Nonlinear evolution of pressure gradient driven modes and anomalous transport in plasmas

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1. Formulation: MHD
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MHD equations

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \nu \Delta \mathbf{v} \]

\[ \frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = \kappa \Delta p \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j}) \]

\[ \mu_0 \mathbf{j} = \nabla \times \mathbf{B} \quad \nabla \cdot \mathbf{B} = 0 \]
Fluid dynamics equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho g z + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \gamma T \frac{\partial \mathbf{v}}{\partial t} + \nu \nabla^2 T$$

$$p = p(\rho, T)$$

with the gravitational field in the negative z direction.
Rayleigh-Taylor instability

\[
\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = 0
\]
\[
\nabla \cdot \mathbf{v} = 0
\]
\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho g \mathbf{z}
\]

incompressibility is assumed
Rayleigh-Taylor instability

often analyzed for two separate fluids

\[ \rho = \rho_1 \quad \text{or} \quad \rho_2 \]
\[ \nabla \cdot \mathbf{v} = 0 \]
\[ \rho_i \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho_i g \mathbf{z} \]
Rayleigh Bénard convection

\[ \nabla \cdot \mathbf{v} = 0 \]

\[ \rho_0 \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho g \hat{z} + \nu \Delta \mathbf{v} \]

\[ \rho(T) = \rho_0 \left[ 1 - \alpha (T - T_0) \right] \]

\[ \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \Delta T \]

Boussinesq approximation
Rayleigh Bénard convection

2D convection: rolls

\[ T = T_1 < T_0 \]
\[ T = T_0 \]

Side view

Top view

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http://www.esam.northwestern.edu/~riecke/Vorlesungen/412/1999/rb.html
http://hmf.enseeiht.fr/travaux/CD0001/travaux/optmfn/h1/01pa/hyb72/rb/rb.htm
Rayleigh Bénard Convection

3D convection: hexagons (with some defects)

top views
Rayleigh Bénard Convection

3D convections: more complicated patterns can appear
Spirals and some related patterns

top views
Defect chaos in Rayleigh-Benard convection

Stephen Morris at the University of Toronto  http://cnls.lanl.gov/~nbt/movies.html
Rayleigh Bénard convection
Equilibrium Solution

\[ \nabla \cdot \mathbf{v} = 0 \]

\[ \rho_0 \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho g \hat{z} + \nu \Delta \mathbf{v} \]

\[ \rho(T) = \rho_0 \left[ 1 - \alpha (T - T_0) \right] \]

\[ \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \Delta T \]

Set \( \mathbf{v} = 0, \quad \frac{\partial}{\partial t} = 0, \quad z \) dependent only

\( \text{(with} \quad \nu = \kappa = 0) \)
Rayleigh Bénard convection
Equilibrium Solution

$$\bar{T} = T_0 - \beta z, \quad \beta = \frac{T_0 - T_1}{d}$$  \hspace{1cm} \text{(temperature gradient)}

$$\bar{p} = p_0 + \rho_0 g z \left(1 + \frac{\alpha \beta}{2} z\right)$$
Rayleigh Bénard convection equations for perturbation

By setting \( \theta = T - \bar{T}(z) \) \( \tilde{p} = p - \bar{p}(z) \)

\( \nabla \cdot \mathbf{v} = 0 \)

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho_0} + g \alpha \theta z + \nu \Delta \mathbf{v}
\]

we have rewritten

\( \frac{\nu}{\rho_0} \rightarrow \nu \)

\( \frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \beta w + \kappa \Delta \theta \)

\( w = \mathbf{v} \cdot \mathbf{z} \) (i.e., z component of \( \mathbf{v} \))

\( \theta(t, x, y, 0) = \theta(t, x, y, d) = 0 \) (Boundary conditions)
Rayleigh Bénard convection

Normalized equations

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + R \theta \mathbf{\hat{z}} + \Delta \mathbf{v} \]

\[ \frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \frac{1}{\nu} \nabla^2 \theta - \frac{1}{P} \frac{\partial p}{\partial z} \]

