Computation of Tokamak Edge Turbulence

techniques and typical results

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# Outline

- **Basics of Low-frequency Drift Dynamics**
  - low frequency basics, energy transfer in turbulence, equilibrium

- **Methods**
  - numerical techniques, mathematical treatment of magnetic geometry

- **Electromagnetic Nonlinear Character**
  - energy transfer, nonlinear saturation, turbulence is not a collection of instabilities

- **Sheared ExB Flows**
  - basic mechanisms, self consistency, toroidal compression

- **Gyrofluid Edge Turbulence**
  - self consistent evolution of MHD equilibrium
  - scale separation, role of gyro-Bohm scaling
  - global burst behaviour, edge/SOL interface, edge to SOL causality

- **Some Important Lessons**
Magnetic Confinement

\[ \mathbf{J} \times \mathbf{B} = c \nabla p \]

MHD equilibrium

- strong magnetic field, small gyroradius
- closed magnetic flux surfaces
- \( \rightarrow \) confined plasma

However . . . turbulence \( \rightarrow \) losses

eddies, few gyroradii
Magnetic Field

Tokamak Magnetic Field

axisymmetric MHD equilibrium

toroidal, poloidal components

mainly toroidal

ratio of components --> pitch parameter “q” $\frac{B_\zeta}{B_\theta}$
Low Frequency Drift Motion

magnetic field

general

sense of gyration for ions

low frequencies

$\omega << \Omega$

drift of gyrocenters

(v_{\perp} << v_{\parallel})

v-space details: “gyrokinetic”

few moments: “gyrofluid”
Low Pressure (Beta) Dynamics

- low “beta”
  \[ p \ll B^2/8\pi \]

- “flute mode”
  vortices/filaments
  \[ k_{||} \ll k_\perp \]

- low frequencies
  \[ \omega \ll k_\perp v_A \]

- magnetic field \( B \)

- pressure disturbance \( \tilde{p} \)
- magnetic disturbance \( \tilde{B} \) (parallel to \( B \))

- \( \nabla (\tilde{p} + 4\pi \tilde{B} B) \sim 0 \)

- \( \omega \sim k_{||} v_A \)

- strict perpendicular force balance

- electromagnetic parallel dynamics
computations: align coordinates to magnetic field (sheared, curved)
(only one contravariant component of B is nonvanishing)
(nonorthogonal, takes advantage of slowly varying B)

ExB Drift at Finite Gyroradius

\[ \mathbf{v}_E = \frac{\mathbf{c}}{B^2} \mathbf{B} \times \nabla \phi \]

\[ k \rho \ll 1 \]

\[ \phi(x,y) > 0 \]

\[ \mathbf{v}_E \rightarrow \mathbf{u}_E \]

\[ \mathbf{u}_E = \frac{\mathbf{c}}{B^2} \mathbf{B} \times \nabla J \phi \]

\[ k \rho \sim 1 \]

\[ \phi(x,y) > 0 \]
Phase Shifts and Transport

$p$ and $\phi$ in phase

$\implies$ no net transport

Phase shift $\implies$ net transport

Phase shift $\implies$ net transport down gradient

$\implies$ free energy drive
Role of Parallel Forces on Electrons

equation of motion for electrons parallel to $B$

$$n_e e \left( \frac{1}{c} \dot{A}_\parallel + \nabla_\parallel \phi + \eta_\parallel J_\parallel \right) = \nabla_\parallel p_e + \text{inertia}$$

Alfven (MHD) coupling

adiabatic (fluid compression) coupling

a “two fluid” effect

static balance of gradients $\rightarrow$ “adiabatic electrons”

general: response of currents to static imbalance

controls possible phase shifts

$\tilde{p}_e \leftrightarrow \tilde{\phi}$
Drift (Alfven) Wave Dynamics

\[ \vec{V}_p \]

ion current

\[ \sim \vec{p} \]

electron current

sound waves

\[ \tilde{\phi} \] coupled to \( \tilde{p} \) through Alfven dynamics

\[ \tilde{\phi} \] continually excites \( \tilde{p} \) in the gradient

\[ \text{structure drifts} \]

(M Wakatani A Hasegawa Phys Fluids 1984)

(B Scott Plasma Phys Contr Fusion 1997)
High resolution, long runs (> 1000 "gyro–Bohm" times) are necessary for equal temperatures, space scale range includes ion gyroradius. The slowest time scale reflects the flow/equilibrium component.

