Tokamak disruptions represent a serious drawback for fusion magnetic confinement systems and for the development of a fusion reactor concept.

Nuclear fusion power plants require steady state operation of quiescent plasmas and no disruptions at all are allowed. In present tokamaks, however, disruptions are almost unavoidable especially for high performances plasmas conditions.

In these lectures I will present an overview of the known, open and critical issues, both from an experimental and a theoretical perspective.

I will mainly concentrate on the magneto-hydro-dynamical (MHD) aspects only briefly mentioning the important issues related to disruptions mitigation using gas injection systems and runaway electrons.

9th ITER International School 20-24 March 2017 Aix en Provence (France)
OUTLINE

• Introduction to disruption phenomenology
• Causes and effects of disruptions
• Equilibrium and vertical stability
• Symmetric and non symmetric halo currents
• Boundary conditions
• Hiro and surface currents
• Halo/hiro/eddy currents and flux conservation
• Current asymmetry rotation
• Virial Theorem and angular momentum
• Open Issues for ITER
• M3D simulations various results
• The mistery of the TQ
• FR scenarios and disruptivity
• Plasma rotation mistery
• Disruptions control and RMP
• Radiation and disruption mitigation
• Runaways electrons
• Conclusions
I would like to live in Theory Country.. why?
Because in Theory everything works!
**Total Energy at any one time matters! (Damage)**

- Tokamaks have explored up to ~10 Megajoules plasma kinetic energy
- Long pulse tokamaks have not dealt with instantaneous energy above a Megajoule level, although removal of ~1 Gigajoule of energy over long timescales has been demonstrated.

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<td>600 MJ</td>
<td>steady</td>
<td>10-20 MA</td>
<td>helium</td>
<td>100 MW</td>
<td>500-1500 m^3</td>
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**TABLE 1**

G.A. Wurden 2011 PPPL workshop
How much energy are we talking about?

- 60 MJ of runaways,
- 400 MJ of thermal quench,
- 600 MJ of poloidal magnetic field energy

600 MJ will melt ~ one ton of copper

15 MJ is released by 7 sticks of TNT

10 GJoule ≈ A380 flying at 700 km/h

100 MJ: F-14 Tomcat launched by steam catapult

Melting point of copper: 1356 K
Specific heat capacity of copper: 385 Jkg⁻¹K⁻¹
Specific latent heat of fusion (energy required to convert a solid at its melting point into a liquid at the same temperature): 205000 Jkg⁻¹
So to melt 1 kg of copper we need \(1000 \times 385 + 205000\) J = 611,560 J.
\[ \frac{W}{\tau_E} = V_\phi I + P_{\text{add}} + P_{fus} - P_\Omega - P_{\text{rad}} \]

- The global (volumetric) energy balance is at the basis of the plasma confinement.

- A sudden non compensated deficit in this balance can lead to disruptions.

Key elements are (neglecting convection losses):

- \( \tau_E \) : the plasma energy confinement time
- \( V_\phi I \) : the transformer ohmic input power (→ 0)
- \( P_{fus} \) : fusion power (\( V \, n_a \, n_T < \sigma > E_{fus} \))
- \( P_{\text{add}} \) : the additional heating power
- \( P_\Omega \) : the plasma ohmic dissipation (\( \approx V \, J^2 \, Z \, T^{-3/2} \))
- \( P_{\text{rad}} \) : \( V \Sigma n \, n_z L_z \) (with brehms \( \approx n^2 T^{1/2} \))

Required \((n \, \tau)\) vs imp. content for break-even @ 10 KeV

Spectral lines+ brehms.
ITER Machine and Divertor System

Divertor system main functions:

- Minimize the helium and impurities content in the plasma
- Exhaust part of the plasma thermal power
Divertor and SOL Layer convective losses

To maintain a clean plasma and to limit the plasma wall interactions diverted open magnetic field are created in the SOL layer where radiation and convective losses are the main sinks of energy.

- main disadvantage is the limited divertor plates surface
The unfavourable Surface to Volume ratio of the torus

One issue which is not well addressed, in my opinion, is that the S/V ratio scales unfavourably with R.

- The neutrons per unit area increase with R (since the number of neutrons is proportional to the plasma volume).
- The divertor area is in any case a fraction of the total surface also the thermal load per unit area increases with R both at the divertor plates and also in general on the entire wall.

From TABLE 1 data.
The unfavourable scaling of disruption forces

Assuming that the forces on the structures (later shown to be reasonable) scale like:

\[ F = \alpha \ I_p B_\phi \]

with \( \alpha = 0.2 \)

there is almost a factor of 10 between actual experiment and ITER.

This simple (but realistic) assumption also show that the scaling to larger current or magnetic field devices is quite unfavourable in case of disruptions.

From TABLE 1 data
What is a tokamak disruption?

It is a **SUDDEN RELEASE** of this stored internal energy that produces **4 main consequences**:

- Large Transient Electromagnetic Loads on vessel components
- Large Transient surface tile heating due to plasma radiation
- Large Transient surface tile heating due to plasma convection
- Large Transient volumetric tile heating in localized places due to runaway electron beam impact.

G.A. Wurden 2011 PPPL workshop
Which is the «typical» disruption phenomenology?

- Thermal quench and current quench
- Consequences heat + EM loads, VDE, halos, runaways

JET

- Pre-disruption energy loss, precursors

T. Hender 2010 CCFE workshop
Which are the important characteristic parameters?

- **The Thermal quench (TQ) and current quench (CQ) times**: \( t_q \) and \( t_c \)

These times determine the power losses. Generally \( t_q \ll t_c \) with \( t_q \) of the order of **few ms** and \( t_c \) of the order of **few tens ms**

In turn the shorter these times the stronger the effects of the heat deposition (**melting of plasma facing component**) and of the electromagnetic consequences (**induced eddy currents and stresses on metallic structures**), and available energy to accelerate electrons (**Runaway Electrons**).

(*too long CQ times can also be an issue -> RE & momentum impulse*)

Moreover the **TQ** can produce the loss of the vertical stability and induces the so called **Vertical Displacement Events (VDEs)** and the generation of large **plasma edge halo currents**

- **Avoidance/Mitigation actions are therefore required**
How to explain the Voltage spike and current behavior?

From Wesson et al NF (1990)

\[ V = -L_v \frac{d}{dt} (I_p + I_v) \]

with \( I_v = \frac{V}{R} \)

\[ V = -R \exp\left(-\frac{R}{L_v}t\right) \int_0^t \exp\left(\frac{R}{L_v}t'\right) \frac{dI_p}{dt'} \, dt' \]

therefore a negative Voltage implies a positive current derivative. On the other hand an internal instability flatten the current profile and decreases the internal inductance (next slide).

If we assume that only a fraction \( f \) of the internal energy goes in increasing the current (and not dissipated), we have:

\[ \frac{1}{2} L_p \frac{dI_p^2}{dt} = -f \frac{1}{2} I_p^2 \frac{dL_p}{dt} \]

Therefore a decrease of the internal inductance can explain the plasma current increase.

To explain the delay between the TQ and the current increase a negative current spike diffusion process is also invoked (similar to the surface current model to be discussed later)
Inductance vs current peaking

From Stacey: Fusion Plasma Physics (Wiley 2012)

\[ p = p_0 \left(1 - \frac{r^2}{a^2}\right) \]
\[ j = j_0 \left(1 - \frac{r^2}{a^2}\right)^\nu \]

\[ l_i = \frac{\tilde{B}_\theta}{B_{\theta a}^2} = \frac{2 \int_0^a B^2_\theta r \, dr}{a^2 B_{\theta a}^2} \]
\[ l_i = \ln(1.65 + 0.89\nu) \]

\[ q_a/q_0 = \nu + 1 \]

\[ j \]
\[ \nu = 2 \]
\[ \nu = 8 \]

higher shear →

Current peaking →
What causes disruptions?

from P. de Vries et.al. NF 51 (2011) 53018
What causes disruptions? an old issue!


Figure 12. A scheme of possible initiating events and precursor scenarios leading to a prequench state with deficient edge.
Disruptions effects

June 2008 Alcator C-Mod, in-vessel inspection
localized melt damage most likely due to runaways

Melt damage at upper edges

“Far away” diagnostic harness burned/melted by runaways

Operated by Los Alamos National Security, LLC for NNSA
Disruption effects in JET ILW


From Lehnen et al JNM (2015)

**Figure 1.** Bulk beryllium melting on the ridge of the JET inner wall limiter (4X).

**Figure 2.** (a) Image of the melted edge of the special tungsten lamella. The lamellas are 5.5 mm wide and 60 mm long. (b) Detail of layering of the migrated material and a small ~150 μm diameter droplet adhered to the side the lamella. (c) Higher resolution image showing layering and cracking of the main droplet.

