Physical Processes
in the Tokamak Edge/Pedestal

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International ITER School, Hefei, Dec 2015
MHD equilibrium

strong magnetic field, small gyroradius

closed magnetic flux surfaces

---> confined plasma, steep "pedestal" at edge

however . . . turbulence, MHD events ---> losses

\[ J \times B = c \nabla p \]
Profile Examples

L−mode

with pedestal

H−mode

no pedestal
tokamak edge pedestal distinctly

• scales – parameters
  — steep gradient, \( R/L_\perp > 30 \), even find \( L_\perp/R < \rho_s/L_\perp \)
  — electron transit frequencies comparable to turbulence, \( c_s/L_\perp > V_e/qR \)

• dynamics
  — low frequency/beta \( \rightarrow \) magnetic stiffness \( \rightarrow \) reduced, gyrokinetic equations
  — electromagnetic turbulence character \( c_s/L_\perp > v_A/qR \)

• equilibration
  — neoclassical time scale separation is affected
  — collisional relaxation in the regime of \( L_\perp/R < \rho_s/L_\perp \sim \{20, 30\}^{-1} \)

the scale ratio regime determines
edge/pedestal dynamical character
space scales

- meaning of steep gradients:

  profile scale \( L_\perp \ll \) toroidal major radius \( R \)

- field line pitch parameter in a conventional tokamak

  \[ q \sim 3 \]

- parallel scale (field line connection length) is the largest

  \[ qR/L_\perp \sim 200 \]

- the local rho-star is not smaller than the perp scale ratio

  \[ \rho_s/L_\perp \sim 1/30 \quad L_\perp/R \sim 1/50 \]
time scales

• transit frequencies for electrons, magnetic field
  ◦ thermal or Alfvén velocity and parallel scale

• turbulence spectrum range, acoustic frequencies go with sound speed

• parallel scales have consequences for time scale separation

\[
\frac{c_s}{L_\perp} > \frac{v_A}{qR} \sim \frac{V_e}{qR} \gg \frac{c_s}{R} > \frac{c_s}{qR}
\]

• the space scale ratio affects relaxation

several \(10^2 \frac{L_\perp}{c_s}\) \(\sim \nu_i^{-1}\) and in some cases several \(\nu_i^{-1} \sim \frac{L_\perp^2}{\chi_i}\)
what determines the edge?

• mainly, the electron thermal nonadiabaticity condition: \( c_s/L_\perp > V_e/qR \), or \( \hat{\mu} > 1 \)

\[
\text{def: } \hat{\mu} = \frac{m_e}{M_D} \left( \frac{qR}{L_\perp} \right)^2
\]

• consider the boundary, \( \hat{\mu} = 1 \), then \( c_s/L_\perp = V_e/qR \) → edge/core bndy

• solve this for the profile scale length

\[
L_\perp = \sqrt{m_e/M_D qR}
\]

• for linear profile gradients this is typically about 8 cm
  ○ and it holds over about the last 4 cm within the LCFS

on the other hand,
if a pedestal exists, the top is the edge/core boundary
Edge Layer Extent

\[ \hat{\mu} = 1 \text{ at } r = r_1 \]
Edge Layer Extent

\[ \hat{\mu} = 1 \text{ at } r = r_1 \]

L–mode

H–mode
Low Pressure (Beta) Dynamics

low ‘‘beta’’
\[ p << \frac{B^2}{8\pi} \]

‘‘flute mode’’
vortices/filaments
\[ k_\parallel << k_\perp \]

low frequencies
\[ \omega << k_\perp v_A \]

magnetic field \( B \)

pressure disturbance \( \tilde{p} \)
magnetic disturbance \( \tilde{B} \) (parallel to \( B \))

\[ \nabla \left( 4\pi \tilde{p} + \tilde{B} \tilde{B} \right) \]

\[ \omega \sim k_\parallel v_A \]

\[ \rightarrow \text{strict perpendicular force balance} \]

\[ \rightarrow \text{electromagnetic parallel dynamics} \]
**low frequency drift regime**

- fluid $\rightarrow$ reduced equations

  $$
  E \rightarrow -\frac{1}{c} \frac{\partial A_\parallel}{\partial t} b - \nabla \phi \quad \nabla \cdot u_\perp \sim \frac{u_\perp}{R} \ll u \cdot \nabla
  $$

  $$
  v_E = \frac{c}{B^2} B \times \nabla \phi
  $$

  $$
  B^2 = \frac{B_0^2 R_0^2}{R^2} \left[ 1 + O \left( \frac{a}{qR}, \beta \right) \right] \rightarrow \frac{I^2}{R^2}
  $$