For equal temperatures, the space scale range includes the ion gyroradius. A broad range of both time and space scales is necessary.

(B Scott Plasma Phys Contr Fusion 2003)
Numerical Methods

• nonlinearities have the form of brackets

\[ \frac{\partial f}{\partial t} + [\psi, f]_{xy} + \cdots = 0 \]

with

\[ [\psi, f]_{xy} = \frac{\partial \psi}{\partial x} \frac{\partial f}{\partial y} - \frac{\partial f}{\partial x} \frac{\partial \psi}{\partial y} \]

• spatial discretisation:
centred-diff for linear terms, Arakawa (J Comput Phys 1966) scheme for brackets
○ basic properties of bracket satisfied to machine accuracy

\[ [\phi, f]_{xy} = \frac{1}{3} \left( J^{++} + J^{+\times} + J^{\times+} \right) \]

• temporal discretisation:
○ both sides expanded \( \implies \) all mixed terms in Taylor expansion present
○ one evaluation per time step
○ tested on turbulence and coherent vortices (Naulin and Nielsen, SIAM J Math 2003)

\[ \frac{\partial f}{\partial t} = S \quad \text{with} \quad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} = \sum_{j=1}^{3} \beta_j S_j \]
(details 1: origin of brackets)

- basic structure of gyrocenter continuity equation (similar to gyrokinetic equation)

\[
\frac{\partial n}{\partial t} + \nabla \phi \cdot \frac{cF}{B^2} \cdot \nabla n + B \nabla \left| \frac{n u}{B} \right| + \nabla \log B^2 \cdot \frac{cF}{B^2} \cdot \left( n \nabla \phi + \frac{1}{e} \nabla p \right) = 0
\]

- define bracket

\[\nabla n \cdot \frac{cB}{B^2} \times \nabla = \nabla \phi \cdot \frac{cF}{B^2} \cdot \nabla n \equiv [\phi, n] \quad \text{using} \quad F = \epsilon \cdot B\]

- local approximations: use $L_{\perp} k_{\perp} \gg 1$ ordering
  - linearise everywhere except in bracket-structure quadratic nonlinearities

\[
\frac{\partial n}{\partial t} + [\phi, n] + n_0 B \nabla \left| \frac{u}{B} \right| + \left[ \log B^2, \left( n_0 \phi + \frac{T_0}{e} n + \frac{n_0}{e} T \right) \right] = 0
\]

- adjust brackets to be divergence-free structures (preserves energetics)
\[ [\phi, f] = \frac{c}{B_0} \frac{1}{r} \left( \frac{\partial \phi}{\partial r} \frac{\partial f}{\partial \theta} - \frac{\partial \phi}{\partial \theta} \frac{\partial f}{\partial r} \right) \]

- parallel derivatives (see geometry, below), simplified form (linear term shown)
  - do these via centred differences (or 4th order if you wish)

\[ B \nabla_{\|} f = \frac{B_0}{2\pi R_0} \left( \frac{\partial f}{\partial \zeta} + \frac{1}{q} \frac{\partial f}{\partial \theta} \right) \]

- dissipation terms are simple, e.g.,

\[ \frac{1}{c} \frac{\partial A_{\|}}{\partial t} = \cdots - \eta_{\|} J_{\|} \quad \text{or} \quad \frac{1}{2} \frac{\partial T_{\|}}{\partial t} = \cdots - \frac{\nu}{3\eta_0} (T_{\|} - T_{\bot}) \]
Representation of Tokamak Geometry