By setting

\[ \begin{align*}
    &\mathbf{v} \\
    &\theta \\
    &p \\
    &\rho \ll
\end{align*} \]

Rayleigh number

Prandtl number

\[ (x, y, z) \rightarrow \tilde{d}(x, y, z) \]

\[ \begin{align*}
    &\mathbf{v} \\
    &\theta \\
    &p \\
    &\rho \\
    &\nu \\
    &\kappa \\
    &\tilde{d} \\
    &t
\end{align*} \]
Consider only 2D solutions:

\[ \mathbf{v} = \nabla \phi(y, z) \times \hat{x} = \left( 0, \frac{\partial \phi}{\partial z}, -\frac{\partial \phi}{\partial y} \right) \]

\[
\begin{cases}
\frac{d}{dt} \Delta \phi = -R \frac{\partial \theta}{\partial y} + \Delta^2 \phi \\
\frac{d\theta}{dt} = -\frac{1}{P} \frac{\partial \phi}{\partial y} + \frac{1}{P} \Delta \theta
\end{cases}
\]
Rayleigh Bénard convection
Stability Analysis

\[ \gamma_c^2 = \frac{\pi^2}{2} \]

\[ R_c = \frac{27}{4} \pi^4 \]

Rayleigh number
Rayleigh Bénard convection

Heat Transfer

\[ Nu = \frac{\text{total heat transfer}}{\text{heat transfer at rest}} = \frac{H}{\kappa \rho_0 c_p (T_0 - T_1)/d} \]

Nusselt Number

Fig. 2.6. Some experimental results on the heat transfer in various fluids in various containers. The Nusselt number is plotted against the Rayleigh number; ○ water; + heptane; × ethylene glycol; ● silicone oil AK 3; ▲ silicone oil AK 350; Δ air; □ mercury. After Silveston 1958 and Rossby 1969.

From “Hydrodynamic stability” (Drazin & Reid)
Resistive interchange mode

\[
\rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla p + j \times B + \nabla \cdot \Pi
\]

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B - \eta j) \quad \mu_0 j = \nabla \times B \quad \nabla \cdot B = 0
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0
\]

\[
\frac{\partial p}{\partial t} + v \cdot \nabla p + \gamma p \nabla \cdot v = \left( \gamma - 1 \left[ \eta j \cdot j + \nabla \cdot (\kappa \nabla T) - \Pi : \nabla v \right] \right)
\]

\[
p = \rho T
\]

\[
\Pi = -3 \mu_\parallel \left( \frac{\hat{b} \hat{b}}{3} - \frac{1}{3} I \right) \lambda - \mu_\perp \sigma \quad \lambda = \hat{b} \cdot \left[ \left( \hat{b} \cdot \nabla \right) v \right] - \frac{1}{3} \nabla \cdot v
\]

\[
\sigma_{ij} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot v
\]
Resistive interchange mode

Equations for the mean fields

$$\nabla p_0 = j_0 \times B_0$$
$$\frac{\partial B_0}{\partial t} = \nabla \times (u_0 \times B_0 + \varepsilon - \eta j_0) \quad \mu_0 j_0 = \nabla \times B_0 \quad \nabla \cdot B_0 = 0$$
$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot (\rho_0 u_0) = 0$$
$$\frac{\partial p_0}{\partial t} + u_0 \cdot \nabla p_0 + \gamma p_0 \nabla \cdot u_0 = \left( \gamma - 1 \left[ \eta j_0 \cdot j_0 + \nabla \cdot (\kappa \nabla T_0) - \nabla \cdot (p_0 \langle s_1 v_1 \rangle) + f \right] \right)$$

$$u_0 = v_0 + \left( \frac{1}{\rho_0} \right) \langle \rho_1 v_1 \rangle \quad s = \left[ \frac{1}{(\gamma - 1)} \right] \log \left( \frac{p}{\rho^\gamma} \right)$$
$$\varepsilon = \langle v_1 \times B_1 \rangle - \left( \frac{1}{\rho_0} \right) \langle \rho_1 v_1 \rangle \times B_0$$
$$f = -\nabla \cdot \left( \left( p_1 + B_0 \cdot B_1 \right) v_1 - \left( v_1 \cdot B_1 \right) B_0 + \left( \frac{1}{2} \right) \rho_0 v^2 \cdot v \right)$$
Resistive interchange mode