- kinetic $\rightarrow$ gyrokinetic

  $$
  A \rightarrow A_{eq} + A_\parallel b \quad b \rightarrow R \nabla \varphi \quad \psi_T \rightarrow \psi_{eq} + A_\parallel R
  $$

  def: $\Omega_E = \frac{c}{B} \nabla^2 \perp \phi \ll \frac{eB}{mc}$

  $$
  \mu = \frac{mv_\perp^2}{2B} \rightarrow \text{conserved, i.e.,} \quad \frac{d\mu}{dt} = 0
  $$

  then: $f = f(R, z, \mu, t)$ but dynamics (exc. coll.) is in space of $\{R, z\}$
example – magnetic compressibility

• write MHD equations for $\mathbf{u}$ and $\mathbf{B}$

$$
\rho_M \frac{d\mathbf{u}}{dt} = \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p \\
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})
$$

• in tokamak geometry, $\mathbf{B} = F_{\text{dia}} \nabla \psi + \nabla \psi_T \times \nabla \varphi$, and in general: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$

• form equations for $\nabla \cdot \mathbf{u}$ and $F_{\text{dia}}$

$$
\nabla \cdot \mu_0 \rho_M R^2 \frac{d\mathbf{u}}{dt} = -\nabla^2 \frac{F_{\text{dia}}^2}{2} - \nabla \cdot (\Delta^* \psi_T \nabla \psi_T + \mu_0 R^2 \nabla p) \\
\frac{\partial}{\partial t} \frac{F_{\text{dia}}}{R^2} = -F_{\text{dia}} \nabla \cdot \frac{\mathbf{u}}{R^2} - \frac{\mathbf{u}}{R^2} \cdot \nabla F_{\text{dia}} + \mathbf{B} \cdot \nabla (\mathbf{u} \cdot \nabla \varphi)
$$

• for $k_\perp v_A \gg \partial / \partial t$ and reasonable velocities these reduce to

$$
F_{\text{dia}} \rightarrow I = \text{constant} \quad \nabla \cdot \frac{\mathbf{u}}{R^2} \rightarrow 0 \rightarrow \text{drifts}
$$
drifts in reduced model

• solve for \( u \) in Lorentz force instead of inertia since \( ZeB/mc \gg \partial/\partial t \)

\[
nm \frac{du}{dt} + \nabla \cdot \Pi^* = nZe \left( E + \frac{u}{c} \times B \right) - \nabla p
\]

• find drifts

\[
u_\perp = \frac{c}{B^2} B \times \left( \nabla \phi + \frac{1}{nZe} \nabla p \right) + \frac{mc}{ZeB^2} B \times \left( \frac{du}{dt} + \frac{1}{nm} \nabla \cdot \Pi^* \right)
\]

• last term is polarisation velocity, and total inertia \( \rightarrow \) polarisation current

all inertia is polarisation (incl. diamag. momentum flux)
vorticity in reduced model

- vorticity instead of fluid inertia (all inertia becomes polarisation)

$$\frac{\partial \mathcal{w}}{\partial t} + [\phi, \mathcal{w}] = B \nabla_{\parallel} \frac{J_{\parallel}}{B} - \mathcal{K}(p) \quad \iff \quad \nabla \cdot \mathbf{J} = 0$$

- geometry and ordering under $L_{\perp} \ll R$ and $nmu^2_E \ll p$ and $\rho_s \ll L_{\perp}$

$$[f, g] = \frac{cR^0}{B^0} \nabla \varphi \cdot (\nabla f \times \nabla g) \quad B \nabla_{\parallel} f = B \cdot \nabla f = B^0 \cdot \nabla f - \nabla \varphi \cdot (\nabla A_{\parallel} R \times \nabla f)$$

$$\mathcal{K}(f) = [(R/R_0)^2, f] \quad \nabla \cdot f \mathbf{v}_E \rightarrow [\phi, f] - f_0 \mathcal{K}(\phi)$$

- vorticity in the reduced MHD limit

$$\mathcal{w} = \nabla \cdot \frac{\rho_M c^2}{B^2} \nabla_{\perp} \phi \quad \rightarrow \quad \frac{\rho_M 0 c^2}{B^2} \nabla^2_{\perp} \phi \quad \rightarrow \text{normalisation} \rightarrow \quad \rho_s^2 \nabla^2_{\perp} \phi$$
normalisation and scales

- low-freq, pressure driven $\rightarrow$ sound speed normalisation, not an Alfvénic one

$$n \leftrightarrow n_0 \quad \phi \leftrightarrow \frac{T_e}{e} \quad u_{\parallel} \leftrightarrow c_s \quad J_{\parallel} \leftrightarrow n_0 e c_s \quad A_{\parallel} \leftrightarrow B_0 \rho_s$$

- for vorticity, divide apparent charge density by $n_0 e$, use above, to find

$$\varpi \leftrightarrow \frac{\rho M_0 c^2}{B_0^2} \frac{T_e}{n_0 e^2} \nabla^2 \phi \rightarrow \rho_s^2 \nabla^2 \phi \quad \rho_s^2 = \frac{c^2 M_i T_e}{e^2 B_0^2} \quad \rightarrow \quad \rho_s = \frac{c_s}{\Omega_i}$$

