Nonlinear Modelling of Fast Ion Driven Instabilities in Fusion Plasmas

SD Pinches
ITER Organization

The views and opinions expressed herein do not necessarily reflect those of the ITER Organization
Outline of Talk

• Introduction to fast ions and fast ion driven modes
• Overview of the HAGIS code
• Nonlinear modelling of fast ion driven instabilities
  – Growth and saturation
  – Multiple modes interacting
  – Pitchfork splitting
  – Frequency sweeping modes
  – Fishbones
  – Tornado modes
• Summary
ITER Mission

• The overall programmatic objective:
  − to demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes

• The principal goal:
  − to design, construct and operate a tokamak experiment at a scale which satisfies this objective

• ITER is designed to confine a Deuterium-Tritium plasma in which \(\alpha\)-particle heating dominates all other forms of plasma heating:

  \[ \Rightarrow \text{a burning plasma experiment} \]
ITER Mission

Physics:

• Produce a significant fusion power amplification factor \((Q \geq 10)\) in long-pulse operation \((300 – 500 \text{ s})\)

• Aim to achieve steady-state operation of a tokamak \((Q \geq 5, \leq 3000 \text{ s})\)

• Retain the possibility of exploring ‘controlled ignition’ \((Q \geq 30)\)

Technology:

• Demonstrate integrated operation of technologies for a fusion power plant

• Test components required for a fusion power plant

• Test concepts for a tritium breeding module
Burning plasma physics in ITER

• Access to plasmas which are dominated by $\alpha$-particle heating will open up new areas of fusion physics research, in particular:
  - confinement of $\alpha$-particles in plasma
  - response of plasma to $\alpha$-heating
  - influence of $\alpha$-particles on stability

• Experiments in existing tokamaks have already provided some positive evidence
  - ‘energetic particles’ (including $\alpha$-particles) are well confined in the plasma
  - ‘energetic particle’ populations interact with the background plasma and transfer their energy as predicted by theory
  - but ‘energetic particles’ can drive instabilities (Alfvén eigenmodes) - for ITER parameters at $Q=10$, the impact is predicted to be tolerable
ITER Baseline Reference Scenarios

- The set of DT reference scenarios in ITER is specified via illustrative cases in the *Project Requirements* ⇒ *Design Basis scenarios*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inductive Operation</th>
<th>Hybrid Operation</th>
<th>Non-inductive Operation</th>
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<tbody>
<tr>
<td>Plasma Current, $I_p$ (MA)</td>
<td>15</td>
<td>13.8</td>
<td>9</td>
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<tr>
<td>Safety Factor, $q_{95}$</td>
<td>3.0</td>
<td>3.3</td>
<td>5.3</td>
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<td>Confinement Time, $\tau_E$ (s)</td>
<td>3.4</td>
<td>2.7</td>
<td>3.1</td>
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<tr>
<td>Fusion Power, $P_{fus}$ (MW)</td>
<td>500</td>
<td>400</td>
<td>360</td>
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<tr>
<td>Power Multiplication, $Q$</td>
<td>10</td>
<td>5.4</td>
<td>6</td>
</tr>
<tr>
<td>Burn time (s)</td>
<td>300 – 500</td>
<td>1000</td>
<td>3000</td>
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</table>

In addition, a range of non-active (H, He) and D plasma scenarios must be supported for commissioning purposes to support rapid transition to DT operation.
As the alpha power rises in high-Q plasmas, the plasma will enter a novel regime

- Plasma behaviour dominated by $\alpha$-particle heating

⇒ Burning plasma regime
Sources of Energetic Particles

• Nuclear fusion
  – Isotropic slowing-down distribution
  – For DT fusion, $\alpha$-particle birth energy of 3.5 MeV

• Neutral beam injection (NBI)
  – Anisotropic slowing-down distribution
  – Well defined $E_b$

• Radio Frequency (RF)
  – E.g. Ion Cyclotron (ICRH)
  – No well defined characteristic energy
  – Anisotropic
## ITER Heating and Current Drive Systems