Equations for fluctuating fields

\[ \zeta |^0 b |^0 b = q \quad (\Delta |^0 b) q - \Delta \equiv ^T \Delta \]

\[ q^\Lambda - q \times \phi ^T \Delta = i \Lambda \]

\[ q \times i d ^T \Delta - q (\nabla ^T \Delta) - = i \mathcal{I} \]

\[ q i d - q \times \nabla ^T \Delta = i b \]
Resistive interchange mode

Equations for fluctuating fields

\[
\begin{align*}
\frac{d \tilde{A}}{d \tau} &= \frac{\partial \tilde{\phi}}{\partial \Theta} + \frac{1}{R} \Delta_\perp \tilde{A} \\
\frac{d}{d \tau} \Delta_\perp \tilde{\phi} &= \frac{\partial}{\partial \Theta} \Delta_\perp \tilde{A} + \{ \tilde{A}, \Delta_\perp \tilde{A} \} - \frac{\partial \tilde{p}}{\partial y} + M \Delta_\perp^2 \tilde{\phi} \\
\frac{d \tilde{p}}{d \tau} &= -D \frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_\perp \tilde{p} \\
\frac{d \tilde{v}}{d \tau} &= \frac{\partial \tilde{p}}{\partial \Theta} + \{ \tilde{A}, \tilde{p} \} + D \frac{\partial \tilde{A}}{\partial y} + M \Delta_\perp \tilde{v} \\
\frac{d}{d \tau} &= \frac{\partial}{\partial \tau} + \{ \phi, \ \} \\
D &\propto -p'_0 = -\left. \frac{dp_0}{dr} \right|_{r=r_0}
\end{align*}
\]
Resistive interchange mode

Normalization

\[ x = \left| \sigma \right|^{1/2} \left[ \left( r - r_0 \right) / r_0 \right] \]

\[ y = \left( B_\theta / r_0 B \right) \left| \sigma \right|^{1/2} \left[ z - \mu(r) \theta \right] \]

\[ \tilde{\theta} = \left| \sigma \right| \theta \]

\[ \tau = \left( B_\theta \left| \sigma \right| / r_0 \sqrt{\rho_0} \right) t \]

\[ \mu(r) = \frac{r B_z}{B_\theta} \quad \sigma = \frac{B_\theta \mu'}{B} \]

parameters are all evaluated at \( r = r_0 \)
Resistive interchange mode

\[ \frac{d \tilde{A}}{d \tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R} \Delta \tilde{A} \]

\[ \frac{d}{d \tau} \Delta \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}} \Delta \tilde{A} + \{ \tilde{A}, \Delta \tilde{A} \} - \frac{\partial \tilde{p}}{\partial \tilde{y}} + M \Delta^2 \tilde{\phi} \]

\[ \frac{d \tilde{p}}{d \tau} = -D \frac{\partial \tilde{\phi}}{\partial \tilde{y}} + \chi \Delta \tilde{p} \]

\[ \frac{d}{d \tau} = \frac{\partial}{\partial \tau} + \{ \phi, \ \} \]