- the scale $\rho_s$ is demanded by eventual $n_e e \nabla_{\parallel} \phi \sim \nabla_{\parallel} p_e \quad \rightarrow \quad T_e/e$ for $\phi$
  - this is the main neglect done by the MHD model

- time scale is nominal space scale divided by $c_s$, so using $L_{\perp}$ you have

$$t \leftrightarrow \frac{L_{\perp}}{c_s} \quad \delta = \frac{\rho_s}{L_{\perp}} \quad \text{and} \quad \mathcal{K}, \quad [f, g] \rightarrow O(\delta) \quad \text{while} \quad \rho_s^2 \nabla^2_{\perp} \rightarrow O(\delta^2)$$
MHD dynamics in reduced model

- in one-fluid MHD (familiar) you neglect diamagnetic effects
  - all appearances of $\nabla p$ next to $\nabla \phi$
  - implicitly assumes smallness of pressure fluctuations/dynamics

- solve for vorticity, electron density, and electron and ion parallel dynamics
  - any model used for learning purposes is isothermal

\[
\frac{\partial \varpi}{\partial t} + [\phi, \varpi] = B \nabla_\parallel \frac{J_\parallel}{B} - \mathcal{K}(n)
\]
\[
\frac{\partial n}{\partial t} + [\phi, n] + B \nabla_\parallel \frac{u_\parallel}{B} = \mathcal{K}(\phi)
\]
\[
\frac{\partial A_\parallel}{\partial t} = -\nabla_\parallel \phi - \eta_\parallel J_\parallel
\]
\[
\frac{\partial u_\parallel}{\partial t} + [\phi, u_\parallel] = -\nabla_\parallel n + \mu_\parallel \nabla_\parallel^2 u_\parallel
\]

- with $\varpi = \rho_s^2 \nabla_\perp^2 \phi$ and $J_\parallel = -(\rho_s^2/\beta_e)\nabla_\perp^2 A_\parallel$ as self-consistent field equations
Reynolds stress and acoustic oscillations

- Reynolds stress is the same as polarisation nonlinearity

\[
\frac{\partial \varpi}{\partial t} + \mathbf{v}_E \cdot \nabla \varpi = \cdots \quad \text{zonal component} \quad \Rightarrow \quad \frac{\partial}{\partial t} \langle \varpi \rangle = \frac{\rho_s}{L} \rho_s^2 \frac{\partial^2}{\partial x^2} \left\langle \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \right\rangle + \cdots
\]

- zonal flow energy (grows if stress is aligned to zonal vorticity)

\[
- \langle \phi \rangle \frac{\partial}{\partial t} \langle \varpi \rangle = \nabla \cdot (\cdots) + \frac{\rho_s}{L} \langle \varpi \rangle \left\langle - \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial y} \right\rangle + \cdots
\]

- toroidicity gives rise to geodesic acoustic oscillation
  - take zonal component, find mode frequency
  - factors of \( \rho_s \partial/\partial x \) cancel, and the \( 1/2 \) is the average of \( \sin^2 \theta \)
  - thermal dynamics add coefficients, ion parallel dynamics adds corrections

\[
\frac{\partial \varpi}{\partial t} + \cdots = -K(n) \quad \frac{\partial n}{\partial t} + \cdots = K(\phi) \quad \Rightarrow \quad \frac{\partial^2}{\partial t^2} = -\frac{1}{2} \left( \frac{2c_s}{R} \right)^2
\]
new things at the fluid level

• in edge turbulence or any pedestal dynamics, never neglect $\nabla p$ against $\nabla \phi$
  ○ this makes everything two-fluid, at least

• adiabatic coupling

  \[
  \begin{align*}
  \text{dynamics} & \quad \tilde{p}_e \quad \leftrightarrow \quad J_\parallel \quad \leftrightarrow \quad \tilde{\phi} \\
  \text{sidebands} & \quad \langle p_e \sin \theta \rangle \quad \leftrightarrow \quad \langle J_\parallel \cos \theta \rangle \quad \leftrightarrow \quad \langle \phi \sin \theta \rangle
  \end{align*}
  \]

• diamagnetic compression

  \[
  \begin{align*}
  \text{sidebands} & \quad \langle p \sin \theta \rangle \quad \leftrightarrow \quad \langle p \rangle
  \end{align*}
  \]

• MHD and acoustic systems coupled by both processes

• flow dynamics in simple models drastically altered
dynamics in reduced two-fluid model

- now you never neglect diamagnetic effects
  - pressure dynamics is never “small” compared to flows, currents