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>IC</th>
<th>EC</th>
<th>LH</th>
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<tr>
<td></td>
<td>Neutral Beam -1 MeV</td>
<td>Ion Cyclotron 40 – 55 MHz</td>
<td>Electron Cyclotron 170 GHz</td>
<td>Lower Hybrid ~5 GHz</td>
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<td></td>
<td><img src="image1.png" alt="Neutral Beam Diagram" /></td>
<td><img src="image2.png" alt="Ion Cyclotron Diagram" /></td>
<td><img src="image3.png" alt="Electron Cyclotron Diagram" /></td>
<td><img src="image4.png" alt="Lower Hybrid Diagram" /></td>
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<tr>
<td></td>
<td>33MW*</td>
<td>20MW*</td>
<td>20MW*</td>
<td>0MW*</td>
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<tr>
<td></td>
<td>+16.5MW#</td>
<td>+20MW#</td>
<td>+20MW#</td>
<td>+40MW#</td>
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<td>Bulk current drive limited modulation</td>
<td>Sawtooth control modulation &lt; 1 kHz</td>
<td>NTM/sawtooth control modulation up to 5 kHz</td>
<td>Off-axis bulk current drive</td>
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</tbody>
</table>

*Baseline Power  #Possible Upgrade
Fast Ion Orbits

Various natural frequencies associated with particle motion

Toroidal direction $\omega_\phi$

Fast ion trajectory

Projection of poloidally trapped ion trajectory

Poloidal direction $\omega_\theta$

Ion gyro-motion $\omega_{ci}$
Burning Plasmas

- New physics element in burning plasmas:
  - Plasma is self-heated by fusion alpha particles

ITER parameters

\[ v_{Ti} = 0.9 \times 10^6 \text{ m/s} \]
\[ v_{A} = 8 \times 10^6 \text{ m/s} \]
\[ v_{\alpha} = 12 \times 10^6 \text{ m/s} \]
\[ v_{Te} = 59 \times 10^6 \text{ m/s} \]
Alfvén waves and αs

Alfvén wave is *very* weakly damped by background plasma

Fusion products (αs) interact with Alfvén waves *much* better than thermal plasma

3.5 MeV

10 keV

10 keV

α

i

e
Loss of Fast Particles

• Loss of bulk plasma heating
  – Clearly unacceptable for an efficient power plant
• Damage to first wall
  – Can only tolerate losses of a few % in a reactor
Reasons for Loss

• Imperfections in confining magnetic field
  – Ripple due to finite number of field coils, TBMs, ELM coils

48 superconducting coils

<table>
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<tr>
<th>System</th>
<th>Energy GJ</th>
<th>Peak Field</th>
<th>Total MAT</th>
<th>Cond length km</th>
<th>Total weight t</th>
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<td>Toroidal Field TF</td>
<td>41</td>
<td>11.8</td>
<td>164</td>
<td>82.2</td>
<td>6540</td>
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<td>Central Solenoid</td>
<td>6.4</td>
<td>13.0</td>
<td>147</td>
<td>35.6</td>
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<td>Poloidal Field PF</td>
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<td>6.0</td>
<td>58.2</td>
<td>61.4</td>
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<td>Correction Coils CC</td>
<td>-</td>
<td>4.2</td>
<td>3.6</td>
<td>8.2</td>
<td>85</td>
</tr>
</tbody>
</table>

• Self-generated field imperfections
  – Collective instabilities
Wave Induced Losses in TFTR

• Specially designed experiments
  – Low field, $B_t = 1$ T
  – Deuterium NBI, $E_b(0D^2) = 100$ keV
  – $v_b \sim v_A$

• Modes observed for $P_{\text{NBI}} > 5$ MW

• Correlated with neutron reduction
  – Neutron yield dominated by beam-plasma reactions
  \[ \Rightarrow \text{Fast ion loss} \]

Alfvén Waves

- Analogous to waves on a string
  - \( v_A = \frac{B}{\sqrt{\mu_0 m_i n_i}} \)
  - \( \omega^2 = \omega_A^2(r) = k_{||}^2 v_A^2(r) \)
  - Form continuum of waves in inhomogeneous plasma
  - Damped due to phase mixing with neighbouring waves

\[
\omega^2 \approx \frac{k_{||}^2 v_A^2(r)}{r^2}
\]

Frequency continuum
Alfvén Waves and Eigenmodes

- Current carrying inhomogeneous cylinder:
  - Helical field
  - $k_{||} = k_{||}(r)$
  - Continuum has extremum
  - Global Alfvén Eigenmode (GAE)

