Transport of Fast Particles in Turbulent Fields

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Motivation: Fast Particles

Fast particles in the Iter plasma:

- fusion-born $\alpha$ particles: 3.5 MeV, total power $\sim$ 100 MW
- injected neutrals: 1 MeV, total power $\sim$ 50 MW
Tracer Particles vs. Alfvén Eigenmodes

Eigenmodes
- fast ions on resonant surfaces
- destabilization of modes

Tracers
- turbulent thermal species
- fast ions are deflected

⇒ both mechanisms can play an important role in fast ion transport

(JET)

(Padberg et al.)
ASDEX Upgrade discharges with significant NBI heating:

\[ D_{\text{fast}} = 0 \]
\[ D_{\text{fast}} = 0.5 \frac{T_e^{3/2}(r)}{T_e^{3/2}(0)} \text{m}^2/\text{s} \]

⇒ need an efficient **radial transport mechanism**
to explain the observed profiles during/after the NBI phase
1 Fast Tracers in Turbulence

2 Plasma Microturbulence

3 Simulations with Fast Particles

Source & further reading:
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Characteristics of Turbulent Fields

**Structure formation:**

Turbulence: nondeterministic, need **statistical** approaches

- **Power law spectra:**

- **Kubo number:**
  
  \[ K = \frac{\tau_c}{\tau_\lambda} = \bar{v} \frac{\tau_c}{\lambda_c} \]

- **Correlations:**
  - time: \( \tau_c \)
  - length: \( \lambda_c \),
    
    but generally \( \lambda_x \neq \lambda_y \)
Fusion reactors: toroidal, poloidal magnetic field

**tokamak:**
\[ B(z) \sim (1 + \epsilon_t \cos z)^{-1} \]

**stellarator:**

**shear:**
\[ \hat{s} = \frac{(R_0/q_0)dq(r)}{dr} \]
Particle Motion in Tori

**pitch angle:** $\eta \equiv v_{\parallel}/v$

### Trapping

- **magnetic field:** inhomogeneous
  
  $(\partial B/\partial z \neq 0)$

- **$\eta \rightarrow 0$: mirror force**
  
  $F_{\text{mirr}} \propto \partial f/\partial v_{\parallel}$

  traps particle in magnetic well

- **banana orbit**

**Additionally:** toroidal

**precession drift** affects both trapped, passing particles
Decorrelation

- short times: ballistic, superdiffusive movement, before turbulent structure is felt
- if a particle is caught by a structure: subdiffusive movement, localized
- only if memory of previous movement (correlation) is lost: diffusive behavior

Diffusivity:

\[ D(t) = \frac{1}{2} \frac{d}{dt} \langle \Delta x^2(t) \rangle \]
Basic idea: if particle orbits move slowly through turbulent structures, orbits can be averaged over

- removes orbit time scale
- particles feel little of averaged-over turbulence
- predicts fast fall-off of diffusivity with particle energy
- turns out to be valid only in low-shear regimes of tokamaks

Conditions for validity:

\[ T_{\text{orbit}} \bar{V}_{\text{drift}} = T_{\text{orbit}} \frac{V_{\text{drift}}}{(2\sqrt{\pi} \Delta x / \lambda_c)^{1/2}} < \lambda_c \]

\[ T_{\text{orbit}} < \tau_c \]
### A Different Approach

If orbit averaging is valid:
- little influence of turbulent eddies on fast particles
- decorrelation through **time evolution** of turbulence

If orbit averaging is **not** valid \((T_{\text{orbit}} \bar{V}_{\text{drift}} > \lambda_c)\):
- particles leave eddie in a **single orbit**
- \(T_{\text{orbit}}\) is a good measure for the decorrelation time

**Note**: alternatively, parallel motion can decorrelate with \(\tau_{||}\)

Typically, however,

\[
T_{\text{orbit}} < \tau_{||} < \tau_c
\]
For electrostatic turbulence, take $E \times B$ drift velocity $v_E$, get

$$D_{fi,p}^{es}(E) \approx \frac{v_E^2 \lambda_V}{3 \eta^2} \left( \frac{E}{T_e} \right)^{-1}$$

$$D_{fi,t}^{es}(E) \approx \frac{1.73 v_E^2 \lambda_c \lambda_V \epsilon_t^{1/2}}{12 \pi^{1/2} \eta^2 (1 - \eta^2)} \left( \frac{E}{T_e} \right)^{-3/2}$$