**Figure 4.** (a) Plasma current versus time for JET pulse #86801 in which a runaway electron (RE) plateau characterized by hard x-ray emission is produced when argon is injected by DMV1(4.7 bar). More argon is injected by DMV2(12.7 bar) in an unsuccessful attempt to mitigate the REs. (b) In-vessel image of melt damage due runaway electrons from pulse #86801 in which REs hit the tops of the inner wall limiters about 60 ms after they are created. The castellations are 12 mm².
Disruption effects
To maintain a tokamak plasma in equilibrium the following equation should be satisfied:

\[ \mathbf{J} \times \mathbf{B} = \nabla p \]

where \( \mathbf{J} \) and \( \mathbf{B} \) are the current and magnetic fields in the plasma region and \( p \) is the plasma Pressure.

However a plasma equilibrium is not possible without external currents!

**Theorem:**
A magnetofluid cannot stay in MHD equilibrium by forces generated only by its own internal currents
A tokamak equilibrium needs external currents!

The reason is very fundamental: it is related to the so called **VIRIAL THEOREM**

Starting from the equation of motion written in conservative form:

\[
\frac{\partial \rho \mathbf{V}}{\partial t} = -\nabla \cdot \mathbf{T}
\]

where \( \mathbf{V} \) is the magnetofluid velocity and \( \mathbf{T} \) the stress tensor. It can be shown that:

\[
\frac{dI}{dt} = \int_V \rho \mathbf{V} \cdot \mathbf{r} \, dV
\]

is the moment of inertia and:

\[
\frac{d^2 I}{dt^2} = \int_V \left( \rho \mathbf{V}^2 + 3p \frac{B^2}{8\pi} - \frac{(\nabla \phi)^2}{8\pi G} \right) \, dV
\]

where

- Kinetic energy: \( \mathcal{E}_V \geq 0 \)
- Internal energy: \( \mathcal{E}_p \geq 0 \)
- Magnetic energy: \( \mathcal{E}_B \geq 0 \)
- Gravitational energy: \( \mathcal{E}_g \leq 0 \) (only possible negative term!)

In an equilibrium, this expression must equal zero:

\[
0 = 2\mathcal{E}_V + 3(\gamma - 1)\mathcal{E}_p + \mathcal{E}_B + \mathcal{E}_g > 0
\]
Vertical stability of an elongated (why?) tokamak (1)

- Distance to go around poloidally is larger

\[ q = \frac{2\pi r^2 B_t}{\mu_0 RI} = \frac{2AB_t}{\mu_0 RI} \]

\[ A = \pi ab = \pi a^2 \kappa \quad \kappa = \frac{b}{a} \]

For the same plasma current:

\[ q_{\text{ellip}} = q_{\text{circ}} \kappa \]

- If \( q = 3-4 \) is the stability limit of operation one can run a larger current in an elliptically shaped plasma
- ..also easily to be DIVERTED

from www2.warwick.ac.uk
Considering a wire model (as in figure) and an elongated plasma kept in equilibrium by $I_{x,y}$.

Imposing at the plasma boundary: $A_z(0, \kappa \alpha) = A_z(a,0)$

Taking into account that: $A_{z,j} = \frac{\mu_0 I_j}{2\pi} \ln(r_j)$ with $r_j$ the radial distance from each wire to the surface and summing up:

$$I_y \ln \left(\frac{c^2 - \kappa^2 a^2}{c^2 + a^2}\right) + I_x \ln \left(\frac{c^2 + \kappa^2 a^2}{c^2 - a^2}\right) - I \ln \kappa = 0.$$  

and for $c >> a$:

$$I_x - I_y = \frac{c^2}{a^2} \frac{\ln \kappa}{1 + \kappa^2} I.$$

Calculating the forces between wires as:

$$\mathbf{F}_{ij} = -\left(\mu_0 L I_i I_j / 2\pi r_{ij}\right)e_{ij}$$

where $L$ is the length and $e_{ij}$ is the radial versor pointing from wire $i$ to $j$.

Therefore the force on the plasma wire is:

$$F_y = \frac{\mu_0 LI}{2\pi} \left( -\frac{I_y}{c - \xi} + \frac{I_y}{c + \xi} + 2 \frac{\xi I_x}{(c^2 + \xi^2)^{1/2}} \right)$$
Vertical stability of an elongated tokamak (3)

By linearizing:

$$\delta F_y = \frac{\mu_0 LI^2}{\pi} \left( \frac{I_x - I_y}{I} \right) \frac{\xi}{c^2}$$

The condition for stability is (restoring force): $\delta F_y < 0$

and finally:

$$\frac{\ln \kappa}{1 + \kappa^2} < 0$$

(remembering the relation for $c >> a$ for $l_{x,y}$)

Therefore $\kappa > 1$ is always **UNSTABLE VERTICALLY**. (if $\kappa < 1$ UNSTABLE HORIZONTALLY)

Hence it is clear that an **active control** is needed to maintain the plasma **STABLE**.

**Any failure in the control system or any sudden change in plasma shape or internal conditions**

can result in a loss of the control and therefore can produce a:

**Vertical Displacement Event (VDE)** and a plasma disruption.
Vertical stability of an elongated tokamak with a perfectly conducting wall (4)

(Assuming that the field of the wires has penetrated the wall before it becomes Ideal!)

Assuming: \( h = \frac{\kappa^2 b^2}{\xi} \) (in this way the wall is a flux surface)

In presence of a wall eddy current the force becomes:

\[
\delta F_y = \frac{\mu_0 L I^2}{2\pi} \left[ 2 \left( \frac{I_x - I_y}{I} \right) \frac{\xi}{c^2} + \left( \frac{I'}{I} \right) \frac{1}{h} \right]
\]

For stability:

\[
2 \left( \frac{I_x - I_y}{I} \right) \frac{\xi}{c^2} + \left( \frac{I'}{I} \right) \frac{1}{h} < 0
\]

After substitution of \( I_{x,y} \) and \( I' = -I \), it follows:

\[
\frac{2 \kappa^2}{1 + \kappa^2} \ln \kappa \leq \frac{1}{w^2}
\]

with \( w = b/a \)

For more realistic models with \( R_0/a \approx 3 \), an elongation of \( \kappa \approx 1.8 \) requires \( w < 1.2 - 1.4 \).
Vertical stability can be discussed introducing a radial field (see fig.)

Introducing the stability index: \( n = - \frac{R \partial B_z}{B_z \partial R} \)

Since in vacuum \( \frac{\partial B_z}{\partial R} = - \frac{\partial B_R}{\partial Z} \) and stability requires \( \frac{\partial B_R}{\partial Z} > 0 \) (see fig.)

and \( B_v \) is in negative z direction (for positive plasma current) i.e. \( n > 0 \) for vertical stability.

To elongate the plasma \( n < 0 \) (equal currents up and down as seen above)

with \( B_z = \alpha_S \frac{\mu_0 I_p}{4\pi R_0} \) (\( \alpha_S \) depends on the plasma conditions: \( \beta_p, l_i \))

Since:

\[
F_{destab} = 2\pi R_0 I_p \frac{\partial B_R}{\partial Z} |_{(R_0, z_0)} (z - z_0) = -2\pi I_p B_z n(z - z_0)
\]

Assuming further \((z - z_0) \propto \exp(\gamma t)\) it follows:

\[
m_p \frac{d^2 z}{dt^2} = F_{destab} = -n \alpha_S \frac{\mu_0 I_p^2}{2R_0} (z - z_0) \rightarrow \gamma^2 = - \frac{v_{A,pol}^2}{R_0^2} \alpha_S n
\]

with \( V_{A,pol} = \left( \frac{\mu_0 I_p}{2\pi a} \right)^{-1} \frac{1}{\sqrt{\mu_0 e}} \)
Vertical stability of an elongated tokamak with a real conducting wall (6)

From the last equation it is clear that for unstable cases \((n<0)\) the growth rate is of the order of the Alfvèn velocity \(\rightarrow \textbf{too fast} \rightarrow\) non accessible for feedback systems!