Alfvén Waves in Tori

- Tokamak plasma:
  - Fourier decomposition:
    - $A \sim \exp[i(n\phi - m\theta - \omega t)]$
    - $B \approx B_0 R_0 / R \approx B_0 (1 - r/R_0 \cos \theta)$
  - Neighbouring poloidal harmonics couple due to toroidicity
  - Gaps in frequency continuum
  - Toroidal Alfvén Eigenmodes (TAE) exist in frequency gap
    - Weakly damped
  - $f_{TAE} \sim v_A / (2qR)$

Alfvén Eigenmodes

- Exist in frequency gaps
- Comprise of two primary harmonics, \( m \) and \( m + L \)
  - Wave-particle resonance condition:
    \[
    \omega - n \omega_\phi + (m \pm 1) \omega_\theta = 0
    \]
  - **TAE**: \( L = 1 \)
  - **EAE**: \( L = 2 \)
  - **NAE**: \( L = 3 \)

\[
v_{||} = \pm \frac{L}{2 \pm L} v_A
\]
TAE in JET driven by ICRH accelerated ions

- TAE have constant amplitude and fine frequency splitting
  ⇒ Nonlinear effect
Fast Particle Drive

- Collective instabilities
  - Fast particle gradients act as source of free energy
    - Non-Maxwellian distribution
      - $\gamma \sim \omega \frac{\partial f}{\partial E} + n \frac{\partial f}{\partial P_\phi}$
      - $\sim \omega \frac{\partial f}{\partial E} - n \frac{\partial f}{\partial \psi}$
    - Negative radial gradient
      - $\Rightarrow$ Drive ($n>0$)
    - Negative energy gradient
      - $\Rightarrow$ Damping
HOW CAN WE MODEL NONLINEAR FAST ION DRIVEN INSTABILITIES IN FUSION PLASMAS?
The HAGIS Code

Equilibrium Representation

- Straight field line (Boozer) coordinates $\psi_p, \theta, \zeta$

\[ j \wedge B = \nabla_p \]

General toroidal geometry

\[
\begin{align*}
B &= \delta(\psi_p, \theta) \nabla \psi_p + I(\psi_p) \nabla \theta + g(\psi_p) \nabla \zeta, \\
B &= \nabla \psi \wedge \nabla \theta - \nabla \psi_p \wedge \nabla \zeta, \\
\Rightarrow A &= \psi \nabla \theta - \psi_p \nabla \zeta.
\end{align*}
\]
Evolution of Energetic Particles

Exact particle Lagrangian, $L_{\text{exact}} = \sum_{ep} \frac{1}{2} m V^2 + e V \cdot A - e \phi$

is gyro-averaged and written in the form,

$$L_{ep} = \sum_{j=1}^{n_p} P_{\theta_j} \dot{\theta}_j + P_{\zeta_j} \dot{\zeta}_j - H_j$$

with

$$H_j = \frac{1}{2} m_j v_{||}^2 + \mu_j B_j + e_j \phi_j$$

leading to $4 \times n_p$ equations

[Diagram of particle trajectory and guiding centre trajectory]
Equations of Motion

Derived from total system Hamiltonian for each particle:

\[
\begin{align*}
\dot{\theta} &= \frac{1}{D} \left[ \rho_{\parallel} B^2 (1 - \rho_c g' - g\tilde{\alpha}') + g \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} \right], \\
\dot{\zeta} &= \frac{1}{D} \left[ \rho_{\parallel} B^2 (\rho_c I' + q + I\tilde{\alpha}') - I \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} \right], \\
\dot{\psi}_p &= \frac{1}{D} \left[ \rho_{\parallel} B^2 \left( g \frac{\partial\tilde{\alpha}}{\partial \theta} - I \frac{\partial\tilde{\alpha}}{\partial \zeta} \right) - \left( g \frac{\partial\tilde{\Phi}}{\partial \theta} - I \frac{\partial\tilde{\Phi}}{\partial \zeta} \right) - g(\rho_{\parallel}^2 B + \mu) \frac{\partial B}{\partial \theta} \right], \\
\dot{\rho}_{\parallel} &= \frac{1}{D} \left[ \left( I \frac{\partial\tilde{\alpha}}{\partial \zeta} - g \frac{\partial\tilde{\alpha}}{\partial \theta} \right) \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} - (q + \rho_c I' + I\tilde{\alpha}') \frac{\partial\tilde{\Phi}}{\partial \zeta} \\
&\quad + (\rho_c g' - 1 + g\tilde{\alpha}') \left\{ (\rho_{\parallel}^2 B + \mu) \frac{\partial B}{\partial \theta} + \frac{\partial\tilde{\Phi}}{\partial \theta} \right\} \right] - \frac{\partial\tilde{\alpha}}{\partial t},
\end{align*}
\]