**Passing** particles ($\eta \to 1$): for very large $E$, eventually $D_{fi,p} \propto E^{-3/2}$ when gyroaveraging becomes important

**Trapped** particles ($\eta \to 0$): particles drift farther during one $T_{orbit}$

$\Rightarrow$ orbit averaging breaks down more quickly
**Tracers in Magnetic Fields**

**Magnetic**: replace $v_E$ with $v_B = v_{\parallel} B_x / B_0$

(deflection on perturbed field lines: *magnetic flutter*)

$$D_{fi,p}^{em}(E) \approx \left( \frac{B_x}{B_0} \right)^2 \frac{\lambda_B}{3}$$

$$D_{fi,t}^{em}(E) \approx \frac{1.73(B_x/B_0)^2 \lambda_B^2 \epsilon^{1/2} \eta}{12\pi^{1/2}(1 - \eta^2)} \left( \frac{E}{T_e} \right)^{-1/2}$$

**Fluctuation strength**

$$\frac{B_x}{B_0} = C \frac{\beta}{\beta_{crit}} \frac{\rho_s c_s}{R_0}, \quad C \sim \mathcal{O}(0.1 - 1) \quad \text{(from simulations)}$$

$\Rightarrow$ no or only slow fall-off, at odds with orbit averaging

**Magnetic transport**: can produce $D_{fi} \sim 0.5 \text{ m}^2/\text{s}$, which is necessary for explaining ASDEX Upgrade results
Runaway Electrons: Motivation

Plasma disruptions

- sudden loss of confinement
- large electric fields
- can accelerate electrons very efficiently

**Fast electrons**: very low collisionality, $\nu_e \propto v^{-3}$

$\Rightarrow$ acceleration “without bounds” to $O(10 - 1000)$ MeV

*Previous theories*: overestimating radial diffusion of runaway electrons

(Tore Supra)
Runaway Electrons: Results

- **relativistic** treatment
  \( E \gg m_e c^2 \)
- typically, \( \eta \sim 1 \)
- only \( D_{em} \) of interest
  (more efficient than \( D_{es} \))

**Runaway electron diffusivity**

\[
D_{em} = \frac{2}{3} \left( \frac{B_x}{B_0} \right)^2 \frac{\lambda_B e B_0 R_0}{m_0} \frac{1}{\gamma_e}
\]

thus

\[
D_{em} = \propto E^{-1}
\]

\( \Rightarrow \) consistent with observations, but \( B_x^2 \) dependence can make quantitative comparisons difficult

- at higher energies,
  \( D_{em} \propto E^{-2} \)
  (gyroaveraging effects)
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Confinement and Anomalous Transport

Energy confinement time $\tau_E$ (as in $nT\tau_E \gtrsim 10^{21}$ keV s/m$^3$): governed by radially outward heat transport

Transport channels

- collisional (negligible)
- neoclassical (banana orbits)
- radiative
- MHD turbulent \textit{(but: equilibrium)}
- microturbulent

$\Rightarrow$ core confinement is limited by Larmor radius scale (kinetic) turbulence
The Gyrokinetic Framework

Kinetic (non-Maxwellian) phenomena generally require **full phase space** description via the Vlasov equation:

\[
\text{3D real space} + \text{3D velocity space} + \text{1D time}
\]

Gyrokinetics

To make computations feasible: **eliminate fast gyromotion**

\[
\langle \ldots \rangle = \int_{0}^{2\pi} \ldots \, d\theta
\]

⇒ going from 6D to 5D, speed-up by factor \(\mathcal{O}(10 - 100)\)
Gyrokinetic Vlasov Equation

\[
\frac{dg_j}{dt} = 0 = \frac{\partial g_j}{\partial t} + \frac{B_0}{B_0^*} \left[ \omega_n + \omega_{Tj} \left( v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \right] F_{j0} \frac{\partial \chi}{\partial y} -
\]

\[
- \frac{v_{Tj} v_\parallel}{JB_0} \frac{\partial G_j}{\partial z} + \frac{B_0}{B_0^*} \left( \frac{\partial \chi}{\partial y} \frac{\partial g_j}{\partial x} + \frac{\partial \chi}{\partial x} \frac{\partial g_j}{\partial y} \right) + \frac{B_0}{B_0^*} \frac{T_{j0} \left( 2v_\parallel^2 + \mu B_0 \right)}{q_j B_0}.
\]