- solve for vorticity, electron density, and electron and ion parallel dynamics
  - any model used for learning purposes is isothermal
  - write density as isothermal $p_e$ to remember the physics, write normed masses $\mu_{i,e}$

\[
\begin{align*}
\frac{\partial \varpi}{\partial t} + [\phi, \varpi] &= B \nabla || \frac{J||}{B} - \mathcal{K}(p_e) \\
\frac{\partial p_e}{\partial t} + [\phi, p_e] + B \nabla || \frac{u|| - J||}{B} &= \mathcal{K}(\phi - p_e) \\
\frac{\partial}{\partial t} (A|| + \mu_e J||) + [\phi, \mu_e J||] &= \nabla || (p_e - \phi) - 0.51 \mu_e \nu_e J|| \\
\mu_i \frac{\partial u||}{\partial t} + [\phi, \mu_i u||] &= -\nabla || p_e + \mu || \nabla^2 u||
\end{align*}
\]

- with $\varpi = \rho_s^2 \nabla_\perp^2 \phi$ and $J|| = - (\rho_s^2 / \beta_e) \nabla_\perp^2 A||$ as self-consistent field equations
turbulence energetics

- adiabatic response allows ExB and thermal coupling through $J_\parallel$
  - as well as the curvature coupling that exists in one-fluid model

- free energy construction – identify transfer effects as pieces of total divergences
  - multiply by $-\phi$ and $J_\parallel$ and $p_e$ respectively

\[ \frac{1}{2} \frac{\partial}{\partial t} \left| \rho_s \nabla \phi \right|^2 + \nabla \cdot (\cdot) = -\phi B \nabla \frac{J_\parallel}{B} + \phi \mathcal{K}(p_e) \]

\[ \frac{1}{2} \frac{\partial}{\partial t} \left( \beta_e^{-1} \left| \rho_s \nabla A_\parallel \right|^2 + \mu_e J_\parallel^2 \right) + \nabla \cdot (\cdot) = \frac{J_\parallel}{B} B \nabla \left( p_e - \phi \right) - 0.51 \mu_e \nu_e J_\parallel^2 \]

\[ \frac{1}{2} \frac{\partial}{\partial t} p_e^2 = p_e B \nabla \frac{J_\parallel}{B} + p_e \mathcal{K}(\phi - p_e) \]

- processes in two-fluid models only
  - electron adiabatic compression $J_\parallel \nabla p_e$
  - diamagnetic compression $p_e \mathcal{K}(p_e)$

- processes in all models: ExB compression, $p_e \mathcal{K}(\phi)$, and sound waves
Energy Transfer: electromagnetic turbulence

\[ \tilde{\phi} \quad \text{nonlinear} \quad \tilde{\phi} \]

\[ \tilde{\mathbf{j}} \quad \text{sink} \quad \tilde{\mathbf{j}} \]

\[ \tilde{p} \quad \text{nonlinear} \quad \tilde{p} \]

\[ \text{thermal gradient} \]


(S Camargo et al Phys Plasmas 1995 and 1996)
Nonlinear Free Energy Cascade

- Direct cascade in delta-$f$ entropy
  - Energy taken out of larger scales
  - $\rightarrow$ Nonlinear drive at small scales

- Inverse cascade in ExB energy
  - $\rightarrow$ Nonlinear drive of long-wave MHD component

- Spectrum tied together, scalings are affected
turbulence signatures

- basic turbulence with finite-beta drift wave mode structure

- spectra: mesoscale MHD activity, vorticity (‘w’) extends to ion gyroradius

- envelopes: ion temperature (magenta) largest fluctuation, most strongly ballooned
  - potential (blue) is flat: shear-Alfvén signature, one of the dissipation channels

- fluxes: moderate, not extreme, ballooning (2 to 1 is common)
Phase Shifts and their Measurement

one dependent variable quantity leads another in the drift direction

\[ \Rightarrow | \alpha | \Leftarrow \]

complex numbers: \[ n = (A \exp^{-i\alpha}) \phi \] in k-space

amplitude/phase are real numbers

how to calculate \( \alpha \): \[ \alpha = \text{Im} \log n^* \phi \]

significance: a positive phase-shift implies a positive down-gradient flux

(B Scott Plasma Phys Contr Fusion 1997)
Nonlinear Transition

linear mode structure wiped out by turbulence after saturation

linear regime phase shift part of the eigenmode for each $k_y$
linear mode structure destroyed by the turbulence during saturation
mode structure at late times is the turbulence one: DW mode structure
Relevance Range for Linear Instabilities

dispersion space bounded by ideal interchange and diamagnetic rates

if the linear growth rate is below the red line then the instability is irrelevant
usually, this is not the case anywhere in the spectrum (unless: MHD threshold)
this situation is a direct consequence of very large $R/L_T >> 1$ in the edge

phase shifts – RMHD versus full gyrofluid

- gyrofluid turbulence is driven by both $\nabla T_e$ and $\nabla T_i$
  - despite the ballooned structure of $T_i$
  - the $n_e \leftrightarrow \phi$ phase shifts remain drift-wave like, $\alpha \gtrsim 0$

- this is completely different from RMHD, which has $\alpha \sim \pi/2$
parallel structure – RMHD vs full gyrofluid