Fast Particle Orbits

- ICRH ions in JET deep shear reversal
  - On axis heating†:
    \[ \Lambda = \mu B_0 / E = 1 \]
    - \( E = 500 \text{ keV} \)
- Produces predominately potato orbits
- Particle trajectories verified through comparison with other codes and analytic solutions

†J. Hedin, PhD Thesis 1999
Calculation of AE Eigenfunctions

Wave Lagrangian:

\[ \mathcal{L}_w = \sum \left[ \frac{1}{2} mv^2 + e \left( \mathbf{A} \cdot \mathbf{v} - \phi \right) \right] + \frac{1}{2\mu_0} \int_V \left( \frac{1}{c^2} E^2 - B^2 \right) dx^3 \]

Expanding in perturbed field powers:

- \( \mathcal{L}^{(0)} \) describes the equilibrium and is solved by, for example, HELENA
- \( \mathcal{L}^{(1)} \) describes first order force balance
- \( \mathcal{L}^{(2)} \) describes fixed amplitude Alfvén Eigenmodes and is solved by appropriate linear codes, e.g. CASTOR, MISHKA, PHOENIX, or LIGKA
Wave Evolution

• Linear eigenmode structure is assumed to remain fixed throughout simulations

• Each wave is allowed two degrees of freedom, amplitude and phase-shift; $A_k$ and $\alpha_k$

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

• The wave Lagrangian can then be written as

$$L_w = \sum_{k=1}^{n_w} \frac{E_k}{\omega_k} \frac{A_k^2}{\alpha_k},$$

where

$$E_k = \frac{1}{2\mu_0} \int_V \frac{\nabla \cdot \tilde{\Phi}_k}{v_A^2} d^3x,$$

and $n_w$ is the number of eigenmodes in the system
Wave Equations

- Linear eigenstructure assumed invariant
- Introduce slowly varying amplitude and phase:
  \[ \tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi)e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))} \]
- Gives wave equations as:
  \[ \dot{\chi}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k\|m\|_j - \omega_k) S_{jkm} + \chi_k \gamma_d, \]
  \[ \dot{\gamma}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k\|m\|_j - \omega_k) C_{jkm} + \gamma_k \gamma_d, \]
- where
  \[ \chi_k \equiv A_k \cos(\alpha_k), \quad C_{jkm} \equiv \Re[\tilde{\phi}_{km}(\psi_j)e^{i\Theta_{jkm}}] \]
  \[ \gamma_k \equiv A_k \sin(\alpha_k), \quad S_{jkm} \equiv \Im[\tilde{\phi}_{km}(\psi_j)e^{i\Theta_{jkm}}] \]
  \[ \Theta_{jkm} \equiv n_k\zeta_j - m\theta_j - \omega_k t \]
Distribution Function

• Represented by a finite number of markers
• Markers represent deviation from initial distribution function - so-called $\delta f$ method
  – Dramatically reduces numerical noise

$$f = f_0(\mathcal{E}, P_\zeta; \mu) + \delta f(\Gamma^{(p)}, t)$$

$$\frac{df}{dt} = 0 \Rightarrow \dot{\delta f} = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}} - \nu_{eff} \delta f$$

$$\int f g d\Gamma^{(p)} \leftrightarrow \int f_0 g d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j g_j$$

where $\delta n_j(t) \equiv \delta f_j(t) \Delta \Gamma^{(p)}_j(t)$
Marker Loading

- Number of particles represented by a marker:

\[ \delta n_j(t) \equiv \delta f_j(t) \Delta \Gamma_j^{(p)}(t) \]

- Physical volume element associated with a marker:

\[ \Delta \Gamma_j^{(p)} \equiv J_j^{(pc)}(t) J_j^{(cu)}(0) \Delta U_j \]

\[ \Delta \Gamma_j^{(c)}(0) \]

**Uniformly loaded space**  
**Canonical phase space**  
**Physical phase space**  

**Incompressible volume elements**  
**Time dependent volume elements**
Quiet Start Method

- Markers are uniformly loaded using Hammersley’s sequence:
  \[ x_i = \{i/N, \phi_2(i), \phi_3(i), \phi_5(i), \phi_7(i)\}. \]
- If integer \( i \) is written in base \( r \):
  \[ i = a_0 + a_1 r + a_2 r^2 + \cdots \]
  \[ \phi_r(i) = a_0 r^{-1} + a_1 r^{-2} + a_2 r^{-3} + \cdots \]