\[
\cdot \left[ \kappa_x \frac{\partial G_j}{\partial x} + \kappa_y \frac{\partial G_j}{\partial y} - \kappa_x F_{j0} \left( \omega_n + \omega_{Tj} \left( v_\parallel^2 + \mu B_0 - \frac{3}{2} \right) \right) \right] -
\]

\[
- \frac{B_0}{B_0^*} \frac{T_{j0} v_\parallel^2}{q_j B_0^2} \beta \left( \sum_j n_{j0} T_{j0} \left( \omega_n + \omega_{Tj} \right) \right) \frac{\partial G_j}{\partial y} + \frac{v_{Tj} \mu}{2JB_0} \frac{\partial B_0}{\partial z} \beta \frac{\partial f_j}{\partial v_\parallel}
\]

(with the generalized potential \( \chi = \bar{\Phi}_j - v_{Tj} v_\parallel \bar{A}_{1||j} + T_{j0}/q_j \mu \bar{B}_{1||j} \), complemented by the field equations for \( \bar{\Phi}, \bar{A}_{1||}, \bar{B}_{1||} \))
Field Equations

\[ A_{1\parallel} = \left( \sum_j \frac{\beta}{2} q_j n_{j0} v_{Tj} \pi B_0 \int v_{\parallel} J_0(\lambda_j) g_j dv_{\parallel} d\mu \right) \times \]

\[ \times \left( k_{\perp}^2 + \sum_j \frac{\beta q_j}{m_j} n_{j0} \pi B_0 \int v_{\parallel}^2 J_0(\lambda_j) F_{j0} dv_{\parallel} d\mu \right)^{-1} \]

\[ \Phi = \left( \sum_j q_j n_{j0} \pi B_0 \int J_0(\lambda_j) g_j dv_{\parallel} d\mu \right) \times \]

\[ \times \left( k_{\perp}^2 \lambda_D^2 + \sum_j \frac{q_j}{T_{j0}} n_{j0} (1 - \Gamma_0(b_j)) \right)^{-1} \]

(small plasma pressure \( \beta \) limit; otherwise, need more complicated, coupled \( \Phi-B_{1\parallel} \) system)
Microturbulence is driven by free energy sources, mostly temperature and density gradients.

**ITG modes**
- ion temperature gradient driven
- zonal flows

**ETG modes**
- electron temperature gradient driven
- streamers
### Trapped Electron Mode (TEMs)

- $B_0$ inhomogeneity: banana orbits
- $\nabla T_e$ or $\nabla n_e$ driven
- Electron mode on $\sim \rho_i$ scales

### Other modes

- **Microtearing**
  - large toroidal scales
  - Restructures magnetic field
  - $\nabla T_e$ driven

- **Kinetic Ballooning**
  - Kinetic version of MHD ballooning mode
  - Strict $\beta$ limit
  - $\nabla T_i$ is important

- Less well-understood modes
Electromagnetic Effects

**Electromagnetic**: finite plasma pressure $\beta$, leading to magnetic field fluctuations $A_\parallel \rightarrow B_x, B_y$

**Consequences of high pressure**
- essential for high *reaction rates*
- important for *bootstrap fraction*, semi-continuous operation of tokamaks
- has complicated impact on microturbulent instabilities (stabilizing/destabilizing)
- eventually leads to loss of confinement at $\beta_{\text{crit}}$ (ballooning threshold)
- responsible for new heat/particle transport channels: *electromagnetic flutter* due to perturbed field

*area of ongoing research*
Magnetic Transport

Model for electron heat transport along **perturbed field lines** (Rechester & Rosenbluth 1978):

Test particle transport model

\[ q_{e\parallel} = -n_{e0}\chi_{e\parallel} \left( \frac{dT_{e\parallel}}{dz} + \frac{B_x}{B_{\text{ref}}} \frac{dT_{e\parallel}}{dx} + \frac{B_x}{B_{\text{ref}}} \frac{dT_{e0}}{dx} \right) \]

Turbulence: third term dominates first, second \( \Rightarrow Q_{e}^\text{em} \propto \beta^2 \)

Heat diffusivity:

\[ \chi_{e}^\text{em} = q_0 R \left( \frac{T_e}{m_e} \right)^{1/2} \langle (B_x/B_{\text{ref}})^2 \rangle \]