- gyrofluid turbulence is driven by both $\nabla T_e$ and $\nabla T_i$
  - despite the ballooned structure of $T_i$
  - the nonadiabatic part of $p_e$ is almost flat

- this is **completely different from RMHD**, 
  - which violates its assumption that $n_e e \nabla \phi \gg \nabla p_e$ (this always happens)
summary – edge turbulence basics

• ions say ITG, electrons say DW, \( \tilde{\phi} \) says trans-MHD \( \rightarrow \) all are present/active

• gyrofluid edge-ITG signatures:
  ◦ \( T_i \) is largest and most ballooned (\( T_{i\perp} \) a little more than \( T_{i\parallel} \))
  ◦ \( H = n_e - \phi \) is the flattest
  ◦ Alfvén signature: \( \phi \) flatter than \( T_e \) which is flatter than \( n_e \)
  ◦ nevertheless, the electron and ion ExB fluxes are comparable

• these features are why in your model for edge turbulence ...

you need to keep \( p_e \leftrightarrow J_{\parallel} \leftrightarrow \phi \) adiabatic response

you need \( T_i \) distinct from \( p \)

you need to resolve \( \rho_i \) in any computations
flow energetics in a tokamak

• main element in a tokamak: geodesic curvature
  ○ poloidal gradient in magnetic field in perpendicular drift dynamics
  ○ absent in a 2D/interchange model

• main two-fluid element for flows:
  ○ coupling between acoustic and Alfvén branches

• fate of zonal flow energy: transfer to dissipation channels via conservative processes
  ○ turbulence $\rightarrow$ incoherent mixing
  ○ adiabatic sideband compression $\rightarrow$ parallel electron dissipation

• how to do the analysis: sideband decomposition
  ○ zonal and $\sin \theta$ components of state variables ($\varpi, p_e$)
  ○ $\sin \cos$ component of flux variables ($J_\parallel, u_\parallel$)
  ○ keep NL only as source/sink effects via turbulence fluxes, flow stress ($\Gamma, R_E$)
  ○ don’t order $\partial/\partial t$
Zonal Flow, Toroidal Compression


Zonal Flow, Toroidal Compression

Zonal Flow, Toroidal Compression

zonal flow

compression at top

< p sin θ >

pressure sideband

divergence at bottom

zonal flow exchanges conservatively with pressure sideband

---> transfer pathway, equipartition
detailed example – continuity equation

- electrons: start with

\[
\frac{\partial p_e}{\partial t} + [\phi, p_e] + B \nabla_\parallel \frac{u_\parallel - J_\parallel}{B} = \mathcal{K} (\phi - p_e)
\]

- zonal component: flux surface average, \( \text{fsavg} \), annihilates linear \( B \nabla_\parallel \)

- the bracket terms represent turbulent flux (transport)

\[
\langle [\phi, p_e] \rangle + \left\langle B \nabla_\parallel \frac{u_\parallel - J_\parallel}{B} \right\rangle = \frac{\partial}{\partial x} \langle \Gamma \rangle
\]

- geodesic curvature has \( \sin \theta \) component, only one to survive \( \text{fsavg} \)

\[
\frac{\partial}{\partial t} \langle p_e \rangle + \frac{\partial}{\partial x} \langle \Gamma \rangle = \omega_B \frac{\partial}{\partial x} \langle (\phi - p_e) \sin \theta \rangle \quad \omega_B = \frac{2\rho_s}{R}
\]

- the sideband is the disturbance maintained by gradients and finite collisionality
detailed example – continuity sideband

- electrons: sideband component: multiply by \( \sin \theta \) and then take \( \text{fsavg} \)

- integrate \( \partial / \partial \theta \) in \( \nabla_\parallel \) by parts under \( \text{fsavg} \)

\[
\langle \sin \theta \nabla_\parallel u_\parallel \rangle = - \langle u_\parallel \nabla_\parallel \sin \theta \rangle = - k_\parallel \langle u_\parallel \cos \theta \rangle \quad k_\parallel = \frac{L_\perp}{qR}
\]

- in the curvature term approximate \( \text{fsavg} \) of \( \sin^2 \theta \) by \( 1/2 \)

\[
\langle \sin \theta K(\phi) \rangle = \omega_B \left\langle \sin^2 \theta \frac{\partial \phi}{\partial x} \right\rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi \rangle
\]

- resulting pressure sideband equation (treat fluxes as in zonal part)

\[
\frac{\partial}{\partial t} \langle p_e \sin \theta \rangle + \frac{\partial}{\partial x} \langle \Gamma \sin \theta \rangle - k_\parallel \langle (u_\parallel - J_\parallel) \cos \theta \rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi - p_e \rangle
\]
detailed example – parallel forces

• electrons: start with

\[
\frac{\partial}{\partial t} (A_{\|} + \mu_e J_{\|}) + [\phi, \mu_e J_{\|}] = \nabla_{\|} (p_e - \phi) - 0.51 \mu_e \nu_e J_{\|}
\]