\( D \propto -p'_0 = \left. -\frac{dp_0}{dr} \right|_{r=r_0} \)
Resistive interchange mode

electrostatic limit

\[
\frac{d \tilde{A}}{d\tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R} \Delta_{\perp} \tilde{A} \n\]
\[
\frac{d}{d\tau} \Delta_{\perp} \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}} \Delta_{\perp} \tilde{A} + \{ \tilde{\Delta}, \Delta_{\perp} \tilde{A} \} - \frac{\partial \tilde{p}}{\partial y} + M \Delta_{\perp}^2 \tilde{\phi} \n\]
\[
\frac{d \tilde{p}}{d\tau} = -D \frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_{\perp} \tilde{p} \n\]

\[
D \propto -p_0' = -\left. \frac{dp_0}{dr} \right|_{r=r_0} \n\]
Resistive interchange mode

electrostatic limit

\[
\frac{d}{d\tau} \Delta_\bot \tilde{\phi} = -R \frac{\partial^2}{\partial \tilde{\Theta}^2} \tilde{\phi} - \frac{\partial \tilde{p}}{\partial y} + M \Delta^2_\bot \tilde{\phi}
\]

\[
\frac{d \tilde{p}}{d\tau} = -D \frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_\bot \tilde{p}
\]
Resistive interchange mode

electrostatic limit

\[ \Delta \phi = -R \frac{\partial^2 \phi}{\partial y^2} + M \Delta^2 \phi \]

\[ \Delta = - \frac{\partial \theta}{\partial y} + \Delta^2 \phi \]

2D Rayleigh Bénard Convection

\[ \frac{d}{dt} \Delta \phi = -R \frac{\partial \phi}{\partial y} + \Delta^2 \phi \]

\[ \frac{d}{dt} \Delta \theta = - \frac{1}{P} \frac{\partial \theta}{\partial y} \]
Resistive interchange mode

Alfvén wave

\[ d\tilde{A} = \frac{1}{R} \frac{\partial\tilde{\phi}}{\partial\tilde{\theta}} \]

\[ \frac{d}{d\tau} \Delta_{\perp} \tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}} \left( \Delta_{\perp} \tilde{A} \right) + \frac{T}{R} \Delta_{\parallel} \tilde{A} \]

\[ \frac{d}{d\tau} \tilde{\phi} = \frac{\partial}{\partial\tilde{\theta}} \tilde{\phi} + \Delta_{\parallel} \tilde{\phi} \]

\[ \frac{d\tilde{p}}{d\tau} = -D \frac{\partial\tilde{\phi}}{\partial\tilde{y}} + \Delta_{\parallel} \tilde{p} \]
Resistive interchange mode

Tearing mode

\[
\frac{d \tilde{A}}{d \tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R} \Delta_{\perp} \tilde{A}
\]

\[
\frac{d}{d \tau} \Delta_{\perp} \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}} \Delta_{\perp} \tilde{A} + \{ \tilde{A}, \Delta_{\perp} \tilde{A} \} - \frac{\partial \tilde{p}}{\partial y} + M \Delta_{\perp}^2 \tilde{\phi}
\]

\[
\frac{d \tilde{p}}{d \tau} = -D \frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_{\perp} \tilde{p}
\]
Resistive interchange mode

bifurcation analysis

\[
\frac{d \tilde{A}}{d \tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\theta}} + \frac{1}{R} \Delta_{\perp} \tilde{A}
\]

\[
\frac{d}{d \tau} \Delta_{\perp} \tilde{\phi} = \frac{\partial}{\partial \tilde{\theta}} \Delta_{\perp} \tilde{A} + \{ \tilde{A}, \Delta_{\perp} \tilde{A} \} - \frac{\partial \tilde{p}}{\partial y} + M \Delta_{\perp}^{2} \tilde{\phi}
\]

\[
\frac{d \tilde{p}}{d \tau} = -D \frac{\partial \tilde{\phi}}{\partial y} + \chi \Delta_{\perp} \tilde{p}
\]

Consider the case \( \frac{\partial}{\partial t} = 0 \) : steady state (i.e., mode saturation)
Resistive interchange mode