**Some sort of passive stabilizing wall is therefore needed!**

Flux balance for the conductor reacting to the plasma current, \(I_p\):

\[
\psi_c = M_{cp} I_p + L_c I_c \quad \text{(1)}
\]

Due to a change of the plasma vertical position:

\[
\frac{d\psi_c}{dt} = I_p \frac{\partial M_{cp}}{\partial z} \frac{dz}{dt} + L_c \frac{dI_c}{dt} = -R_c I_c
\]

Therefore the conductor current changes as:

\[
I_c = -I_p \frac{\partial M_{cp}}{\partial z} \frac{z}{L_c} \frac{\gamma \tau_R}{\gamma \tau_R + 1}
\]

with \(\tau_R = \frac{L_c}{R_c}\)

Considering an eq. like (1) for the plasma (with \(M_{cp} = M_{pc}\)) the induced (by \(I_c\)) radial field is:

\[
B_R = -\frac{1}{2\pi R_0} \frac{\partial \psi_p}{\partial z} = -\frac{1}{2\pi R_0} \frac{\partial M_{pc}}{\partial z} I_c = \alpha_c \frac{\gamma \tau_R}{\gamma \tau_R + 1} \frac{z}{R_0} \frac{\mu_0 I_p}{4\pi R_0}
\]

where:

\[
\alpha_c = \frac{2R_0}{\mu_0 L_c} \left( \frac{\partial M_{cp}}{\partial z} \right)^2
\]
Vertical stability of an elongated tokamak with a real conducting wall (7)

The stabilizing force due to this current can be calculated as:
\[ F_{stab} = -2\pi R_o B R I_p \]

and finally the dispersion relation becomes:
\[ \gamma^2 \tau_{A,pol}^2 + \alpha_S n + \alpha_c \frac{\gamma \tau_R}{\gamma \tau_R + 1} = 0 \]

- Natural elongation in toroidal geometry (with a pure vertical field) doesn’t need feedback if:
  \[ \kappa < \kappa_{nat} = 1 + \frac{1}{2(A-1)} \quad \text{with} \ A = \frac{R}{a} \]

- This calculations assume no change in shape i.e plasma rigidity → not completely true!
The standard model of halo currents consists in a layer (pink area) of poloidal currents that circulate in the open field line region at the boundary of the vertically moving plasma.

- 2D codes (like DINA, TSC) contain specific models to describe the halo current layer evolution, in terms of width and temperature of the halo.

- The halo free parameters are adjusted to match the experimental data.

From Nakamura et al. 37th EPS Conf. (2010)
In DINA code the **halo flux** is defined as a fraction \( w \) of the flux inside the plasma:

\[
\Delta \Psi_{halo} = \Psi_b - \Psi_s = w(\Psi_m - \Psi_b)
\]

\( w \) is calculated setting: \( \gamma = 1 \) with:

\[
\gamma(t, w) = \frac{S_0}{S(t, w)} \left[ C + \left( \frac{I_p(t, w)}{I_{p0}} \right) \right] \frac{1}{C + 1}
\]

where ‘o’ means before the thermal quench time, \( S \) is the total area (plasma+halo) and \( C \) is a free constant: **for large \( C \) the total \( S \) is conserved** and \( S_p \) shrinks \( S_h \) grows. For \( C=1 \):

\[
\frac{S}{S_0} = \frac{I_p}{I_{p0}}
\]

It is just an **empirical relation**

---

*from M. Windridge phd Thesis (2009)*
Symmetric VDEs and halo currents (3)


Vertical forces (a) during the VDE and vessel rolling motion (b) in JET due to dampers (MVP)

In JET vertical forces up to 3-4 MN lasting for several (10-50) ms have been measured.

The force scales with the poloidal halo current crossed with the toroidal magnetic field:

\[ F_v \approx I_{halo} B_\phi \]

with \( I_{halo} = \text{f} I_p \) and \( \text{f} \approx 0.1 - 0.2 \)
VDEs symmetric and non symmetric events (1)


- Tilted/shifted \((m=1,n=1)\) wire model and sideway forces
In JET lateral support to withstand to sideways forces have been installed in 1996 after a vessel serious damage in 1994.

After this event it has become clear that sideways forces due to toroidal non axi-symmetric halo currents distribution are extremely dangerous and should be avoided.

The fact that an n=1 mode could explain the observations led to hypothesize that the responsible agent could be an MHD mode grown at relatively high amplitude.
Horizontal force components (tilted/shifted wire)

From Bachmann, ITER report (2007)

- \( F_1 \approx I_{pla} \sin(\phi) B_{tor} \) (due to the vertical comp. of the current)
- \( F_2 \approx I_{pla} \left( B_{pol}(y + \Delta y) - B_{pol}(y - \Delta y) \right) \) (due to the poloidal field variation with \( y \) : shift)
- \( F_3 \approx I_{pla} \left( B_{pol}(z + \Delta z) - B_{pol}(z - \Delta z) \right) \) (due to the poloidal field variation with \( z \) : tilt)

\[
F_1 >> F_{2,3} \quad (30-40 \text{ MN} >> 2-3 \text{ MN}) \text{ in ITER}
\]

\[
F_{\text{horizontal, VV}} = \sqrt{(F_{\text{horizontal,1}} - F_{\text{horizontal,2}})^2 + F_{\text{horizontal,3}}^2}
\]
To characterize using a simple parameter the occurrence of non symmetric disruptions the Toroidal Peaking Factor (TPF) was introduced:

\[ TPF = \frac{\text{Max}(I_h(\phi))}{< I_h(\phi) >} \]

while \( hf = \frac{I_{h,\text{max}}}{I_p} \) is said the halo fraction

In ITER the product \( (TPF*hf) \) should remain below 0.75 (see Fig.)
Experimental characterization of non symmetric events

Not only is important the amount of non axi-symmetry but also how long it lasts: a parameter $A$ is defined to this purpose and it measures the severity of the impulse

$$A = \int A_p^{\text{asym}} \, dt$$

$$A_p^{\text{asym}} = I_p^{\text{asym}} \left| I_p^{\text{dis}} \right|$$

$$I_p^{\text{asym}} = \sqrt{(I_{p7} - I_{p3})^2 + (I_{p5} - I_{p1})^2}$$
A simple model for TPF vs Halo

From Pumphrey et al. NF (1998)

2/1 mode plasma-wall Interaction along the torus

Assuming a 2/1 mode and a force free plasma at the boundary:

\[ X = R + r \cos(\theta) \quad Z = r \sin(\theta) \]

\[ r^2 = \frac{a^2 \rho}{2} ((\kappa^2 + 1) - (\kappa^2 - 1) \cos(2\theta - \phi)) \]

\[ J = \lambda B \]

\[ I_{pol} = \int_{0}^{2\pi} I_{pol}(\phi) d\phi \quad TPF^{2,1} = \frac{i_{h,\text{max}}}{\langle i^h \rangle} \]
Defining the elevation \( \alpha = \frac{\Delta z}{R_o} \) (y goes in toroidal direction) of the current ring and assuming also a shift in x direction, \( \Delta x \). For **small tilt and shift**, the magnetic field at \( R \) can be expressed as:

\[
\begin{align*}
B_R & \approx B_0 R_0 (\Delta x \sin \phi - \alpha z \cos \phi)/R^2 \\
B_\phi & \approx B_0 R_0 (R - \Delta x \cos \phi - \alpha z \sin \phi)/R^2 \\
B_z & \approx B_0 R_0 \alpha \cos \phi/R
\end{align*}
\]

The element force is:

\[
\delta F_x = \int \delta I \delta I_0 B_z(R,\phi) \cos \phi R d\phi \approx \pi \delta I_0 B_0 R_0 \alpha
\]

and the total:

\[
F_x \approx \pi I_0 B_0 R_0 \alpha = \pi I_0 B_0 \Delta z
\]

can be expressed as:

\[
F_x \approx \frac{\pi}{2} \Delta M_z B_0 \quad \text{(Noll formula)}
\]

where \( \Delta M_z \) is the difference between the current moment at \( \phi = -\frac{\pi}{2} \) and \( \phi = \frac{\pi}{2} \).
An equivalent (more clear) way of calculating the dominant force of the tilted-shifted wire, considering the Lorentz force between the vertical (z-component) of the current and the toroidal magnetic field.

Therefore (similarly to the symmetric case):

\[ F_{\text{hor}} \approx f' I_p B_\phi \]

with \( f' \approx 0.1 - 0.3 \)
A good approximation for the vector potential of a non tilted current loop is given by:

\[ A_\phi (y, z) = \left( \frac{\mu_0}{4\pi} \right) \frac{(\pi a^2 I_c y)}{(a^2 + y^2 + z^2)^{3/2}} \left( 1 + \frac{15 a^2 y^2}{8 (a^2+y^2+z^2)^2} \right) \]

With the transformation below, the z axis is rotated by the angle \( \alpha \):

\[ y' = y \cos(\alpha) - z \sin(\alpha) \]
\[ z' = y \sin(\alpha) + z \cos(\alpha) \]

\( A_\phi(y', z') \) contours are plotted in the figure with the magnetic field vector obtained from:

\[ B_y = -\frac{\partial A_\phi}{\partial z} \quad \text{and} \quad B_z = \frac{\partial A_\phi}{\partial y} \]

\(^\wedge\) without using elliptic integrals
In JET it has been observed that the horizontal force is well approximated by the Noll formula.

Also it is observed that there is a linear correlation between the current moment and the current asymmetry (see figure).

This correlation is not completely obvious since it could be expected that the current becomes lower when the plasma touches the wall, instead exactly the opposite is observed.