• flux variable sideband component: multiply by \( \cos \theta \) and then take \( \text{fsavg} \)
  o integrate \( \partial / \partial \theta \) in \( \nabla_{\|} \) by parts under \( \text{fsavg} \) (watch signs)
  o the nonlinear terms in this equation are small (MHD stable regime)

\[
\frac{\partial}{\partial t} \langle (A_{\|} + \mu_e J_{\|}) \cos \theta \rangle = k_{\|} \langle (p_e - \phi) \sin \theta \rangle - 0.51 \mu_e \nu_e \langle J_{\|} \cos \theta \rangle
\]
detailed example – parallel forces

- ions: start with
  \[ \mu_i \frac{\partial}{\partial t} u_\parallel + [\phi, \mu_i u_\parallel] = -\nabla_\parallel p_e + \mu_\parallel \nabla^2_\parallel u_\parallel \]

- flux variable sideband component: multiply by \cos \theta\) and then take \text{fsavg}
  - integrate \frac{\partial}{\partial \theta} in \nabla_\parallel by parts under \text{fsavg} (watch signs)
  - the nonlinear terms in this equation are small (subsonic regime)

\[ \frac{\partial}{\partial t} \langle \mu_i u_\parallel \cos \theta \rangle = -k_\parallel \langle p_e \sin \theta \rangle - \mu_\parallel k_\parallel^2 \langle u_\parallel \cos \theta \rangle \]
detailed example – charge conservation

• charge: start with the vorticity equation

\[
\frac{\partial \varpi}{\partial t} + \left[ \phi, \varpi \right] = B \nabla \| J \| \frac{J}{B} - \mathcal{K}(p_e)
\]

• zonal component: treat as in the zonal continuity equation
  ◦ the bracket term represents Reynolds/Maxwell stresses (ExB forcing)

\[
\left\langle \left[ \phi, \varpi \right] \right\rangle - \left\langle B \nabla \| J \| \frac{J}{B} \right\rangle = \frac{\partial^2}{\partial x^2} \left\langle R_E \right\rangle
\]

• geodesic curvature has sin θ component, only one to survive \( fsavg \)

\[
\frac{\partial}{\partial t} \left\langle \varpi \right\rangle = - \frac{\partial^2}{\partial x^2} \left\langle R_E \right\rangle - \omega_B \frac{\partial}{\partial x} \left\langle p_e \sin \theta \right\rangle
\]

• the sideband is the disturbance maintained by gradients and finite collisionality
Sideband Dynamics for Flows/Currents

- zonal vorticity, pressure sideband, sound wave sideband

\[
\frac{\partial}{\partial t} \langle \omega \rangle = - \frac{\partial^2}{\partial x^2} \langle R_E \rangle - \omega_B \frac{\partial}{\partial x} \langle p_e \sin \theta \rangle
\]

\[
\frac{\partial}{\partial t} \langle p_e \sin \theta \rangle + \frac{\partial}{\partial x} \langle \Gamma \sin \theta \rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi \rangle - \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle p_e \rangle + k_{||} \langle u_{||} \cos \theta \rangle - k_{||} \langle J_{||} \cos \theta \rangle
\]

\[
\frac{\partial}{\partial t} \mu_i \langle u_{||} \cos \theta \rangle = -k_{||} \langle p_e \sin \theta \rangle - \mu_{||} k_{||}^2 \langle u_{||} \cos \theta \rangle
\]

- Alfvén sideband, flow sideband, zonal pressure

\[
\frac{\partial}{\partial t} \langle (A_{||} + \mu_e J_{||}) \cos \theta \rangle = k_{||} \langle p_e \sin \theta \rangle - k_{||} \langle \phi \sin \theta \rangle - 0.51 \mu_e \nu_e \langle J_{||} \cos \theta \rangle
\]

\[
\frac{\partial}{\partial t} \langle \omega \sin \theta \rangle = - \frac{\partial^2}{\partial x^2} \langle R_E \sin \theta \rangle - k_{||} \langle J_{||} \cos \theta \rangle - \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle p_e \rangle
\]

\[
\frac{\partial}{\partial t} \langle p_e \rangle + \frac{\partial}{\partial x} \langle \Gamma \rangle = \omega_B \frac{\partial}{\partial x} \langle \phi \sin \theta \rangle - \omega_B \frac{\partial}{\partial x} \langle p_e \sin \theta \rangle
\]
zonal flow energetics

• the equations don’t all fit on the page, but this is what you do . . .
  ○ here, one equation per page

• for the zonal vorticity multiply by $f_{\text{avg}}$ of $-\phi$ and integrate over $d\mathcal{V}$

\[ \int d\mathcal{V} \times \langle -\phi \rangle \frac{\partial}{\partial t} \langle \omega \rangle = \langle \phi \rangle \frac{\partial^2}{\partial x^2} \langle R_E \rangle + \langle \phi \rangle \frac{\partial}{\partial x} \omega_B \langle p_e \sin \theta \rangle \]

\[ \int d\mathcal{V} \times \frac{1}{2} \frac{\partial}{\partial t} \left( \rho_s \nabla \perp \phi \right)^2 = \langle \omega \rangle \langle R_E \rangle - \omega_B \langle p_e \sin \theta \rangle \frac{\partial}{\partial x} \langle \phi \rangle \]