![Projections of uniformly loaded 5-D hypercube](image)

- This achieves a discrepancy \( \propto 1/N \), where a random distribution has a discrepancy \( \propto 1/\sqrt{N} \).
Example of Linear Growth and Saturation of a TAE

- Equilibrium:
  - \( a/R_0 = 0.3 \)
  - \( q_0 = 1.1 \)
  - \( E_0 = 3.5 \) MeV

\( (m,n) = (3, 3) \)

\( (m,n) = (4, 3) \)

Linear Growthrate

- $\langle \beta_f \rangle = 3 \times 10^{-4}$

$\frac{\delta B}{B_0} = \frac{n_p}{52,500}$

Mode saturates at $\delta B/B \approx 10^{-3}$

$\gamma_d/\omega_0 = 2.7\%$
Fast Ion Redistribution due to TAE
Multiple KTAE in JET

- Multiple KTAE \((n = 5 \rightarrow 9)\) in JET interacting through the driving alpha particle distribution
INCLUDING DISSIPATION
Nonlinear Theory and Dissipative Effects

- When modes are near marginal stability then there are various competing effects
  - Drive from fast ions, $\gamma_L$
  - Damping from background plasma, $\gamma_D$
  - Reconstitution of profiles, $\nu_{\text{eff}}$

\[ \left| \gamma_L - \gamma_D \right| \sim \nu_{\text{eff}} \ll \gamma_L, \gamma_D \]
Nonlinear Theory

• Nonlinear cubic equation describes Alfvén eigenmodes near threshold
  – \( \nu \) is the collision frequency for fast particles

\[
\frac{dA}{d\tau} = A(\tau) - \frac{1}{2} \int_0^{\tau/2} dz z^2 A(\tau - z) \times \int_0^{\tau-2z} dx \exp[-\nu(2z + x)] \times A(\tau - z - x)A(\tau - 2z - x)
\]

Closer look at TAE…

- Resonant particles relax through collisions
- Single mode undergoes pitchfork splitting
  - Used to determine $\gamma$ and $\nu$

Frequency Sweeping

• Occurs when mode is close to marginality
  – Damping balancing drive

• Structures form in fast particle distribution function
  – Holes and clumps

• These support long-lived nonlinear BGK waves

• Background dissipation is balanced by frequency sweeping

Experimental Observations

- Frequency sweeping in MAST #5568

Simultaneous upwards and downwards frequency sweeping, $\delta\omega/\omega_0 \sim 20\%$

JET Observations

- Shear optimised D-T pulse
- TAE modes during current ramp phase

Frequency sweep $\delta\omega/\omega_0 \sim 5\%$
Using Theory for Diagnostic Purposes

- Trapping frequency is related to TAE amplitude
  \[ \omega_{b,l}(t) \propto |\delta B|^{1/2} \]

- Frequency sweep is related to trapping frequency
  \[ \delta \omega \propto \omega_b^{3/2} t^{1/2} \]

- Amplitude related to frequency sweep

\[ \frac{\delta B}{B} = \frac{1}{C_1^2} \left( \frac{\delta \omega^2}{C_2^2 t} \right)^{2/3} \]

[Analytic estimates give correct order of magnitude. Numerical simulation required for more accurate estimate.]

Validation of Nonlinear Modelling

• Use experimentally observed rate of frequency sweeping to determine wave amplitude and compare with independent measurements
  – In general, numerical modelling is needed to establish the form factor that relates $\delta\omega$ and $\delta B$
  – Verify HAGIS for model case
  – Employ HAGIS to establish $\delta B$ in general case
    • General geometry (including tight-aspect ratio)
    • Mode structure: global mode analysis
Recall \( n = 3 \) TAE example

- \( \gamma_d / \omega_0 = 0, \langle \beta_f \rangle = 3 \times 10^{-4} \)

\( n_p = 52,500 \)

Mode saturates at \( \delta B / B \sim 10^{-3} \)