This interesting and simple observation has led to different attempts of interpretation:

- a surface current model (remaining Wesson’s paper)
- a nonlinear MHD model
- a passive structures model

All these models claim to be able to explain the JET observations.

From Gerasimov et al NF (2014)
Boundary conditions at ideal wall

- Plasma is surrounded by a **perfectly conducting wall** ($n$ is outward-pointing normal vector)

\[
\begin{align*}
- (n \times E)|_{r_{wall}} &= 0 \\
- (n \cdot B)|_{r_{wall}} &= 0 \\
- n \times (E + c^{-1}V \times B)|_{r_{wall}} &= 0 \Rightarrow (n \cdot V)|_{r_{wall}} = 0 \Rightarrow (n \cdot \tilde{\xi})|_{r_{wall}} &= 0
\end{align*}
\]

Not appropriate to study VDEs
Boundary conditions for plasma – vacuum interface:

- Plasma is surrounded by a vacuum region (which is described by $\nabla \times \hat{B} = \nabla \cdot \hat{B} = 0$, with $\hat{B}$ the vacuum magnetic field)
  
  - $(n \cdot \hat{B})|_{r_{wall}} = 0$
  
  - Plasma surface is free to move $\Rightarrow (n \cdot \tilde{\xi})|_{r_{plasma}} = \text{arbitrary}$
  
  - $[[n \cdot B]]|_{r_{plasma}} = 0$ (with $[[\cdots]]$ denoting a jump across the plasma surface)
  
  - $[[n \times B]]|_{r_{plasma}} = (4\pi/c)K$, with $K$ the surface current density
  
  - $[[p + B^2/8\pi]]|_{r_{plasma}} = 0$

BC appropriate for KTM / surface current
Boundary conditions at a thin resistive wall

\[
[n \cdot B]_{rwall} = 0 \quad [n \times B]_{rwall} = \mu_0 \delta \hat{J} = \mu_0 \frac{\delta E}{\eta_{wall}}
\]

Can be rewritten more explicitly as:

\[
\hat{n} \cdot B_v = \hat{n} \cdot B_p
\]

\[
B_v = \nabla \phi_v \quad \nabla^2 \phi_v = 0
\]

\[
\frac{\partial}{\partial t} \hat{n} \cdot B = \frac{\partial}{\partial t} \frac{\eta_w}{\delta} \left[ B_p \cdot \hat{l} - B_v \cdot \hat{l} \right] + \frac{1}{R} \frac{\partial}{\partial \phi} \frac{\eta_w}{\delta} \left[ B_p \cdot \phi - B_v \cdot \phi \right]
\]

\[
B_v = \text{magnetic field on vacuum side of wall}
\]

\[
B_p = \text{magnetic field on plasma side of wall}
\]

\[
\phi_v = \text{magnetic scalar potential in wall}
\]

\[
\eta_w = \text{ resistivity of wall}
\]

\[
\delta = \text{ thickness of wall}
\]

good for VDEs studies without surface currents
(only wall currents are allowed)

Generally also assuming for \( v \):

\[
(n \cdot V)|_{rwall} = 0
\]
Plasma-Vacuum BC are often replaced in nonlinear codes (as in M3D) by a two region model assuming a thin and cold plasma layer at the edge between the hot core and the wall.

From Biskamp «Nonlinear MHD»

Hence effectively in cases in which a surface current plays a fundamental role on the dynamics, a cold plasma layer model could not be considered equivalent to a vacuum-plasma model.
The surface current model (Kink Touching Mode) (1)

Magnetic diagnostics provide the **vertical position of current centroid**, not the position of the geometrical plasma centre!

The m/n=1/1 surface currents hide the actual plasma displacement from magnetic diagnostics.
The surface current model (Kink Touching Mode) (2)


The idea is that the plasma reacts to the $1/1$ kink deformation by a surface current which tend to slow down the kink achieving a quasi-equilibrium state.

Where the first term contributes to the kinked MHD equilibrium, while the second shields the eddy currents from the wall at the position of the surface current layer.

The deduced force is consistent with Noll's formula:

$$F_{x}^{\text{theory}} = \pi B_{\phi} I_{pl} \left( 1 - \frac{\lambda}{q_{a}} \right) \delta z = \left( 1 - \frac{\lambda}{q_{a}} \right) F_{x}^{\text{Noll}}$$
Using the nonlinear MHD code M3D (described later) and defining:

\[
\Delta I = \frac{1}{V} \left( \int d\phi \bar{r}^2 \right)^{1/2},
\]

\[
\Delta M_{IZ} = \frac{1}{V} \left( \int d\phi \bar{M}_{IZ}^2 \right)^{1/2},
\]

\[
V = (2\pi)^{1/2} \int dR dZ.
\]

The results plotted in the figure, that seem consistent with the JET data are obtained (sin and cos components of \(\Delta I\) and \(\Delta M\) are shown):
Eddy currents model in the JET wall

From Roccella et al. NF (2016)

Providing a detailed description of the JET wall a model is developed that can explain the toroidal current asymmetry correlation with the magnetic moment as a result of eddy currents flowing from the wall to the plasma where the plasma touches (and short circuit) some wall elements.

Halo current and toroidal current asymmetry are 90 degrees phase shifted as in JET measurements.

The emphasis of the model is on the necessity of a detailed description of the passive structures surrounding the plasma.
An interesting observation is that **Halo, Hiro or even Eddy** currents are originated by the attempt of the plasma (for halo and hiros) and of the external conductors (for eddies) to **oppose** the **flux variations** associated with the plasma movements/rearrangements due to the MHD phenomena.

**Halo, Hiro and Eddy** are all **stabilizing currents** that tend to slow down and **counteract** (to some extent at least) the **plasma Instabilities**.

**Halo and Hiro rise to preserve the magnetic flux in the plasma region**, while **eddy currents screen the plasma region flux variation to the outside world**.

**The amount of these currents depends critically** on the **plasma edge electrical conductivity** (for halo and hiro) and on the **wall conductivity** (for eddy).
The flux conservation in an **ideally conducting plasma** can be written as:

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B}) \rightarrow \frac{\partial}{\partial t} \iint \mathbf{B} \cdot dS - \oint (\mathbf{v} \times \mathbf{B}) \cdot dS = 0
\]

or

\[
\frac{\partial}{\partial t} \iint \mathbf{B} \cdot dS + \oint (\mathbf{v} \times dS) \cdot \mathbf{B} = 0 \text{ i.e the FROZEN IN CONDITION}
\]

The physical meaning of the expression is that:

- the magnetic field is comoving with the fluid in ideal MHD
- i.e. the flux through every flux tube is constant as the tube moves around in space
- i.e. the field lines are attached to the fluids
- i.e. the magnetic field cannot change its topology,
- i.e the fluid cannot move across the magnetic field (it is only free to slide along B)

All the above **is not true in a resistive plasma**: but the higher is the plasma temperature the better the **ideal condition** is satisfied
Flux conservation: Halos, Hiros and Eddies currents (3)

- **The case of halos:**
  
  Assume that the plasma shrinks: and the toroidal flux decreases (!) in the plasma region a poloidal current will rise in the halo region to oppose the flux variation.

- **The case of hiros:**
  
  If a similar shrinkage happens but no halo is formed outside the plasma, i.e. a true vacuum region surrounds the plasma the only way to preserve the flux is that a surface current rises at the boundary (so the J current is now flowing in a narrow layer on the plasma-vacuum «moving» interface).

- **The case of eddies:**
  
  The case of eddies is similar to the case of halos with the metal wall playing (Lenz’s law) the same role as the halo region.

What happens if a metal wall is present at the same time as halos or hiros?
Some consideration about Hiro or surface currents (1)

from Knoepfel «Magnetic Fields»

If a step field $H_0$ is turned on around a cylindrical metallic conductor, the eddy current that arise on the conductor surface is:

$$i_z = j_z d = -2H_0 e^{-t/\tau_d} \sin\phi$$

with $\tau_d = \frac{\mu_0 d a}{2\eta_w}$ being the diffusion time through the metal wall.

- Therefore it can be seen that the current decays in a time of the order of $\tau_d$.
- Also $\tau_d$ is shorter for a narrow wall (dissipation increases).
- Similar things can be expected to happen in a «real» plasma since temperature is high but finite and the layers containing the reaction currents are expected to be thin.

Assuming in a plasma $d=1$ cm $a=1$ m and $T=10$ eV what is $\tau_d$?
Some consideration about Hiro or surface currents (2)

Assuming a more **realistic** linear model: **Wall + Vacuum + ideal Plasma**

an **analytic dispersion relation** has been deduced for various current profiles (from flat $\alpha=0$ to parabolic $\alpha=1$)

The main results are that the **surface currents**:

- depend on the equilibrium $\mathbf{J}$ profile
- are strongly reduced by the presence of a wall
- are linearly dependent on $nq_a$

Dashed lines have $r_w/r_p = 1.1$

For plain lines there is no wall
Rotation of the current asymmetry in JET

From S. Gerasimov et al NF (2014)

In JET rotation of the current asymmetry has been detected. The asymmetry is seen to make a few toroidal turns with a relatively low 100 Hz frequency.