• flow energy driven by Reynolds/Maxwell stress (corr. with zonal vorticity)

• flow energy is depleted by geodesic compression

| turbulence driven zonal flows saturate at low levels |
pressure sideband energetics

- for the pressure sideband multiply by \(2\langle pe \sin \theta\rangle\) and integrate over \(dV\)

\[
\int dV \times 2 \langle pe \sin \theta\rangle \frac{\partial}{\partial t} \langle pe \sin \theta\rangle = \cdots \\
+ \omega_B \langle pe \sin \theta\rangle \frac{\partial}{\partial x} \langle \phi \rangle - 2 \langle pe \sin \theta\rangle \frac{\partial}{\partial x} \langle \Gamma \sin \theta \rangle - 2k|| \langle pe \sin \theta\rangle \langle J|| \cos \theta \rangle
\]

\[
\int dV \times \frac{\partial}{\partial t} \langle pe \sin \theta\rangle^2 = \cdots \\
+ \omega_B \langle pe \sin \theta\rangle \frac{\partial}{\partial x} \langle \phi \rangle - 2 \langle \Gamma \sin \theta \rangle \left\langle -\frac{\partial p_e}{\partial x} \sin \theta \right\rangle - 2k|| \langle pe \sin \theta\rangle \langle J|| \cos \theta \rangle
\]

- drive by geodesic compression (conservative transfer from zonal flow energy)
- sink by turbulent mixing and adiabatic compression \(\cdots\)

- turbulent mixing is one of the main sinks for zonal flow energy
Ohmic sideband energetics

• for the current sideband multiply by $2 \langle J_\parallel \cos \theta \rangle$ and integrate over $dV$

$$2 \langle J_\parallel \cos \theta \rangle \frac{\partial}{\partial t} \langle A_\parallel \cos \theta \rangle = \ldots + 2k_\parallel \langle J_\parallel \cos \theta \rangle \langle p_e \sin \theta \rangle - 2\eta_\parallel \langle J_\parallel \cos \theta \rangle^2$$

$$\frac{\partial}{\partial t} \beta_e^{-1} \langle \rho_s (\nabla_\perp A_\parallel) \cos \theta \rangle^2 = \ldots + 2k_\parallel \langle p_e \sin \theta \rangle \langle J_\parallel \cos \theta \rangle - 2\eta_\parallel \langle J_\parallel \cos \theta \rangle^2$$

• magnetic and parallel electron kinetic energy (the $\mu_e J_\parallel$ term, not shown)

• drive by adiabatic compression (conservative transfer from pressure sideband energy)
  ○ the other process, Alfvénic compression, leads to the Pfirsch-Schlüter current (equil.)

• sink by dissipation (here, resistivity)

the sink by dissipation is the other main sink for zonal flow energy
Energy Transfer: flows and currents

- Ion dissipation
- Transport
- Diamagnetic compression
- Adiabatic compression
- 2-fluid effects
- MHD effects
- Reynolds stress
- P-S current
- Resistivity

Coupling to Zonal Flows

turbulence regulated by flows, regulated by toroidal compression

eddy Reynolds stress --> energy transfer from turbulence to flows

turbulence moderately weakened but not suppressed

toroidal compression --> energy loss channel to pressure, turbulence

entire system in self regulated statistical equilibrium (turb, flows, mag eq)

Turbulence vs Geodesic Curvature

- no geodesic curvature $\implies$ no sideband dynamics
  - details: *New J Phys* 7 (2005) 92

  ![Graph 1](basic_toroidal.png)

  ![Graph 2](toroidal_no_gd_curv.png)

- self-generated flows are held down by geodesic compression ...
  - $\implies$ coupling back to turbulence, coupling to dissipative currents

- flows have weak effect on saturated energy, no role for predator-prey mechanism
Energy Transfer: flows and currents in 2D

MHD effects

Reynolds stress

no compression of the zonal flow

transport

generic dissipation

Energy Transfer: flows and currents in 2D

what is gyrokinetic

- low frequency approximations, usually also low-$\beta$ and small $a/qR$
- polarisation density, not polarisation current
- gyrocenter charge density $\leftrightarrow$ vorticity, polarisation current
- ambipolarity of particle charge density holds
- ambipolarity of gyrocenter charge density holds only in steady state $\partial/\partial t = 0$
- gauge transformation (coordinate changes, addition of pure divergences)
  - addition of a pure divergence $\Rightarrow$ integrations by parts
- equivalent to gyroaveraging over a ring orbit only at linear order
- Lagrangian/Hamiltonian support $\rightarrow$ automatic energetic consistency
  - works in practice only if field equations are obtained from the same Lagrangian
what is gyrofluid