\( \gamma_d / \omega_0 = 2.7\% \)
...with additional damping

- $\gamma_d/\omega_0 = 2\%$, $\langle \beta_f \rangle = 3 \times 10^{-4}$

Mode saturates at much lower level, $\delta B/B \approx 10^{-4}$

$n_p = 210,000$
Frequency Sweeping

- Fourier spectrum of evolving mode

\[ \delta \omega = 0.44 \gamma_L^{3/2} t^{1/2} \]

Frequency sweep \( \delta \omega/\omega_0 \sim 10\% \)
• Obtain factor relating $\omega_b$ and $\delta B$

$E_b = 40$ keV  
$a/R_0 = 0.7$  
$B_0 = 0.5$ T  
$R_0 = 0.77$ m

Global $n=1$ TAE

Monotonic q-profile
Particle Trapping in MAST

- Particles trapped in TAE wave
  - All particles have same
    \[ H' = E - \omega/n \, P_\zeta \]
    \[ = 20 \text{ keV} \]
  - TAE amplitude:
    \[ \delta B/B = 10^{-3} \]
Scaling of Nonlinear Bounce Frequency

- Monotonic $q$ profile
- $H' = 20$ keV

$$\omega_b = 1.156 \times 10^6 \sqrt{\frac{\delta B}{B}}$$
TAE Amplitude in MAST

\[
\frac{\delta B}{B} = \frac{1}{(1.156 \times 10^6)^2} \left( \frac{32 \, \delta f^2}{\delta t} \right)^{2/3} = 4 \times 10^{-4}
\]

\( df = 18 \text{ kHz} \)
\( dt = 0.8 \text{ ms} \)
Consider again our $n = 3$ TAE case

- **Equilibrium:**
  - $a/R_0 = 0.3$
  - $E_0 = 3.5$ MeV
  - $q_0 = 1.1$
  - $\beta_f = 3 \times 10^{-4}$

Radially peaked fast ion profile

Radially peaked fast ion profile

Slowing down distribution

Growth rate has a maximum (~6%) at ~70% of original frequency
Effect of damping

- \( n_p = 262,500, \frac{\gamma_d}{\omega_0} = 6\% \)

Long term symmetric frequency sweeping, \( \delta \omega \sim t^{1/2} \)
HAGIS Code: Fast Particle Drag

• Introducing drag into the kinetic equation:

\[
\dot{f} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \nu_{ei} \frac{\partial}{\partial \mathbf{v}}(\mathbf{v} f) + S
\]

Drag term, C

• Manifests itself through a change in the characteristics of the kinetic equation (marker trajectories)

\[
\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{x}}(\mathbf{v} f) + \frac{\partial}{\partial \mathbf{v}}(Df) = 0
\]

Model allows both \(v\) dependent and constant drag

Fast ion source
HAGIS Code: Fast Particle Drag

- Including drag necessitates the inclusion of a fast ion source to maintain initial steady-state conditions.

\[ f_0 : \text{Analytic function or numerical representation} \]

\[ \delta f : \text{Represented by ensemble of markers} \]

Markers follow guiding centre trajectories and slow down. They may be removed from simulation at low \( v \).

High energy source of ions: fusion alphas or beam ions.

Marker source is handled by loading markers to high \( v \).
Perturbation to distribution moves through phase space affecting gradients and stability.

- Slowing down due to drag
- Krook relaxation

Phase space with perturbation moves through, affecting gradients and stability.
Super-Alfvénic ion source and effect of drag

Bump-on-tail distribution

Drag moves flat spot leading to increased drive and explosive growth!
Effect of (Krook) relaxation

• If $\nu_{eff}$ is $\sim 1\%$ of $\gamma_L$ then frequency sweeping structures are destroyed after $\sim 100 \gamma_L t$

• Increasing Krook relaxation to 10% almost completely eradicates any mode sweeping
Nonlinear Behaviour: Drag + Krook

- \( n_p = 262,500, \gamma_L/\omega_0 = 6.12\%, \gamma_d/\omega_0 = 6\%, \nu_{ei}/\omega_0 = 0.3\%, \nu_{eff}/\omega_0 = 1\% \)

- Asymmetric, repetitive, frequency sweeps: \( \delta \omega/\omega_0 \sim \pm 30\% \)
Fast Ion Redistribution: Drag + Krook

- Changes to fast ion distribution due to nonlinear self-consistent wave-particle interaction:
  - Extensive and sustained redistribution