- flux coordinates with nested flux surfaces (S Hamada, 1958, Nucl Fusion 1962)
  - surface label minor radius $r$, poloidal/toroidal angles $\theta, \zeta$ periodic on unit torus

$$\mathbf{B} \cdot \nabla r = 0 \quad \mathbf{B} \cdot \nabla \theta = B_0/2\pi qR_0 \quad \mathbf{B} \cdot \nabla \zeta = B_0/2\pi R_0$$

- take advantage of $k_{||} \ll k_\perp$ and align the coordinates to $\mathbf{B}$ — note $q$ is $q(r)$

$$x = r/a \quad y = q\theta - \zeta \quad s = \theta$$

- this ensures that only one contravariant component is nonzero (here: $B^s$)
  - coarse resolution is allowed in that direction (dimension)
  - very high resolution, necessary for both $k_\perp$ dimensions, becomes feasible

- typically MHD ↔ turbulence crosstalk requires 500 or more toroidal modes

- main caveat: global consistency in $\theta$ boundary conditions

$$f(x, y + q, s + 1) = f(x, y, s) \quad \text{ensures} \quad k_{||} qR = m - nq$$
More Work on the Coordinates

- main issue is deformation: large values of $g^{xy} \to$ extra numerical dissipation

- solution: different coordinate system on each “drift plane” $s = s_k = \text{constant}$

\[
x = r/a \quad y_k = q(\theta - s_k) - \zeta - \Delta\alpha_k \quad s = \theta
\]

- non-zero $\nabla r \cdot \nabla \theta$ and $\nabla r \cdot \nabla \zeta$, choose

\[
\alpha_k = q s_k + \Delta\alpha = \alpha_k(r) \quad \frac{\partial}{\partial r} \Delta\alpha = (g^{rr})^{-1} \left( q g^{r\theta} - g^{r\zeta} \right)
\]

- this makes $g_k^{xy} = 0$ at $s = s_k$
  - retaining global field aligning, local orthogonality, exactly

- this “shifted metric” technique is required to treat anything with “slab character”
  - e.g., shear Alfvén turbulence component, global MHD such as tearing

- carry angle periodicity through, exactly, to obtain angle boundary conditions
(details 1: boundary conditions on angles)

- coordinates defined as
  
  \[ \begin{align*}
  x &= r/a \\
  y_k &= q(\theta - s_k) - \zeta - \Delta\alpha_k \\
  s &= \theta
  \end{align*} \]

- already satisfy toroidal periodicity
  - changing \( y_k \) holding \( x, s \) constant is same as changing \( \zeta \) holding \( r, \theta \) constant
    
    \[ f(r, \theta, \zeta + 1) = f(r, \theta, \zeta) \]
    
    becomes
    \[ f(x, y_k - 1, s) = f(x, y_k, s) \]

- now must satisfy poloidal periodicity
  - changing \( \theta \) holding \( r, \zeta \) constant changes both \( y_k \) and \( s \)
    
    \[ f(r, \theta + 1, \zeta) = f(r, \theta, \zeta) \]
    
    becomes
    \[ f(x, y_k + q, s + 1) = f(x, y_k, s) \]

- now put each plane on its own coordinate system
  - \( N \) drift planes: \( s_{k+N} = s_k + 1 \)
    
    \[ f(x, y_k + q, s + 1) = f(x, y_k, s) \]
    
    becomes
    \[ f(x, y_{k+N}, s_{k+N}) = f(x, y_k, s_k) \]
(details 2: parallel derivatives)

- special attention to unperturbed $\nabla \parallel$

\[
B \nabla \parallel f = \frac{\partial f}{\partial s}
\]

- finite difference across drift planes, each on its own coordinate system
  - (equidistant: $s_{k+1} - s_k = h_s$

\[
2 h_s \frac{\partial f}{\partial s} \bigg|_{s=s_k} = f(x, y_k, s_{k+1}) - f(x, y_k, s_{k-1})
\]

\[
= f(x, y_{k+1} - \Delta^+, s_{k+1}) - f(x, y_{k-1} - \Delta^-, s_{k-1})
\]