2D numerical simulation

$D = 0.2 \quad M = 1 \quad K = 5 \times 10^{-3}$

$D < 0.25$ : ideal stability limit by Suydam

Nusselt Number

$$Nu = 1 + p_0 \left< s_1 v_{1r} \right>/ -\kappa' T_0'$$
Resistive interchange mode

2D numerical simulation

Contours of

\( \tilde{\phi}(\tilde{x}, \tilde{y}) \)

\[ D = 0.2 \quad M = 1 \quad K = 5 \times 10^{-3} \]
bifurcation

From the obvious equilibrium \( \tilde{A} = \tilde{\phi} = \tilde{p} = 0 \) to another equilibrium in 2D:

\[
\frac{\partial}{\partial \tilde{\theta}} = \frac{\partial}{\partial z} + x \frac{\partial}{\partial y} \quad \rightarrow \quad x \frac{\partial}{\partial y}
\]

\[
\epsilon^2 = \frac{1}{2} \left\langle |\nabla_\perp \tilde{\phi}|^2 \right\rangle
\]

\[
\tilde{A} = \epsilon \hat{A} \quad \tilde{\phi} = \epsilon \hat{\phi} \quad \tilde{p} = \epsilon \hat{p}
\]
Resistive interchange mode
bifurcation analysis

\[ \varepsilon \{ \hat{\phi}, \hat{A} \} = \hat{x} \frac{\partial \hat{\phi}}{\partial \hat{y}} + \frac{1}{R} \Delta_{\perp} \hat{A} \]

\[ \varepsilon \{ \hat{\phi}, \Delta_{\perp} \hat{\phi} \} = \hat{x} \frac{\partial}{\partial \hat{y}} \Delta_{\perp} \hat{A} + \left\{ \hat{A}, \Delta_{\perp} \hat{A} \right\} - \frac{\partial \hat{p}}{\partial \hat{y}} + M \Delta^2_{\perp} \hat{\phi} \]

\[ \varepsilon \{ \hat{\phi}, \hat{p} \} = -D \frac{\partial \hat{\phi}}{\partial \hat{y}} + \chi \Delta_{\perp} \hat{p} \]
Resistive interchange mode
bifurcation analysis

\[ \hat{A} = \sum_{n=0}^{\infty} \hat{A}_n \varepsilon^n, \quad \cdots \]

\[ D = D_L + \sum_{n=0}^{\infty} D_n \varepsilon^n \]

\[
L \left( \begin{array}{c}
\hat{\phi}_0 \\
\hat{p}_0
\end{array} \right) =
\begin{pmatrix}
-R\tilde{x}^2 \frac{\partial^2 \hat{\phi}_0}{\partial \tilde{y}^2} - \frac{\partial \hat{p}_0}{\partial y} + M \Delta_{\perp}^2 \hat{\phi}_0 \\
\frac{\partial \hat{\phi}_0}{\partial \tilde{y}} - \left( \chi / D_L \right) \Delta_{\perp} \hat{p}_0
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix}
\]
Resistive interchange mode
bifurcation analysis

\[ D_1 = 0 \quad D = D_L + D_2 \varepsilon^2 + \cdots \]

\[
p_0 \langle s_1 v_{1r} \rangle = -\frac{B_\theta B^2 \sigma^2}{(\gamma - 1) \sqrt{\rho_0}} k \langle \hat{p}_{01} \hat{\phi}_{01} \rangle \frac{D - D_L}{D_2}
\]

Nusselt Number
\[ Nu = 1 + p_0 \langle s_1 v_{1r} \rangle / -\kappa_\perp T'_0 \]
Resistive interchange mode
bifurcation analysis and 2D nonlinear simulation

![Graph showing theoretical and simulated Nuerons (Nu) as a function of D.](image)
Resistive interchange mode
bifurcation analysis and 2D nonlinear simulation

\[ \mu - 1 \approx -10^{-2} \]
Resistive interchange mode

3D nonlinear simulation

\[ D = 0.15 \quad M = 1 \quad K = 6 \times 10^{-2} \]
Resistive interchange mode

3D nonlinear simulation

\[ D = 0.23 \quad M = 1 \quad K = 6 \times 10^{-2} \]
Resistive interchange mode
Comparison btw 2D and 3D nonlinear simulations

$M = 1$ $K = 6 \times 10^{-2}$
practical issues for resistive interchange modes and resistive ballooning modes

- strongly nonlinear evolution and turbulence
- effects of sheared flows on nonlinear modes
- flow generation from turbulence
- intermittent bursts of transport
- ...