- \( n_p = 262,500, \gamma_L/\omega_0 = 6.12\%, \gamma_d/\omega_0 = 6\%, \nu_{ei}/\omega_0 = 0.3\%, \nu_{eff}/\omega_0 = 1\% \)
FISHBONES
Fast Particle Losses in JET

- NBI heating
  - $v_b \sim v_A$
- 10% drop in neutron yield due to ‘fishbones’

D.N. Borba et al., Nucl. Fusion 40 (2000)
Fishbone Instability

- Frequency sweeping mode driven by fast particles
- Consistent MHD/kinetic description being developed

Modelling Fishbones in ASDEX Upgrade

- \( m=1, n=1 \) internal kink
- Linear frequency chirps (27.5 → 20 kHz)
- Repetition rate: 1ms
- Slowing down distribution of 60 keV NBI ions
- \(<\beta_{\text{fast}}> = 0.36\%\)
Fishbone Evolution
Fishbone Simulation

ASDEX Upgrade

$t = 0.00\, [\text{ms}]$

Amplitude

$z [\text{m}]$

$R [\text{m}]$

$\text{Radial Current} \times 10^6 \, \text{MHz/s}$

$q = 1$

$t_0 = 1.566 \, [\text{s}]$

# 13921
Current Carrying Ion

- Trapped ion at \( q = 1 \) surface
- Energy, \( E = 55 \text{ keV} \)
- Precession frequency, \( \omega_\phi = 7 \text{ kHz} \)
- Bounce frequency, \( \omega_b = 41 \text{ kHz} \)
Spatial redistribution due to fishbones

- Fast ionsradially expelled towards low field side
Pitch Angle Redistribution

• Change in trapped/passing fast ion distribution
Fast Ion Radial Current

- $\delta f$ simulation with HAGIS code gives $<J^\psi(t)>$ and variation of fast ion distribution function
FAST ION LOSSES DUE TO TORNADO MODES IN JET
Tornado modes in JET

- *Every* “monster” sawtooth crash preceded by tornado modes

![Graph showing sawtooth crash and tornado modes](image)

*On-axis interferometer*

*Magnetics*

- $t = 11.2 - 11.7 \text{ [s]}$
- $t = 12.8 - 13.6 \text{ [s]}$
- $t = 14.9 - 15.7 \text{ [s]}$
- $t = 16.9 - 17.4 \text{ [s]}$

*Sawtooth crash*
Observations of Fast Ion Losses in JET

Loss measurements increase during tornado mode activity

3.1-MeV $\gamma$-ray emission from $^{12}$C(d,p$\gamma$)$^{13}$C;
Deuterons with E>500 keV

Scintillator probe
TAE Mode Structure

- Linear MHD eigenfunctions calculated with CASTOR code
  - Equilibrium from HELENA code
Fast Ion Properties

- Determine natural particle frequencies, $\omega_\phi$ and $\omega_\theta$

![Image of Fast Ion Properties graph]
Resonant ICRH ions

Resonance condition:
- $\Omega_{np} = n \omega_\phi - p \omega_\theta - \omega = 0$

$n = 3$ tornado mode:
- $p = -1 \rightarrow 2$
- $f = 283$ kHz
Resonance Overlap

Particles move along lines given by
\[ E - (\omega/n)P_\phi = K \]

All resonances, \( n = 3 - 7 \)

- Overlap between resonances explains observed loss

\[ P_{\phi,i} \text{[MeV]} \]

\[ \text{Log}(sE/\Omega_{np}) \]

\( n = 5, 7 \) overlapping

Prompt losses

Additional losses due to tornado modes

\( E \text{[MeV]} \)

4.5 5.0 5.5 6.0 6.5 7.0 7.5

-250 -200 -150 -100 -50 0
Summary

• Physics of fast ion driven instabilities well understood
• Fast particles drive instabilities and are in turn re-distributed and, in some cases, lost
  – Consistent \textit{nonlinear} story emerging
• Nonlinear modelling of fast ion driven instabilities
  – Multiple modes interacting through driving fast ion distribution
  – Determination of amplitude of frequency sweeping modes in MAST
  – Radial fast ion current due to fishbones in ASDEX Upgrade
  – Fast ion losses due to tornado modes in JET
• Models start to successfully describe rich nonlinear phenomena near marginal stability
  – Mode saturation, pitchfork splitting and frequency sweeping
• Fast particle driven modes remain a valuable diagnostic tool
  – MHD spectroscopy ($q_{\min}(t)$ from Alfvén cascades)