- shifts come from coordinate definition $y_k = y - \alpha_k$

\[
\Delta^\pm = \alpha_{k\pm 1} - \alpha_k
\]
• transform using tensor rules (simplified form with $\Delta \alpha = 0$)
  o in general the only simplification is evaluation at $s = s_k$

$$\frac{\partial}{\partial r} = \frac{\partial x}{\partial r} \frac{\partial}{\partial x} + \frac{\partial y}{\partial r} \frac{\partial}{\partial y} = \frac{1}{a} \frac{\partial}{\partial x} + \frac{\partial q}{\partial r} (s - s_k) \frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial \theta} = \frac{\partial y}{\partial \theta} \frac{\partial}{\partial y} + \frac{\partial s}{\partial \theta} \frac{\partial}{\partial s} = q \frac{\partial}{\partial y} + \frac{\partial}{\partial s}$$

• fluxtube ordering: $k_\parallel \ll k_\perp$ implies $\partial/\partial s \ll \partial/\partial x$ or $\partial/\partial y$

$$\frac{1}{r} \text{ becomes } \frac{1}{a} \text{ hence } [\phi, f] = \frac{c}{B_0 a^2} \left( \frac{\partial \phi}{\partial x} \frac{\partial f}{\partial y} - \frac{\partial \phi}{\partial y} \frac{\partial f}{\partial x} \right)$$

• caveat on the curvature: $\partial/\partial y = 0$ for $\log B^2$ hence keep $\partial/\partial s$ for it

$$- [\log B^2, f] \rightarrow \mathcal{K}(f) = \frac{c}{B_0 a} \frac{2}{R_0} \left[ (\cos s + g_k^{xy} \sin s) \frac{\partial f}{\partial y} + \sin s \frac{\partial f}{\partial x} \right]$$
form various versions of “Jacobian” operation (straight, rotational, diagonal)
○ evaluated at grid node 00 with + or − neighbours in $xy$ plane

$$J^{++} = \frac{1}{4h^2}[(\phi_{0+} - \phi_{0-})(f_{0+} - f_{0-}) - (\phi_{0+} - \phi_{0-})(f_{0+} - f_{0-})]$$

$$J^{+\times} = \frac{1}{4h^2}[\phi_{0+}(f_{++} - f_{+-}) - \phi_{0-}(f_{--} - f_{-+}) - \phi_{0+}(f_{++} - f_{+-}) + \phi_{0-}(f_{+-} - f_{--})]$$

$$J^{\times+} = \frac{1}{4h^2}[\phi_{++}(f_{0+} - f_{0+}) - \phi_{--}(f_{0-} - f_{0-}) - \phi_{++}(f_{0+} - f_{0+}) + \phi_{--}(f_{0+} - f_{0+})]$$

$$J^{\times\times} = \frac{1}{8h^2}[(\phi_{++} - \phi_{--})(f_{++} - f_{--}) - (\phi_{++} - \phi_{--})(f_{++} - f_{--})]$$

• demand antisymmetry of bracket, conservation of energy and enstrophy, find

$$[\phi, f]_{xy} = \frac{1}{3} (J^{++} + J^{+\times} + J^{\times+})$$
(details 5: Karniadakis’s time step)

- a variant on the Adams/Bashforth theme, expand both sides 3 timesteps deep
  - this recovers all mixed terms in time-space Taylor expansion

\[
\frac{\partial f}{\partial t} = S \quad \text{with} \quad \sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} = \sum_{j=1}^{3} \beta_j S_j
\]

- coefficients for order 3:

\[
\alpha_{1,2,3} = 3 \quad -3/2 \quad 1/3 \quad \beta_{1,2,3} = 3 \quad -3 \quad 1
\]

- incorporation of an implicit dissipation piece \( L \) is straightforward
  - watch out for the factor of \( 6/11 \) (inverse sum over the \( \alpha_j \))
  - NB: always avoid implicit techniques with wave dynamics

\[
\sum_{j=1}^{3} \alpha_j \frac{f_0 - f_j}{\Delta t} + L(f_0) = \sum_{j=1}^{3} \beta_j S_j
\]
Basic Situation in the Tokamak Edge