This effect is worring for ITER, in fact if the frequency will scale to 5-10 Hz it could resonate with mechanical structures eigenfrequency and produce force amplification.

Rotation as been observed in 3D nonlinear MHD (Strauss, PoP (2014 and 2015), while the expansion is difficult considering KTM or passive wall models. In 3D MHD also the cause of the rotation is not easily deconvolved from simulations (I will come back later on this issue).
Previously the role of the virial theorem for the force balance was discussed. In a more interesting and general form for what concern the conservation of angular momentum (in particular in toroidal direction) it can be written as:

\[
\frac{\partial L_\phi}{\partial t} = T
\]

The interesting thing is that the torque \( T \) can be expressed completely by surface contributions:

\[
T = T_R + T_{em} + T_p + T_\Pi
\]
where:

\[
T_R = - \int R \rho u \phi \mathbf{v} \cdot d\mathbf{S}
\]
\[
T_{em} = \int R B \phi \mathbf{B} \cdot d\mathbf{S} - \int R \frac{B^2}{2} \mathbf{B} \cdot d\mathbf{S}
\]
\[
T_p = - \int R p \phi \cdot d\mathbf{S}
\]
\[
T_\Pi = - \int R (\mathbf{\Pi} \cdot \dot{\mathbf{\phi}}) \cdot d\mathbf{S}
\]

The normal \( \mathbf{B} \) to the wall is very important i.e. NONIDEAL (RESISTIVE) WALL bc

The momentum changes if there are non zero torques contributions at the wall either due to kinetic or magnetic terms

with the viscous stress tensor given by:

\[
\tilde{\tau} = \rho \left( uu - \frac{u^2}{3} \mathbf{i} \right)
\]

where the \( <..> \) is an average in velocity space over the particle distribution, and \( \mathbf{u} \) the velocity
The concept of angular momentum conservation is exactly the same as for the boy on the revolving platform:

One central stack connected to the «earth» is necessary to change the angular momentum of the System.

Through the central stack «surface» torques are applied that are able to bring the system in rotation starting from rest.

..however is the situation so clear or are there complications that can arise in electromagnetism?
The answer is: ...

YES

From the point of view of **electromagnetism the answer is quite clear** however the paradox arises because initially the disk is at rest and from a mechanical point of view apparently there are no applied torques.

The point is however that the **electromagnetic field has an intrinsic angular momentum** that is transmitted to the disk.
Feynman disk paradox: the role of an electrostatic electric field

- **microscopic point of view:** the electrons providing the initial current in the wire move in circle and after the current is switched off they can transmit this loss of momentum to the disk through the wire (electrical resistance and collisions)

- **a macroscopic point of view:** from $-\nabla \times E = \frac{\partial B}{\partial t}$ an inductive electric field is generated that acts on the charges on the disk with a force $F = qE$ that brings the disk in rotation

A more quantitative resolution of the paradox can be found in:

This paradox can however help in understanding that in tokamaks the edge conditions including the presence or the birth of **electrostatic electric fields** can be extremely important for the angular momentum balance and therefore to understand plasma rotation (even during disruptions).

In turn it should be remarked that such fields can be originated either by **transport phenomena** that can separate the electron and ion dynamics or even by any **charge accumulation** effects on wall gaps or divertor components.
To summarize regarding non symmetric events several points are still open to predict the ITER behavior:

- the nature of the **current asymmetry**: halos vs hiros vs eddies
- the **role** of the external conductor
- the **duration** of the phenomenon i.e. the impulse transmitted to the structures
- the **nature/origin** and amount of **expected rotation**

Clearly also foreseeing the plasma conditions in ITER after the **thermal quench (TQ) (how fast? T?)** and during the **current quench (CQ) (how long?)** are extremely important to predict the following behavior and therefore to correctly estimate the consequences
ITER before and after the thermal quench

\[ S = \frac{\tau_R}{\tau_A} \]

(Lundquist number)
M3D is a nonlinear (extended) MHD code (with a peculiar model for the parallel transport):

**MHD model**

- Solves MHD equations.

\[
\begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} &= -\nabla p + \mathbf{J} \times \mathbf{B} + \mu \nabla^2 \mathbf{v} \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E}, \quad \mathbf{E} = (\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}), \quad \mathbf{J} = \nabla \times \mathbf{B} \\
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho &= -\gamma \rho \nabla \cdot \mathbf{v} + \rho \nabla \cdot \mathbf{S} (\rho / \rho) \\
\end{align*}
\]

The fast parallel equilibration of $T$ is modeled using wave equations:

\[
\begin{align*}
\frac{\partial T}{\partial t} &= s \mathbf{B} / \rho \cdot \nabla u \\
\frac{\partial u}{\partial t} &= s \mathbf{B} \cdot \nabla T + \nabla^2 u \\
\end{align*}
\]

$s =$ wave speed / $v_A$

**Two-fluid MH3D-T**

(Sugiyama et al.)

- Solves the two fluid equations with gyro-viscosity and neoclassical parallel viscosity terms in a torus.

**Equations**

\[
\begin{align*}
\mathbf{v} &= \mathbf{v}_i - \mathbf{v}_e^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}/en, \\
\mathbf{v}_e^* &= -\mathbf{B} \times \nabla p_e / (en B^2), \quad \mathbf{v}_i^* = \mathbf{v}_e^* + \mathbf{J}/en, \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} + \rho (\mathbf{v}_i^* \cdot \nabla) \mathbf{v}_i &= -\nabla p + \mathbf{J} \times \mathbf{B} - \mathbf{b} \cdot \nabla \Pi_i, \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E}, \quad \mathbf{E} = (\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}) - \nabla p_e / (en - \mathbf{b} \cdot \nabla \Pi_e, \\
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_i) &= 0, \\
\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho &= -\gamma \rho \nabla \cdot \mathbf{v} + \rho \nabla \cdot \mathbf{S}_h (\rho / \rho) \\
&- \mathbf{v}_i^* \cdot \nabla p + (1/en) \mathbf{J} \cdot \nabla p_e \quad - \gamma \rho \nabla \cdot \mathbf{v}_i^* + \gamma p_e J \cdot \nabla (1/en) \\
\frac{\partial p_e}{\partial t} + \mathbf{v} \cdot \nabla p_e &= -\gamma p_e \nabla \cdot \mathbf{v} + \rho \nabla \cdot \mathbf{S}_h (p_e / \rho) \\
&+ (1/en) \mathbf{J} \cdot \nabla p_e - \gamma p_e \mathbf{V} \cdot (\mathbf{v}_e^* - \mathbf{J}_i / en)
\end{align*}
\]
M3D mesh and equilibrium initialization:

Triangular piecewise linear element are used in the poloidal (R,Z) plane for an unstructured mesh, and a pseudospectral Fourier representation is used in toroidal (\( \phi \)) direction.

The open field line vacuum region surrounding the plasma is modeled as a low density high resistivity plasma.

Upwinding and dealiasing provide adequate numerical stabilization.

The initial equilibrium can be read from an *eqdsk file* obtained from real data or as result of an *equilibrium MHD code*.
M3D boundary conditions: 

\begin{equation}
B_n^v = B_n^p,
\end{equation}

where $B_n^v, B_n^p$ are the normal component of magnetic field in the vacuum, and the plasma, adjacent to the wall.