\( \Rightarrow \) magnetic transport can become comparable with electrostatic transport at reactor-relevant \( \beta \)
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Numerical Approach

Solving gyrokinetic Vlasov equation requires advanced code: **Gyrokinetic Electromagnetic Numerical Experiment**

**The GENE Code**
- runs on up to $10^5$ cores
- initial/eigenvalue code
- advanced collision operator
- high-β physics
- tokamaks and stellarators
- worldwide user base
- available to researchers: gene.rzg.mpg.de

**GENE Diagnostics Tool**
- post-processing suite
- standard analyses, e.g.: spectra/profiles/contours
- advanced diagnostics: 3D rendering textures, field line integrator
Passive Species Simulations

Active \((a)\) vs. passive \((p)\) particle species

Vlasov equation used for all species

\[
\frac{\partial g_a}{\partial t} = f(a, \Phi, A_{1||}) \quad \text{and} \quad \frac{\partial g_p}{\partial t} = f(p, \Phi, A_{1||})
\]

while the field equations

\[
\Phi \propto \sum_a \int g_a dv_\parallel d\mu \quad \text{and} \quad A_{1\parallel} \propto \sum_a \int g_a dv_\parallel d\mu
\]

only depend on \(a\)

**Here:** passive fast ion species with \(T_{fi} = 30 - 100T_e\)

\(\Rightarrow\) significant impact on numerical time step!

**Note:** \(fi\) species has Maxwellian distribution, but analyses divide by \(F_{fi0}\)
Simulation Results

\[ \frac{D_{ri}}{(\rho_s c_s)^2 / R_0} \]

\[ \propto E^{-1.5} \]

\[ \propto E^{-1} \]

\[ \propto E^{-0.5} \]
General Geometry

Previous results: simple \( \hat{s}-\alpha \) geometry (circular flux surfaces), but realistic equilibria may affect fast particle diffusion. Experimental results available for ASDEX Upgrade, DIII-D ⇒ take respective discharge geometries.

Note: shaping affects fast ions also via turbulence (e.g.: \( \lambda_c, B_x \)) work in progress . . .
**Weak shaping:**

- Steep falloff,
- \( D_{em} \sim D_{em} \) at rather high \( E \)

**Strong shaping:**

- Lower intersection energy

\[ 3 \text{D} \sim \frac{\text{D} + \text{D}^2}{2} \]

Very high fluxes at *nominal* gradients

\( \Rightarrow \) reduce by

\( \sim 25\%: Q_{\text{sim}} \sim Q_{\text{exp}} \)
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DIII-D

General geometry conclusions *(preliminary!)*

- $D_{em}$ not negligible
- Shaping increases influence of $D_{em}$
- Impact of $\eta$ needs to be considered

Electrostatic-magnetic intersection at low energies:

**Magnetic component important**
Gene database (ASDEX Upgrade, DIII-D, $s$-$\alpha$):

$$\Rightarrow \text{can use simple equations with experimentally available quantities:}$$

$$\frac{2}{3} \nu_E^2 \approx \chi_{\text{total}}^{\text{es}}$$

$$40 \left( \frac{B_x}{(\beta/\beta_{\text{crit}})} \right)^2 \approx \chi_{\text{total}}^{\text{es}}$$

Additionally: avoid $\lambda$s by using passive particle gauge
Summary I

- Fast particles: susceptible to **background turbulence**
- Recent ASDEX Upgrade experiments: significant **radial diffusion**
- Standard model Orbit Averaging cannot explain findings
- Decorrelation occurs through drift motion over a *single* orbit
- Magnetic diffusion of passing fast ions: no $E$ dependence, providing an **efficient transport mechanism**
- **Runaway electrons**: previous theories cannot explain suppressed diffusion
  relativistic treatment shows: $D \propto E^{-2}$ at large energies
  (gyroaveraging)
Background species: gradients drive turbulence on the scale of gyroradii—ITG, TEM, ETG, KBM, MT, . . .

Simulations:
- self-consistent turbulence from *active* species
- *passive* species evolve according to gyrokinetic Vlasov equation

Results show excellent agreement with theory, in particular no energy dependence of $D_{fi,p}^{em}$
Approach has already been used successfully to explain astrophysical phenomena (cosmic rays).

Building up database through simulation evaluation: convenient access to quantities required to obtain diffusivities, using only experimentally available data.

Impact of plasma shaping, pitch angle: work in progress.