm) and $\alpha=0$ (Bohm).

• Theory predicts that when $\rho^*_\rightarrow 0$, the scaling is gyroBohm

$$\chi \equiv \frac{T}{eB} \rho^*_\rightarrow 0$$
An example of gyroBohm scaling

- Simulations where the scale $\rho^*$ is changed by a factor 4
- Agree with $L_c \equiv \rho_c$ and $\chi \equiv (T/eB) \rho_c/a \rightarrow \omega_c \tau_E \equiv \rho_*^{-3} F(\beta, \nu_*)$

Sarazin 07
Scaling law with $\rho_*$ is close to the theoretical expectation

- ITER scaling law
  \[ \omega_c \tau_E \sim \rho_*^{-3.0} \beta^{-2.9} \nu_*^{0.0} \]
- Experiments on DIII-D and JET
  \[ \omega_c \tau_E \sim \rho_*^{-3.0} \beta^{0.0} \nu_*^{-0.35} \]
- Consistent with gyroBohm scaling law for electrostatic turbulence:
  \[ \omega_c \tau_E \sim \rho_*^{-3.0} \beta^{0.0} \nu_*^? \]

Normalised $\tau_E$ vs. gyroradius - JET
Scaling is gyroBohm when $\rho^* \rightarrow 0$

- Gyrokinetic and fluid simulations find that the scaling is gyroBohm when $\rho^* \rightarrow 0$.
- The critical value of $\rho^*$ for Bohm to gyroBohm scaling is still subject to debate.
- Cause for Bohm scaling is controversial.

Lin 02
GyroBohm scaling law is favorable for ITER

- At constant $\beta$ and $\nu^*$ the normalized loss power $P_a^{3/4}$ is a function of only, i.e.
  \[ P_a^{3/4} \equiv [\rho_*]^{\alpha - 5/2} \]

- GyroBohm scaling corresponds to the lowest losses.
Part III
Status of the understanding for each transport channel

- Ion heat transport
- Electron heat transport
- Particle transport
- Momentum transport
Ion heat transport is rather well understood

- ITG dominated: quite well assessed.
- Has become a test for gyrokinetic codes
- Still some issues: turbulence spreading, Dimits shift, etc…

[Graph showing data points and trend lines]
Electron heat transport

- Large contribution from TEMs
- Contribution from ETGs still a debated issue:
  - small for ITG dominated turbulence Candy 06
  - might be significant for TEM/ETG dominant modes Jenko 08
Particle transport

- Particle flux
  \[ \Gamma_e = -D \frac{dn_e}{dr} + Vn_e \]

  - Diffusion is turbulent
    \[ D = D_{turb} \]

  - Pinch velocity = collisions + turbulence
    \[ V = V_{neo} + V_{turb} \]

- In a reactor:
  - Ionisation source localised in the edge \( \rightarrow \Gamma_e = 0 \)
  - \( V_{neo} \sim V_{Ware} = 0 \). Turbulent pinch \( V_{turb} \rightarrow \) density peaking?
Density profile depends on safety factor and temperature

Two additive contributions:

- Curvature pinch, depends on magnetic shear
  \[ \frac{V}{D} \propto s \quad s = \frac{r \ dq}{q \ dr} \]

- Thermo-diffusion
  \[ \frac{V}{D} \propto \frac{\nabla T_e}{T_e} \]
  changes sign when moving from electron to ion turbulence.
Momentum transport and spontaneous spin-up

- A puzzling observation on Alcator C-mod, JET, TS, DIII-D: toroidal rotation without external torque
- Structure of momentum radial flux Diamond 07

\[ \Gamma_\Omega = -D \frac{d\Omega_\phi}{dr} + V\Omega_\phi + S \]

- Still an open issue Hahm 06, Gurcan 06, Peeters 07, Waltz 07
Part IV
Building a Transport Model

• Integrated modelling: important for ITER - preparation of scenarios, safe operation, coherence of data, designing control algorithms

• Reduced models for turbulent transport using the Mixing Length Estimate.

• Combining similarity and mixing-length estimate.

• Critical gradient models.
Mixing-length estimate:
level of fluctuations

- Mixing of the pressure profile by vortex of size $\ell$
  \[
  \frac{\delta p}{p} \approx \frac{\ell}{L_p}
  \]
- With a bit of cooking ...
  \[
  \frac{e \delta \phi}{T} \approx \frac{\delta p}{p} \approx \frac{\gamma \ell}{\omega_r L_p}
  \]
Mixing-length estimate : diffusion

- Quasi-linear diffusion Vedenov 61, Drummond 63, Horton 83

  \[ D = \sum \left| v_{E\ell} \right|^2 \tau_{c\ell} \]

- Combining with mixing-length estimate

  \[ D \approx \gamma_{\text{max}} L_c^2 \]

- Basis of most transport models: GLF23, Weiland, CDBM...