- The plasma is bounded by a *thin resistive wall* of thickness $\delta$, resistivity $\eta_w$. Outside the wall is vacuum. Normal component of magnetic field is continuous at the wall,

- Green’s identity yields other other components of $B^v$, given $B_n^v$. The current in the wall is given by

$$
J_w = \nabla \times B \approx \frac{\hat{n}}{\delta} \times (B^v - B^p).
$$

This allows time advance of

$$
\frac{\partial B_n}{\partial t} = -\hat{n} \cdot \nabla \times \eta_w J = -\frac{\eta_w}{\delta} \nabla \cdot [\hat{n} \times (B^v - B^p)] \times \hat{n}.
$$
M3D boundary conditions: (2)

\[ \mathbf{B}_v = \nabla \psi_v \times \nabla \phi + \nabla \lambda + I_o \nabla \phi \]

Vacuum magnetic field

GRIN Solver:

\[ \left( \frac{\partial \psi_v}{\partial n} \right)_i = \sum_j K_{ij}^0 \psi_{pj} + S_i \]

\[ (\lambda^n)_i = \sum_j K_{ij}^n (\mathbf{B}_p \cdot n)_j \]

Virtual “case” \[ S_i = \frac{\partial \psi_v}{\partial n} \] at the wall

\[ \frac{\partial \psi_w}{\partial t} = \frac{\eta_w}{\mu_o \delta_w} \left[ \frac{\partial \psi_w}{\partial n} \right] \]

\[ \frac{\partial B_{npw}}{\partial t} = \frac{\eta_w}{\delta_w} \left[ \frac{\partial B_{nw}}{\partial n} \right] \]

[ ] \rightarrow Jump
Some of the free parameters in the code are of particular importance for the disruption simulations. In particular:

- $S$, the Lundquist number (It is mainly limited by the achievable numerical resolution)
- $\mu$, the plasma viscosity (smooth the length scales of the turbulence)
- $\eta_{\text{out}}$, the resistivity of the outer plasma layer (from the separatrix to the wall)
- $\eta_{\text{wall}}$, the wall time constant (the longer the slower the penetration the longer the simulation time)
- $s$, the sound wave related parameter (linked to the parallel transport)
- $\chi_{\text{perp}}$, the perpendicular transport coefficient
A relatively fast kink develops in numerical simulations. TPFs and halo fractions are consistent with the experimental database.
M3D simulations results: horizontal force scaling

From Strauss et al PoP (2010)

Normal force at the wall vs poloidal and toroidal angles (x and y axis)

force vs growth time

\( \gamma_{tw} = 2 \)

n=1 structure
Realistic wall effects and horizontal forces

Realistic model of ITER wall

Clearly the forces depend also on the distribution of wall currents and therefore on wall real geometry.
By assuming:

\[ J_\phi = J_{\phi 0}(r - \xi_{VDE} \sin \theta) + J_{\phi 1}(r - \xi_{VDE} \sin \theta) \cos(\theta + \phi) \]

The perturbed toroidal current can be calculated as:

\[ I_{\phi 1} = -\int drrd\theta \frac{dJ_{\phi 1}}{dr} \xi_{VDE} \sin \theta \cos(\theta + \phi) \]
\[ = -\pi \xi_{VDE} \int dr J_{\phi 1} \sin \phi, \]

The magnetic moment is instead:

\[ M_{IZ} = \int d\theta dr r^2 \sin \theta J_{\phi 1} \cos(\theta + \phi) \]
\[ = -\pi \int dr r^2 J_{\phi 1} \sin \phi. \]

Therefore assuming \( J_{\phi 1} = K_\alpha \delta(r - a) \):

\[ \frac{dI_\phi}{d\phi} = \frac{\xi_{VDE}}{a^2} \frac{dM_{IZ}}{d\phi}. \]
On the other hand from $\nabla \cdot \mathbf{J} = 0$ a simple relation can be deduced:

$$\frac{\partial I}{\partial \phi} = - \int J_n Rdl = -I_{halo}$$

Which shows a 90 degree phase shift between the toroidal variation of the toroidal current and the halo current (as noted in experiments at JET).

Analogously from:

$$\nabla \cdot \mathbf{B} = 0, \frac{\partial \Phi}{\partial \phi} = - \int RB_n dl$$

And assuming: $J_n \approx B_n/a$ it can be seen that $\Delta \Phi \approx \Delta I$

again similarly to what observed in experiments at JET.
A correlation has been found in simulations between the VDE vertical displacement and the plasma rotation. And also an analytical theory has been developed:

\[
\dot{L}_\phi = - \frac{R}{B_{\phi 0}} \int \frac{\partial \psi_0}{\partial \theta} p \, d\theta \, d\phi.
\]

Taking into account that:

\[
\frac{\partial \psi_0}{\partial \theta} = \xi_{10} \cos \theta B_{\phi 0},
\]

and that at the second order in perturbation:

\[
p_2 = \frac{p_0'}{2r^2} \frac{\partial}{\partial r} \left( \frac{\partial \xi}{\partial \theta} \right)^2.
\]
M3D simulations: sustained current

ITER case

In MHD simulations TQ and CQ are quite coupled

TQ and CQ can be decoupled in sustained cases, where an external electric field is applied to sustain the plasma current.

IN CONCLUSION:

- progresses have been done in simulating AVDE’s time behavior, forces and also rotation however
- numerical resolution is generally low (up to n = 6-8)
- simulations could not reach realistic collisionality regimes
- kinetic effects are completely neglected
- flow is generally absent from the initial equilibrium
- transport is likely not realistically modelled
Is the fast experimental thermal quench a mystery?

- In experiments the thermal quench is a fast phenomenon sometime without clear precursor, or at least without from the outside measurable big MHD modes.

- Plasma internal energy is suddenly released in msec timescale (or faster).

- In simulations (apart sustained cases) TQ and CQ are simultaneous and follow the modes growth:

  ![Diagram]

  Mode growth → Mode coupling → Stochasticisation → Transport TQ & CQ

So the question is:

Are there different mechanisms that can explain the fast experimental TQ?
An axi-symmetric tokamak has well conserved (2D) flux surfaces. However if there are non symmetric perturbations the magnetic filed can be described by a perturbed hamiltonian like:

\[ H_{tot} = H_o(J) + H(J, \alpha, \varphi) \]

with:

\[ H(J, \alpha, \varphi) = \sum_{m,n} H_{m,n}(J) \cos[m\alpha + (m-n)\varphi] \]

Harmonics overlapping can lead to field line stochasticity

In turn harmonic overlapping depends on the locations and amplitudes of the modes at the resonant radii (determined by the q profile). For 2 modes the threshold is obtained for \( s > 1 \) with:

\[ s = \frac{1}{2} \left( \frac{w_{m,n} + w_{m_1,n_1}}{|r_{m,n} - r_{m_1,n_1}|} \right) \]  

(Chirikov parameter)

The electron thermal diffusion in stochastic fields can be estimated (collisionless) as:

\[ \chi_e = D_{st} v_{th,e} \]  

with \( D_{st} = \langle b_r/B \rangle^2 \) \( L_c \) and \( L_c \approx \pi R \)
• Although the stochastic transport could be quite fast (if \( \frac{b}{B} \approx 10^{-3} \) and \( T_e = 3 \text{ Kev} \), \( \chi \approx 100 \frac{m^2}{s} \)) compared with standard transport, a quantitative estimate is difficult since often the q profile is only approximately reconstructed, the spectrum and the amplitudes of the modes are also not very well known from external measurements.

• Not just for the TQ, but even for more standard phenomena in tokamaks, like the sawtooth crashes that are observed in the core plasma region when the q on axis approaches 1, there is no firm agreement about what is determining the temperature crashes and if they can be linked to an enhanced stochastic thermal diffusion through higher harmonic generation, as found in some simulations.

Nonlinear XTOR simulations showing the generation of high \( n \) mode numbers

The physical parameters are not extremely realistic:
\[ S = 10^6, \chi_\parallel = 100, \chi_\perp = 10^{-5}, Pr = \frac{v}{\eta} = 1 (?) \]

From Luetjens et al JCP (2010)
Experimental results in AUG tokamak have shown that the appearance of chaos during sawtooth activity is very sensitive to the q profile near the axis.

possible alternatives: explosive instability at relatively low $\beta_N$

From A Y Aydemir et al, NF 56 (2016)

CTD spectral toroidal code

I, II both ideally unstable (effect of geometry?)

$\beta_N = 1.4$

Explosive pressure fingers development

High harmonics up to $n=30$ in simulation

High numerical resolution
• Do **FUSION RELEVANT (FR)** low disruptivity scenarios exist?

• Is it possible to **classify FR** scenarios according to **disruptivity**?

• what we know about **scalings to larger devices**?
Density is limited by the so called Murakami/Greenwald limit:

\[
  n_G \ [m^{-3}] = \frac{I \ [MA]}{(\pi A^2 \ [m])}
\]

..not well understood (but likely connected with input/output energy balance)

.. experimentally clear \(\rightarrow\) DISRUPTIONS ABOVE GL (or near to it..)