- edge time scales for electrons
  - collisions $\nu_e$
  - thermal transit $V_e/qR$
  - Alfvén transit $v_A/qR$
  - turbulence $10^{-2}$ to 1 times $c_s/L_T$

- edge time scales for ions
  - collisions $\nu_i$
  - thermal transit $c_s/qR$

  | electron time scales comparable to turbulence |
  | ion time scales much slower |

\[
\beta = \left( \frac{c_s/L_\perp}{v_A/qR} \right)^2 \quad \hat{\mu} = \left( \frac{c_s/L_\perp}{V_e/qR} \right)^2 \quad C = \frac{0.51\nu_e}{c_s/L_\perp} \hat{\mu} \quad \text{all} > 1
\]
Nonlinear Saturation

basic feature of any instability — transition to turbulence

linear drive (n) —> linear growth

moment of saturation — growth rate (T) drops to zero

saturation maintained — nonlinear transfer to subgrid scale dissipation (E)

transport (Q) overshoots, finds saturated balance

(B Scott Phys Plasmas 6/2005)
Nonlinear Cascade in Turbulence

basic statistical character of three wave energy transfer

dominant transfer is through the thermal free energy ($n$), others also active

(S Camargo et al Phys Plasmas 1995, 1996)
Nonlinear Instability

basic feature of drift wave turbulence (edge turbulence test case)

amplitude threshold $\rightarrow$ linear stability

vorticity nonlinearity $\rightarrow$ damped eigenmodes destabilise each other

role of pressure advection nonlinearity $\rightarrow$ saturation

edge turbulence $\rightarrow$ washes out microinstabilities in toroidal magnetic field

Energy Transfer

part of energy theorem governed by vorticity equation

\[-\phi_k \left( \dot{\Omega} + v_E \cdot \nabla \Omega + \text{FLR} \right) = \nabla || J || + \nabla \cdot \left( \frac{c}{B^2 B_x} \nabla p \right) \]

Fourier mode \( k \)

vorticity \( \Omega = (n_e - n_i) e \)

currents: polarisation parallel diamagnetic

free energy: source in pressure equation, transfer in to vorticity equation

pathways: over parallel dynamics or toroidal compression

between modes within ExB energy —— nonlinear advection

direct, in–context measurement of physical mechanism supporting turbulence

(B Scott Phys Plasmas 2000)
Nonlinear Saturation

basic feature of any instability — transition to turbulence

linear drive (n) —> linear growth
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(B Scott Phys Plasmas 6/2005)
Vorticity Energetics —— Transition to Turbulence

turbulence imposes its own mode structure on dynamics

linear interchange mode —— balance between diamagnetic/parallel currents

turbulence —— emergence of nonlinear ExB vorticity advection

developed turbulence —— balance between polarisation/parallel currents

basic mechanism supporting eddies in turbulence differs from linear instability

(B Scott Plasma Phys Contr Fusion 2003)
Energy Transfer: electromagnetic turbulence

(S Camargo et al Phys Plasmas 1995 and 1996)
electromagnetic cases — notes

- nominal value of beta

\[ \hat{\beta} = \frac{4\pi p_e}{B^2} \left( \frac{qR}{L_\perp} \right)^2 = 1.75 \]

- introduction of “flutter” effects

\[ \nabla_\parallel = b^s \frac{\partial}{\partial s} - \hat{\beta}[A_\parallel, \ ] - \nabla_\perp^2 A_\parallel = J_\parallel \leftrightarrow \nabla_\parallel(p_e - \phi) \]

- as \( \hat{\beta} \) rises from zero, transport is relatively insensitive, then rises
  - linearly weak interchange/ballooning modes are available at low-\( k_y \)
  - these are driven via nonlinear cascade, through turbulence vorticity