Nonlinear evolution of tearing modes with multiple rational surfaces

Tearing mode equations

\[
\frac{d \tilde{A}}{d\tau} = \frac{\partial \tilde{\phi}}{\partial \tilde{\Theta}} + \frac{1}{R} \Delta_{\perp} \tilde{A}
\]

\[
\frac{d}{d\tau} \Delta_{\perp} \tilde{\phi} = \frac{\partial}{\partial \tilde{\Theta}} \Delta_{\perp} \tilde{A} + \{ \tilde{A}, \Delta_{\perp} \tilde{A} \} + M \Delta_{\perp}^2 \tilde{\phi}
\]

Furth, Killeen, & Rosenbluth, Phys. Fluids, 6, 459 (1963)
tearing mode

“m=1 tearing mode”, “resistive kink mode”

\[ \gamma = \left[ q'(r_s) k r_s^2 S \right]^{2/3} \tau_R^{-1} \propto \eta^{1/3} q'^{2/3} \]

\[ q(r) = \frac{r B_t}{R B_p} \]

\[ S = \tau_R / \tau_A \]

\[ \tau_R = \frac{a^2}{\eta} \]

\[ \tau_A = \frac{a}{V_A} \quad V_A = \frac{B_t}{\sqrt{\rho_0}} \]

mode structure

Alfvén transit time

r = r_s

0
Multiple $q=m/n$ resonant surfaces may occur with a small distance apart.

Advanced reversed-shear tokamak & two resonant surfaces near $q_{\text{min}}$.

Partial sawtooth collapse with $q_0<1$ and $q \sim 1$ in an annular region.

Multiple Tearing Modes (MTM) may become unstable.
Linear stability analyses:
growth rate vs. poloidal mode number

resistive $q=1$ Double Tearing Mode (DTM)

- Theory [Pritchett et al. 1980] and numerical results agree
- Broad spectrum, dominant high-$m$ modes

comparison between theory and numerical results yields semi-empirical estimate: $m_{\text{peak}} \approx m_{\text{trans}} + 1$
Linear stability analyses: mode structures

\[ m=1 \text{ double kink-tearing mode, } m>1 \text{ coupled tearing modes} \]
Nonlinear drive of $m=1$ mode (*fast trigger*)

Triple Tearing Modes (TTM) drive resistive kink

- Growth rate switches from $\gamma_{\text{lin}}(m=1)$ to about 2 $\gamma_{\text{lin}}(m_{\text{peak}})$
- Nonlinear drive local, but mode structure remains global
Sawtooth crash: Interaction with kink

Internal kink surrounded by MHD turbulence

• Direction of kink flow fluctuates, crash may be delayed
Sawtooth crash:  \textit{Partial reconnection}

Internal kink surrounded by MHD turbulence

- In some cases, the kink saturates and decays temporarily
Summary

- Gravitational driven instabilities of fluids: Rayleigh Taylor instability and Rayleigh Bénard instability (convection)
- Similar instability for magnetically confined plasmas: Resistive interchange instability (Resistive g-mode)
- Nonlinear saturation and bifurcation of resistive interchange instability
- Another example of nonlinear evolution and turbulence: Nonlinear evolution of double and triple tearing modes
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References
theorem

Suppose the range of linear operator $L$ is closed. Then the inhomogeneous equation

$$Lx = a$$

has a solution if and only if $a$ is orthogonal to all solutions $z$ of

$$L^*z = 0$$

with $L^*$ being the adjoint operator of $L$. 