\(\beta\) limited by ideal MHD instabilities (ISL): \(\beta_T \leq C \left(\frac{I}{a_B}\right)\) (or \(\beta_N \leq C\) (3-4))

.. experimentally reasonably confirmed (standard non rev. shear plasmas)

\(\rightarrow\) DISRUPTIONS ABOVE or NEAR ISL
The limits in terms of simple macroscopic parameters

Hugill plot for density limit and \( q(a) \)

IPB-NF 1999 (DIII-D data)

From Stacey: Fusion Plasma Physics (Wiley 2012)

\[
p = p_0 \left(1 - \frac{r^2}{a^2}\right)
\]

\[
j = j_0 \left(1 - \frac{r^2}{a^2}\right)^\nu
\]

\[
l_i = \frac{\bar{B}_\theta}{B_{\theta a}} = \frac{2 \int_0^a B_\theta^2 r \, dr}{a^2 B_{\theta a}^2}
\]

\[
l_i = \ln(1.65 + 0.89 \nu)
\]

higher shear →

and

\[
q_a/q_0 = \nu + 1
\]
ISL in toroidal geometry and shaped plasmas

*Turnbull et.al. NF (1998), β limits*

However limitation are due to:

- broad p enhances $f_{bs}$ at edge and limits achievable $l_i$
- therefore high $l_i$ also depends on possibility of reducing outer pedestal height (lower $f_{bs}$) (..with ELMs control)
- low q(0) could maximize $l_i$ but sawth. limits q(0) to 1

ITER like plasmas in DIII-D

*shape factor $S = \left( \frac{I}{aB} \right) q_{edge}$*

*from Ferron et.al. NF (2015)*
Conditions for high $\beta$ and performing plasmas (i.e. FR)

- High $\beta_N$, High $l_i$, high shear
- High shaping $S$
- as low as possible $q(0)/q_{\text{min}}$
- low outer pedestal $h$. (by ELM control)
- flat pressure

Clearly quite contradictory with:
disruption limits/avoidance and safe tokamak operation!

not to speak of:
- Low plasma rotation (..eventually)
- Plasma wall proximity (see later) in larger devices
Fusion performances and $\beta$ limits

\[ \frac{\beta_T}{\varepsilon \frac{1+\kappa^2}{2}} \]

YQ Liu, Peking University, Feb 16-20, 2009
External ideal kink instability (time scale = microseconds)
- Normally pressure-driven (above no-wall beta limit)
- Resistive wall slows down kink instability to time scale of wall eddy current decay time ➔ RWM (typically milliseconds)

- At high pressure, mode located towards low-field side (kink-ballooning)
- Low toroidal mode number n=1,2,3
- Similar to vertical instability (RWM with n=0)

- Three consequences of slowed down
  ➔ Still unstable ➔ eventually causes disruption
  ➔ Time scale feasible for feedback control
  ➔ Kinetic effects become important

YQ Liu, Peking University, Feb 16-20, 2009
Recent NSTX results seem to enlarge the parameter space.

NSTX reaches high $\beta_N$, low $l_i$ range of next-step STs and the highest $\beta_N/l_i$ is not the least stable.

- High $\beta_N$ for fusion performance, high non-inductive fraction for continuous operation
  - High bootstrap current fraction $\rightarrow$ Broad current profile $\rightarrow$ Low $l_i = \langle B_p \rangle^2 / \langle B_p \rangle^2$
  - Unfavorable for ideal stability since low $l_i$ reduces the ideal n = 1 no-wall beta limit
- The highest $\beta_N/l_i$ is not the least stable in NSTX
  - In the overall database of NSTX disruptions, disruptivity decreases as $\beta_N/l_i$ increases
  - Passive stability of the resistive wall mode (RWM) must be explained

Interpreted by the combined effect of rotation and kinetic stabilization

RWM disruption rate from 45% to 14% at low $l_i$ and high $\beta_N$

But low collisionality plasmas are also susceptible to sudden instability when kinetic profiles change.
Misk code calculated kinetic terms for experimental data showing larger effect at high $\beta_N/\text{li}$

The relevant resonance is between the slowly rotating mode and the thermal ions in their precession drift motion:

$$|\omega_E - \omega_D| \approx 0$$

with:

$$\omega_E = \omega_\phi + \frac{1}{en_i} \frac{d}{d\psi}(n_i T_i)$$

and $\omega_\phi$ is the plasma rotation

Collisionality seems also to play a beneficial role for stable cases: (not for the unstable cases)
Low collisionality stabilization of RWMs due to resistivity?

He et al., PRL (2014)

\[ \gamma \tau_w = -\frac{\delta W^\infty + \delta W_k + \delta W_{RL}}{\delta W^b + \delta W_k + \delta W_{RL}} \]

Ideal terms

ITER equilibrium (\( \beta_N = 4.2 \) b/a = 1.25) CHEASE&MARS-F

No effect of resistivity above interesting S (Lundquist) numbers (no kinetic effects here)
Low li versus High li operation

- low aspect ratio devices operate at larger $f_{bs}$ and broad current (i.e. low li)
- in this case however $\beta_{NW} < \beta_{IDW} \Rightarrow \text{wall stabilization is needed to increase } \beta$
- On the other hand at higher li (peaked current) $\beta_{NW} \approx \beta_{IDW} \Rightarrow \text{wall stab. is less important}$

- Which is the situation in a FR device regarding the wall stabilization effectiveness?
SCALING OF THE SHELL PROXIMITY IN FR CASES
.IS IT A RELEVANT ISSUE?

from J. Freidberg et al PoP 22 (2015)

STANDARD TOKAMAK

..but for stability with an ideal wall:

\[
\frac{b}{a} \leq 1.3 \div 1.4
\]

=>

\[
a \approx 3 \div 4 \ m
\]

This is a serious constraint to minimum a (and R)!

(this is true also for vertical n=0 stability!)

b ≥ 1.2 m

stabilizing wall
Growth vs. $\beta_N$ and ideal wall stability

R/a=2, elong.=1.7 tri=0.2

just as an example..

\[ q \]

\[ \psi \]

\[ \text{pres}/p_0 \]

\[ \gamma_{nw}, \gamma_{id\_wall} \]

\[ \beta_N \]

\[ b/a \]

range of wall position
Plasma rotation mysteries.. and its role in stabilization

from J. E. Rice, *Experimental observations of driven and intrinsic rotation in tokamak plasmas* PPCF 58 (2016):

- a substantial fraction of the rotation observed following NBI is not due to direct drive from the beams

This calls into question the traditional method of determining momentum transport coefficients from observed rotation profiles assuming momentum input (calculated) from the beams

- Regarding LH: once the q profile is modified, the observed rotation is in the opposite direction to the momentum input from the LH waves!

- ICRF waves in the minority heating scheme, observations show rotation in both directions, with complicated profile shapes and agreement with theoretical models isn’t even qualitative.

These results indicate that momentum input from RF waves is not well understood

- For momentum sinks due to locked modes, magnetic braking and NTV the agreement between experiment and theory is often very good

- the comparison between observations and the predictions of neo-classical theory show a huge range of agreement/disagreement from excellent quantitative comparisons to complete disparity! ..not understood residual stresses ?!
plasma rotation & mode stabilization:

*from F. Turco et al, NF 55 (2015)*

**Plasma response:** the model shows a significant discrepancy at the highest $\beta_N$ points. *still missed physics!*

MARS-K vs Exp. DIIIID @ $\beta_N=2.4$ @ I-coil 20Hz

- **plasma rotation stabilize RWM**
  Bondeson&Ward PRL (1994)

However later:

- **the threshold** is at relatively low plasma rotation $\omega_{\text{crit}} \tau_A = 0.3\%$ at the $q=2$ surface (T.Strait et al, PoP 14(2007) )

- kinetic drifts therm. & fast. ions are important but seem not to fully describe the physics in DIIIID
from J. Berkery et al, "Benchmarking kinetic calculations of resistive wall mode stability" PoP 21 (2014)

One must recall that this is an incomplete calculation for ITER, however, as various simplifications have been made in the benchmarking process, including, most notably, the lack of collisions and energetic or alpha particles. Nevertheless, the codes agree in the basic underlying calculation of kinetic effects and all support the present understanding that both high and low rotation kinetic resonances are stabilizing to the RWM, but intermediate plasma rotation is potentially susceptible to instability.

Assuming as in Parra et al, PRL (2012) : $V_\phi = k \frac{T}{I}$ and $\omega_{E_0} \approx \frac{V_\phi}{a}$, ITER@20Kev,10MA will likely be in the dangerous zone (or near to it)!!
RWMs critical issues

- **Rotational stabilization of RWM**
  - *Mode damping physics*: $\omega_r$, $\omega_d$ resonances; reactive closure model (Weiland), neoclassical viscosity (Shaing)
  - *Effect of rotation profile*: rotation shear, damping distribution, role of edge rotation, global parameter for rotation threshold
  - *Toroidal momentum damping* due to RWM
  - *Nonlinear coupling* between RWM, RFA, and rotation: (rotation damps mode, mode damps rotation via RFA)
  - Plasma rotation enough in ITER for RWM stabilization?