- at high-\( k_y \) turbulence vorticity overcomes linear instabilities
  - rough rule of thumb: \( \omega_\ast > \gamma_I \) (diamag freq > MHD interchange growth rate)
  - edge conditions: \( k_y \rho_s > 2L_\perp/R \) since \( \rho_s/L_\perp \gtrsim L_\perp/R \)
synergy: all three transport channels vary together (squares: nominal case)

beta turnup due to long wavelength nonlinear transfer (dashes: 2 × resolution)

in both cases sensitivity is due to nonadiabatic electrons
Suppression of Turbulence by Flows
(Biglari Diamond Terry, Phys Fl B 1991)

edgies tilted into energy-losing relationship to flow vorticity

--- same process as in self generation
Sensitivity to Externally Imposed ExB Shear

standard L-mode cases with ITG gradients: $L_T = 0.5 L_n$

- squares give value at zero shear
  - red/blue/green lines give constant/cos/sech2 profiles for applied vorticity

- rolloff is slow, no steeper than $Q \sim (V')^{-1}$

- max suppression is only about a factor of 4
Main Points — Transport Scaling

- Trends follow nonlinear, not linear, physics

- Effects of ion grad-T must be kept...
  - Accounts for nonlinearly driven longer wavelengths
  - Prevents cutoff of transport towards higher T and grad-T

- Trend with either grad-T or beta always monotonically upward

- Shear flow suppression too weak to overcome beta scaling

no L-to-H transition in fully developed turbulence in local models
Zonal Flow, Toroidal Compression


Zonal Flow, Toroidal Compression

zonal flow

compression at top divergence at bottom

pressure sideband

zonal flow exchanges conservatively with pressure sideband

--> transfer pathway, equipartition
Energy Transfer: flows and currents

- Ion dissipation
- Transport
- MHD effects
- Reynolds stress
- 2-fluid effects
- Adiabatic compression
- Diamagnetic compression
- P–S current
- Resistivity

Coupling to Zonal Flows

turbulence regulated by flows, regulated by toroidal compression

eddy Reynolds stress $\rightarrow$ energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed

toroidal compression $\rightarrow$ energy loss channel to pressure, turbulence

entire system in self regulated statistical equilibrium (turb, flows, mag eq)

Including the Self-consistent Profile Evolution

- allow the turbulence advection (mixing) to evolve the profile
  - here, “profile” is the same as “zonal component”

- now the profile is part of the dependent variable
  - it is acted upon by magnetic curvature (toroidal compressibility of drifts)

- hence the “neoclassical equilibrium” is necessarily a part of the evolution
  - flow balance: zonal flows, geodesic curvature coupling,
    nonlinear transfer to turbulence \(\rightarrow\) zonal flow saturation
  - current balance: Pfirsch-Schlüter current, Shafranov shift

\[
\frac{\partial}{\partial x} \langle A_\parallel \rangle \rightarrow \delta \frac{1}{q} \quad \langle A_\parallel \cos s \rangle \rightarrow \text{Shafranov shift}
\]

all of the above must now be carried self consistently
Incorporation of Magnetic Equilibrium

toroidal equilibration current <-> Shafranov shift

\[ \nabla B \]

P–S current equilibrates toroidal diamagnetic compression

Ampere’s Law --> “Pfirsch–Schlueter magnetic field” --> toroidal shift

current stays in moment variables, magnetic field in coordinate metric
Global Electromagnetic Gyrofluid (GEM):

- turbulence and transport (profile + disturbances)
- self consistent magn eq, geometry (Pf–Sch currents $\rightarrow$ Shafranov shift)

L–Mode Base Case (ASDEX Upgrade generic)
- correct mass ratio, gyroradius
- closed/open flux surfaces, separatrix topology

(B Scott Contrib Plasma Phys 2006)
A Typical Burst Event

- edge
- scrape-off layer (SOL)
- last closed flux surface (LCFS)
- symmetry axis
- major/minor radii

$n_e(r, \theta)$
$t = 235.0$

$\Delta = 0.214$
Scale Separation

- turbulence vorticity scales with $c_s/L_\perp$, velocity with $c_s(\rho_s/L_\perp)$
- transport flux scales with $c_s(\rho_s/L_\perp)^2$, diffusivity with $c_s\rho_s^2/L_\perp$
- this is called “gyro-Bohm” and arises in general from $\rho_s \ll L_\perp$
- edge layer confinement time scales with $L_\perp^2/D$ or $(L_\perp/c_s) \times (L_\perp/\rho_s)^2$
- for edge (not SOL) turbulence this is about 1 msec with $L_\perp/\rho_s \gtrsim 50$

it is vital to get this correct in a computation since the turbulence/equilibrium crosstalk depends on it
Scale Separation Look and Feel

electromagnetic core cases with $a/\rho_s$ of 50, 100, and 200, non-axisymmetric part