- Firmer basis from more refined statistical theories
  Diamond 91, Krommes 97, Itoh 99.
• Rules for correlation length and time:

\[ L_c \equiv \rho_s \quad \gamma \equiv \frac{c_s}{R} \left( \frac{RdT}{Tdr} - \kappa_c \right) \]

• Mixing length estimate:

\[ \chi = \chi_s \rho_e B_e \left( \frac{RdT}{Tdr} - \kappa_c \right) \]

stiffness \quad threshold

• Typical behavior of more complex models: Weiland, GLF23, CDBM, …
A useful, but controversial, concept: marginal stability

- Marginally stable profile

\[ T = T_a e^{\frac{\kappa_c (a-r)}{R}} \]

- Stiffness: tendency of profiles to stay close to marginal stability.
- Central temperature is improved if
  - threshold \( \kappa_c \) is larger
  - edge pedestal \( T_a \) is higher.
Development of reduced models: present status

- Encouraging results
  see lecture by Pr Fukuyama.

- However, still some uncertainty on the prediction of ITER performances.

- Requires an improvement on transport models.
Part V - Beyond the Mixing Length Estimate

- Tendency for producing large scale structures: inverse cascade.
- Large scale transport events: avalanches and streamers: breaks locality and scaling of the correlation length, some link with turbulence spreading.
- Fluctuations of the poloidal flow: Zonal Flows, Geodesic Acoustic Modes. Reduce anomalous transport. Introduce non locality in k space.
- Sources of intermittency.
Large Scale Transport Events

• Events that take place over distances larger than a correlation length

• Identified as
  - avalanches
  - streamers

• May lead to enhanced transport and/or non local effects.
Avalanches

- Profile relaxations at all scales. Diamond & Hahm 95.
- Domino effect.
- Propagate at a fraction of the sound speed.
- Clear link with turbulence spreading.
  Garbet 94, Hahm 04
Streamers

• Convective cells elongated in the radial direction, aligned along the magnetic field. Beyer 00, Champeaux 00.

• Boost the radial transport if the ExB velocity is large enough → controversial. Jenko 00, Labit 03, Idomura 06, Lin 05, Candy 08.
Do avalanches and streamers really exist?

- No direct observation.
- Some hint from fast evolution of temperature profiles.
Zonal flows
Diamond, Itoh, Itoh & Hahm 05

- Fluctuations of the poloidal velocity
- Generated by turbulence via Reynolds stress
- Damping is weak

\[ \partial_t V_\theta' = -\nabla_r \cdot \langle v_{Er} v_{E\theta} \rangle - v V_\theta' \]

Turbulent amplification \( \sim |\phi|^2 V_\theta' \)
Zonal Flows (cont.)

- Strong feed-back on turbulence: shearing of vortices.
- Clearly seen in all turbulence simulations.
- Leads to a self-organized state
Geodesic Acoustic Modes

• \( n=0, \ m=0 \) mode coupled to sidebands \( m=\pm 1, \ m\pm 2, \ldots \)

Hallatschek 01, cluster PPCF 06

• GAM frequency

\[
\omega^2 = \left( 1 + \frac{1}{2q^2} \right) \frac{2\Gamma T_i + T_e}{m_i} \frac{1}{R^2}
\]

• Turbulence self-regulation, however shear effect less efficient than zonal flows.

\[
V_0 = -\frac{\partial_r \phi_0}{B}
\]
Impact on Transport Models

• Mixing-length estimate can be modified to account for Zonal Flows (GLF23, Weiland, …): some cooking!

• Statistical theory accounting for all these beasties still to be fully developed ...

• Why not direct simulations of turbulence, as for weather forecast?
Part VI
Improved confinement

• Shear flow

• Negative magnetic shear

• Transport barriers

• Consequences
Several “regimes” in a tokamak plasma

- **L-mode**: basic plasma, turbulence everywhere.
- **H-mode**: low turbulent transport in the edge, formation of a pedestal.
- **Internal Transport Barrier**: low turbulent transport in the core, steep profiles.
Several mechanisms may lead to improved confinement

- Flow shear: same effect as Zonal Flows
- Magnetic shear
- $T_e/T_i$, $Z_{\text{eff}}$, density gradient, fast particles…: not generic
Shear flow is stabilizing

- $E \times B$ velocity shear tears apart large scale vortices

- Approximate criterion for stabilization

$$\gamma_E = \frac{dV_E}{dr} > \gamma_{lin}$$

Contour lines of electric potential.
Flow shear stabilisation

• Shear rate

\[ \dot{V}_E = \frac{dV_E}{dr} \]