- **Feedback stabilization of RWM**
  - *Systematic toroidal study of RWM dynamics* (PRM) vs. $\beta_N$ and $\omega_{rot}$
  - *Control issues for ITER*: choice of feedback coils, non-ideal effects (voltage saturation, noise, ac-losses of super-conducting coils)
  - *3D wall effect* on RWM control in ITER
  - RWM control for $n \geq 2$

YQ Liu, Peking University, Feb 16-20, 2009
The request for high performances i.e. high \( n \) and high \( \beta \) is equivalent to operate near to the DISRUPTION LIMITS and increase the PROBABILITY of DISRUPTIONS (PD).
Mode Locking and disruptions

From Sweeney NF (2017)

Typical sequence of mode locking

Shots with IRLM ended 76% of the time in a disruption

At high $\beta$ 28% of the disruptions are caused by a detected IRLM (18% at low $\beta$)
Recent experiments in AUG-U of disruption control with RMP

From R. Paccagnella et al, EPS P1.027 (Leuven, 2016)

AUG #33197 10 MW NBI with mode entrainment

2/1 (NTM?) tearing excited by a kink aligned RMP

SHOT 32532 spectrogram

SHOT 32532: amplitude of vacuum field vs poloidal mode m

SHOT 32532: ECE confirms resonant 2/1 island structure in phase with B coils

Island width
What about the modelling of this interesting case?

The dynamics of the $m=2$, $n=1$ tearing mode is simulated by the **cylindrical**, spectral RFXlocking code [1]

- Equations of motion
- Newcomb Equation
- NTV from island determined as in [2]
- Rutherford Equation
- No-slip condition
- Wall resistive diffusion


Modelling in toroidal geometry would be necessary (..but very difficult) **Reduced Models are important**
TM dynamics is simulated by the cylindrical, spectral RFXlocking code, solving:

• **Single-fluid motion equations** with perpendicular viscosity $\mu$ and em. torque $\delta T_{EM}$ localized at the resonant surface $r_{m,n}$

$$\rho \frac{\partial \Omega_\phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \mu r \frac{\partial}{\partial r} \Omega_\phi \right) + \frac{\delta T_{EM,\phi}^{m,n}}{4\pi^2 r^3 R_0} \delta(r - r_{m,n})$$

$$\rho \frac{\partial \Omega_\theta}{\partial t} = \frac{1}{r^3} \frac{\partial}{\partial r} \left( \mu r^3 \frac{\partial}{\partial r} \Omega_\theta \right) - \frac{\rho}{\tau_D} \Omega_\theta + \frac{\delta T_{EM,\theta}^{m,n}}{4\pi^2 r^3 R_0} \delta(r - r_{m,n})$$

• Em. Torque, due to interaction with the passive structures, is modelled exploiting **Newcomb’s equation**
  (parabolic equilibrium current distribution)

+ $NTV$ like torque:

$$\frac{\partial \Omega}{\partial t} = -\mu_\parallel \left( \frac{\delta B_{eff}}{B_0} \right)^2 (\Omega - \Omega_\ast)$$

...also for the $NTV$
- **Newcomb equation**

\[
\frac{\partial}{\partial r} \left[ \frac{r}{H_{m,n}} \frac{\partial \psi^{m,n}}{\partial r} \right] - \left[ \frac{1}{r} + \frac{r G^{m,n}}{H_{m,n} F_{m,n}} \frac{d\sigma}{dr} + \frac{2mn\varepsilon\sigma}{(H_{m,n})^2} - \frac{r \sigma^2}{H_{m,n}} \right] \psi^{m,n} = 0,
\]

\[
F_{m,n}(r) = m B_{\theta 0} - n \varepsilon B_{\phi 0}, \quad G_{m,n}(r) = m B_{\phi 0} + n \varepsilon B_{\theta 0}, \quad H_{m,n}(r) = m^2 + n^2 \varepsilon^2, \quad \varepsilon = r/R_0.
\]

with \( \psi^{m,n}(r, t) \equiv -ib_{r}^{m,n}(r, t) \)

- **Rutherford equation** for the island width

\[
\tau_R \frac{dW}{dt} = 1.22 \Delta'(W)
\]

- **Diffusion equations** for radial field penetration across the passive structures

\[
\mu_0 \sigma \frac{\partial b_r^{m,n}}{\partial t} = \frac{\partial^2 b_r^{m,n}}{\partial r^2}
\]

- Island phase determined by the no-slip condition

(for the present simulations we neglect diamagnetic term)

\[
\frac{d\phi^{m,n}}{dt} = n \Omega_{\phi}(r_{m,n}, t) - m \Omega_{\theta}(r_{m,n}, t)
\]

*From Zanca P PPCF (2010)*
RFX locking estimates for ITER

**TM locking during CQ in ITER**

Cylindrical model and walls:
- Blanket is treated as a EM-thin wall: $\tau_{w1} = 2\,ms$
- VV is treated as 2 EM-thick walls: $\tau_{w2} = 94\,ms$, $\tau_{w3} = 94\,ms$

*Villone F. et al 2010 Nucl. Fusion 50 125011*

**m/n=2/1 TM locking during CQ**

- $\tau_{CQ}$ - Current quench time
- $\tau_R$ - Resistive time
- $\tau_V$ - Viscous time
- $\tau_D$ - Poloidal damping time

$\tau_R \approx \tau_{CQ}$
$\tau_V \geq \tau_{CQ}$
$\tau_D \approx \tau_{ii}$

Graphs show the evolution of $\nu$ and $t/\tau_R$ with $r_s/r_a = 0.9$, $\tau_R = 50\,ms$, $\tau_D = 3 \times 10^{-5}\tau_R$. Additional cases with $\tau_D = 3 \times 10^{-4}\tau_R$ and $\tau_D = 3 \times 10^{-6}\tau_R$ are also shown.
The role of radiation in disruption mitigation

- Disruptions can be triggered by a sudden increase of the radiation losses.
- However, radiation can also be used to mitigate disruption effects: reducing divertor heat loads, asymmetric stresses and runaway electrons.

*From Lehnen et al NF (2013) and (2015)*
Nonlinear simulations of disruption mitigation (1)

from Izzo V et al, NF 55 (2015)

MHD produces radiation asymmetries

Weak points:
- Simplified physics, radiation models, plasma-wall int.
- Transport
- Collisionality
- Num. resolution

Runaways in ITER
$10^{-2}$ (s) est. conf. time

from Izzo V et al, NF 51 (2011)
Nonlinear simulations of disruption mitigation (2)

Recent JOREK simulations (at high S number > $10^7$)

- 2/1 induced by resistivity drop due to MGI
- 3/2 destabilized by current flattening
- nonlinear coupled modes triggered
- plasma stochasticisation and TQ
The critical electric field depends on plasma density (more weakly on temperature)

\[ E_{\text{crit}} = \frac{n q^3 \ln \Lambda}{4 \pi \varepsilon_0^2 mc^2} \]
• The critical electric field is quite small according to theory.

• The experimental data show a much higher electric field threshold: interpreted as an extra loss mechanism beside the collisional drag (e.g., synchrotron rad.)

• Avalanche mechanism likely dominant in ITER at difference with actual experiments.
From Martin-Solis et al NF (2014)

- Predictions for ITER are quite uncertain
- the ratio between the plasma resistive diffusion time (after TQ) and the RE loss time is critical
- Avalanche (for long duration of CQ) could be an issue
- large fraction of plasma magnetic energy could be converted to RE energy

RE represent a serious issue for ITER therefore mitigations and/or control systems are mandatory
About Runways electrons: beam penetration

From Reux et al NF (2015)

In recent JET experiments with ILW:

- RE are suppressed by early (before TQ) gas injection
- are instead produced for a later gas injection

- Runaway mitigation after the beam has been accelerated has been proven unsuccessful at JET, with injections of 663 Pa.m³ to 4340 Pa.m³ of argon, krypton or xenon

- These results confirm globally that runaway physics are similar with a metallic wall and with carbon wall, and that runaway electron suppression should be attempted before the beam is fully developed.
There are several MHD related physical OPEN issues related to disruptions:

- the nature and detailed mechanisms of the TQ
- the duration of the CQ (residual temperature after TQ & RE)
- The halo structure (2D) in symmetric VDEs
- the halo structure (3D) in non symmetric VDEs
- halos vs. hiros (the role of surface currents)
- the role of the passive structures (and eddy currents)
- the nature and origin of plasma rotation and the residual slow mode rotation
- Interaction between plasma and external MPs
- Thermal loads and RE electrons

... for all this issues EXISTING MODELS ARE GENERALLY LACKING
• a lot of interesting physics is related / linked to DISRUPTIONS

• our understanding is still quite incomplete and our modelling capabilities need to be further extended (physics) and increased in capability (resolution)

• Pathological cases (like disruptions) can be very helpful also for the understanding of healthy plasmas: the physics of plasma rotation, mode locking, plasma relaxation and reconnection, transport in stochastic fields are only few examples

• for their effects on the structures and on the containing wall material plasmas completely avoiding them are needed in a fusion plants

• disruptions are really the most serious showstopper for fusion

Runaways and localised plasma wall interactions:
could represent also very serious issues for fusion even in presence of mitigation systems as MGI or fast and massive pellets launchers: NO DISRUPTIONS → NO SIDE EFFECTS