- if you can see the eddies on a global plot they’re too large!
- in the edge you have $L_\perp/\rho_s < 100$ but $2\pi a/q > 10^3 \rho_s$
Ion Flow Sideband Divergences — Small Case

- flow divergence pieces do not balance
Ion Flow Sideband Divergences — Medium Case

- flow divergence pieces almost balance

\[ \nabla v \sin(\theta(r_a)) \]

\[ \nabla v_\perp \sin(\theta(r_a)) \]

\[ \nabla v_\parallel \sin(\theta(r_a))^\perp = 300.0 \]

\[ \nabla v \sin(\theta(r_a)) r_a \]

\[ \phi(r_a) \]

\[ W_i(r_a) \]
Ion Flow Sideband Divergences — Nominal Case

- flow divergence pieces balance closely
Scale Separation and the Profile Decay Rate

- profile (zonal component ion thermal energy) decay for the three cases
Scale Separation and the Spectrum

- density and vorticity spectra for the three cases

- ion heat source and sink spectra for the three cases
Scale Separation in the Edge

- radial extent is narrow, channeled by finite extend of the region where \( c_s/L_\perp > V_e/qR \)

\[
\hat{\mu} = \frac{m_e}{M_i} \left( \frac{qR}{L_\perp} \right)^2 > 1 \quad \text{in edge} \quad L_x \sim 50, 100 \times \rho_s
\]

- extent in drift angle is very large: low \( T_e \rightarrow \) large \( a/\rho_s \)

\[
a \sim 10^3 \times \rho_s \quad \quad L_y = 2\pi a/q \sim 2 \times a
\]

- typical extent for full flux-surface case in medium-sized tokamak

\[
L_x = 128 \rho_s \quad \quad L_y = 2048 \rho_s \quad \quad L_s = 2\pi qR
\]

- typical grid (2\( \rho_s \)-resolution) (no field aligning: \( N_\theta \sim N_\zeta/2 \times \Delta q \) and 16 \( \rightarrow \) 2048)

\[
N_x \times N_y \times N_s = 64 \times 1024 \times 16
\]
Between Bursts

\[ n_e(r, \theta) \]

- LCFS boundary relatively sharp
- despite robust > 10% fluctuations

\[ \Delta = 0.217 \]

During a Burst

\[ n_e(r, \theta) \]

- much more activity into SOL
- medium wavelength structures
- source of activity is edge region

\[ \Delta = 0.214 \]
Flux Temporal Behaviour

\[ F_e(x, t) \]

\[
\begin{align*}
\Delta &= 0.152\text{E}-01 \\
\Delta &= 0.205\text{E}-01 \\
\Delta &= 0.192\text{E}-01 \\
\Delta &= 0.209\text{E}-01 
\end{align*}
\]
Zonal Profiles between Bursts

- electrostatic potential shows nominal shear layer at LCFS ($r_a = 1$)
Ion Flow Sideband Divergences between Bursts

\[ \nabla v_E \sin \theta (r_a) \]

\[ \nabla v_\perp \sin \theta (r_a) \]

\[ \nabla v_\parallel \sin \theta (r_a)^{360.0} \]

\[ \nabla v \sin \theta (r_a) \]

\[ \phi (r_a) \]

\[ W_i (r_a) \]
Spectra between and during Bursts

- amplitudes/energies (left) and fluxes (right), between (top) and during (bottom)
Burst Notes