• Criteria for stabilization

\[ \left( Dk_\theta V'_E \right)^{1/3} > \frac{\tau_c}{\gamma_{lin}} \]

Biglari-Diamond-Terry 90

Waltz 94

Figarella 03
Controlling the flow

• Force balance equation

\[ E_r = \frac{T_i d n_i}{e_i n_i dr} + (1 - k_{\text{neo}}) \frac{d T_i}{e_i dr} + V_{T_i} B_p \]

Fuelling \quad \text{Heating} \quad \text{Toroidal momentum} \quad \rightarrow \text{power threshold!}

• Flow generation

\[ \partial_t V_{\theta} = -\nabla_r \left\langle \vec{V}_{E_r} \vec{V}_{E\theta} \right\rangle - \nu_{\text{neo}} \left( V_{\theta} - V_{\text{eq}} \right) \]
Transport reduction due to shear flow

- Flux reduction factor factor $F(\gamma_E)$

$$F = \frac{1}{1 + \left(\frac{V'_E}{\gamma_{\text{lin}}}\right)^2}$$

or

$$F = 1 - \frac{V'_E}{\gamma_{\text{lin}}}$$

Figarella 03

![Graph showing transport reduction due to shear flow with variables $p_{\text{rms}}$, $V_{E,\text{rms}}$, and $V'_E$.]
A simple model for a bifurcation towards a transport barrier

- Particle flux with ExB shear
  \[
  \Gamma = -D \left( 1 + C \left( \frac{dn}{dr} \right)^4 \right) \frac{dn}{dr}
  \]

- Transition to improved confinement occurs above a critical threshold in flux.

Hinton 92, Itoh 02, Diamond 07
Negative magnetic shear is stabilising

- Magnetic shear:
  \[ s = \frac{r}{q} \frac{dq}{dr} \]
- \( s < 0 \): favourable average of interchange drive \((v_E \cdot \nabla B)(v_E \cdot \nabla p)\) along field lines.
- \( s > 0 \): unstable
- Enhanced by geometry effect.

B.B. Kadomtsev, J. Connor, M. Beer, J. Drake, R. Waltz, A. Dimits, C. Bourdelle…

Vortex distortion
Negative magnetic shear is a robust effect

- Turbulence simulations: stabilization for $s<-0.5$
- Some agreement with electron transport barriers in JET

![Graphs showing safety factor and temperature profiles](image-url)
Internal Transport Barriers

- Transport barriers are layers of plasma where turbulent transport is quenched.
- Requires a minimum amount of power → triggering?

Contour lines of electric potential.

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Dynamics of transport barriers is more complex than s<0 and shear flow

Map of \(-\rho_c \nabla T/T\) : profile steepening

R(m)

\(q=2\)

\(q_{min}\)

s>0

s<0

‘narrow’ ITB

s<0 region

5MW ICRH + 11.5MW NBI

Time(s)

JET - E. Joffrin
Why a special role of \( s=0 \) and rational surfaces?

- At negative shear, slab ITG still unstable \( \rightarrow \) some kind of optimum for \( s=rdq/qdr=0 \).
- Special role of low order rational surfaces:
  - density of resonant surfaces Romanelli 93, Kishimoto 99, Garbet 01
  - MHD mode Joffrin 04
  - Zonal flows Waltz 06
  - convective cell Diamond 06
Consequences for ITER: advanced scenarios

Advanced scenarios where the plasma current is non inductively generated are foreseen in a second phase.

- The objective is to reach a steady-state regime: needs a large fraction of bootstrap current.
- Requires an ITB or some global improvement of the confinement.
Conclusions I

• Huge progress in the understanding of turbulent transport, thanks to theory, turbulence simulations and increasingly refined measurements.

• Some hotly debated issues though: dimensionless scaling laws, electron heat transport, particle and momentum transport.

• Present computational resources do not allow a full scale turbulence simulation for ITER.
Conclusions II

• Reduced transport models are efficient ways of testing theories, analysing experiments, and predicting performances in ITER. Still the accuracy of reduced transport models is not better than 20%.

• Due to the complex dynamics of turbulence: structure formation, intermittency, etc,...

• Improved models on the basis of a better statistical theory (to be done) or direct use of simulations of turbulence?
Conclusions III

• Generic mechanisms to control turbulence → improved confinement. Crucial for ITER.

• Turbulence simulations are good tools to test the validity of various theoretical ideas.

• Still many issues remain unresolved. At the moment, no full ab-initio simulations of L-H transition.

• Long pulse plasmas in ITER with improved confinement will be a challenge.