- electron/ion heat flux variation a factor of about 3
- bursts are strong events but do not completely destroy the neoclassical equilibrium
  - no “new mode” is involved
- edge/SOL transition is sharp, about 10 to 15 $\rho_s$
- vorticity spectrum always reaches to $k_\perp \rho_i > 1$ since if $T_i \sim T_e$ then $\rho_i \sim \rho_s$
- capture of burst phenomenology requires full scale separation, entire flux surface
  - fluxtube cases give “too clean,” too strong bursts (quasiperiodic, factor of 10)
- long-wavelength range $0.01 < k_y \rho_s < 0.1$ necessary as nonlinear energy-dump range
- fluxtube cases can study basic turbulence character
  - but not the self-consistent interaction with neoclassical equilibrium
**Edge versus Core**

- main parameter differences are $\rho_s/L_x$ and $L_y/L_x$ and $R/L_T$
  - edge: $\hat{\mu} > 1$, core: $\hat{\mu} < 1$, following $c_s/L_T$ versus $V_e/qR$ and hence $R/L_T$ (> 50)

- in the edge, electron dynamics is strongly nonadiabatic
  - nevertheless, adiabatic coupling is still strong

- hence neither simplified “adiabatic” or “hydrodynamic” or “MHD” models apply

- spectral ranges of free energy, the fluxes, and vorticity separate
  - dynamics occupies full spectrum, all scales $\rho_s$ to several $L_T$ are involved

- relevance of underlying nonlinear instability physics
  - some strong linear modes are wiped out by native turbulence: $\omega_{\text{rms}} > \gamma_L$
  - weak long-wave linear modes become important either as sinks or as secondary drive
    (e.g., TAE, reconnection, ballooning)
  - a significant fraction of free energy resides in linearly damped modes
    (e.g., dissipative shear Alfvén waves)
  - rule of thumb on relevance of instability: $\gamma_L > \omega_*$ for that $k_y$

- consequences of parameter regime: $k_y\rho_s > \sqrt{L_T/R}$ over most of the drive range
Relevance Range for Linear Instabilities

dispersion space bounded by ideal interchange and diamagnetic rates

if the linear growth rate is above the red line then the instability is relevant
usually, this is not the case anywhere in the spectrum (unless: MHD threshold)
this situation is a direct consequence of very large $R/L_T >> 1$ in the edge

Comparison -- Fluctuation Statistics

Probability distribution of cross phase for each Fourier mode

Unified spectrum, phase shifts between 0 and \( \pi/4 \), in code and TJK experiment

Basic signature of drift wave mode structure (parallel current dynamics)

wavelet analysis of fluctuation induced transport in code and TJK experiment
both results show same phenomenology: regime break in spectrum
evidence of nonlinear cascade overcoming drive?

(N Mahdizadeh et al Phys Plasmas 2004)
Nonlinear Free Energy Cascade

direct cascade

\[ \rightarrow \text{nonlinear drive at small scales} \]
\[ \implies \text{passive scalar regime} \]

frequency/scale correlation

matches with frequency break

evidence for onset of
passive scalar regime
The EFDA Integrated Modelling Effort (TF–ITM)

coordinate and establish standards for European codes in all categories

wide effort led by P Strand

Project 4 – instabilities, transport, turbulence

currently: cross–benchmarking on standard cases

global models automatically face the neoclassical equilibrium

separate issues: neoclassical equilibrium, and then transport

currently:

global core benchmarks on Cyclone base case

local and global edge benchmarks on L–mode base case

incorporation of trapping effects in fluid codes (may be hopeless)
local fluid vs gyrofluid drift-Alfvén
edge, collisional, cold-ion electromagnetic, fluxtube, saturated

Risø TYR (blue), Jülich ATTEMPT (green), GEM (red), DALF3 (pink)
Main Points

basics of energetics a central theme for physical understanding

essence of the physics of edge turbulence is nonlinear

scales separate for different parts, linear modes wiped out, character changes

coupling of turbulence to flows extends to the magnetic equilibrium

self consistency: do the magnetic background inside the turbulence model

new physics themes:

✦ global electromagnetic computation

✦✦ stable reconnection and equilibration currents

incorporation of trapping effects in fluid codes (may be hopeless)

✦✦ nonlocal gyrofluid field theory → edge/core transition

one should expect surprises affecting design of high performance devices