- it is a representation not a closure and not an ordering

- example: Hasegawa-Wakatani (fold the gradient term $n_0$ into $n$)

$$\frac{\partial n}{\partial t} + [\phi, n] = B \nabla || \frac{J_{||}}{B}$$

$$\frac{\partial}{\partial t} \nabla^2 \phi + [\phi, \nabla^2 \phi] = B \nabla || \frac{J_{||}}{B}$$

subtract and define $N$

$$\frac{\partial N}{\partial t} + [\phi, N] = 0$$

$$N = n - \nabla^2 \phi$$

- this is nothing more and nothing less than the simplest gyrofluid model
  - equations for $n$ and $N$ with “polarisation” $-\nabla^2 \perp \phi = N - n$

no polarisation drift for gyrocenters, but a polarisation density
how to treat FLR

• lots of detail, but the simplest long-wavelength version is

\[
\phi_G = \left(1 + \frac{\rho_i^2}{2} \nabla^2 \right) \phi \quad \quad \quad n_G = \left(1 + \frac{\rho_i^2}{2} \nabla^2 \right) N
\]

then

\[
\frac{\partial N}{\partial t} + [\phi_G, N] + \cdots \quad \quad \quad \text{and} \quad \quad \quad - \rho_s^2 \nabla^2 \phi = n_G - n
\]

• lots of algebra including bracket forms such as

\[
\nabla^2 \left[ f, g \right] = \nabla \cdot \left[ f, \nabla g \right] + \nabla \cdot \left[ \nabla g, f \right] \quad \quad \quad \nabla \cdot \left[ f, \nabla g \right] = \left[ \nabla f, \nabla g \right] + \left[ f, \nabla^2 g \right]
\]

• use the fact that \( \tau_i \rho_s^2 = \rho_i^2 \), grind away,

° and recover the fluid polarisation drift divergence, label \( p_i = \tau_i n \)

\[
\frac{\partial n}{\partial t} + [\phi, n] - \nabla \cdot \left( \frac{\partial}{\partial t} + [\phi, \cdot] \right) \nabla (\phi + p_i) + \cdots
\]
fluid vs gyrofluid – same model

- using these methods, can show that gyrofluid FLR covers fluid nonlinear polarisation
- dissipation …
- large-$\nu_i$ limit of $p_{i\parallel} - p_{i\perp}$ represents parallel viscosity
  - including heat flux crossover (arises from $v$-dependence of $C$-operator)
- large-$\nu$ limit of $q_{\parallel\parallel}, q_{\perp\parallel} \rightarrow q_{\parallel}$ covers thermal conduction, parallel thermal force
- result: thermal gyrofluid model covers reduced Braginskii in all aspects

 gyrofluid model is … easier to maintain computationally
 has an obvious descent from underlying gyrokinetic theory
 covers pedestal dynamics in transcollisional regime
gyrokinetics and neoclassics

- basic assumptions of neoclassical theory applied to the gyrokinetic equation system
  - gyrokinetic theory requires only $\rho_L \ll L_\perp$ (forced by $\Omega_E \ll \Omega_i$)
  - conventional neoclassical theory requires $\rho_L (qR/a) \ll L_\perp$

- split $f$ into background $F^M$ and disturbance $\delta f$ arising from thermal gradients

- usually neglect FLR corrections, since $\rho_L \ll L_\perp$
  - note orbit width comes from drifts, not FLR \textit{per se}

- resulting equation set is identical to that used in neoclassical theory

- then, the fully nonlinear set you started with can be used in computations
  - (that is, if you coded them as they stand)
basic assumption of neoclassical theory

- time scales: slow compared to relaxation dynamics, fast compared to transport

\[
\frac{v_A}{qR} > \frac{c_s}{R} > v_i > \frac{\partial}{\partial t} > \frac{\chi_{NC}}{L_{\perp}^2}
\]

- space scales:
  - this also follows from drifts \( \ll \) parallel/collisional relaxation

\[
\rho_B \sim \rho_s \frac{qR}{a} < L_{\perp}
\]

marginal but not strongly violated even in pedestal for conventional tokamaks
neoclassical models

- drift-kinetic equation with small drift piece acting on Maxwellian $F^M(\epsilon, \mu, \psi_c)$
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering

$$v_\parallel \nabla_\parallel \left( \delta f + \frac{Z e}{T} \phi F^M \right) - C(\delta f) = -v_d \cdot \left( \nabla F^M + F^M \frac{Z e}{T} \nabla \phi \right)$$

- flow damping rate $\nu_{NC}$: maximal ordering $v_\parallel/qR \sim \nu_i$ suborderings:
  - banana regime: $v_\parallel/qR > \nu_i$ $\rightarrow$ $\nu_B \propto \nu_i$
  - collisional regime: $v_\parallel/qR < \nu_i$ $\rightarrow$ $\nu_{PS} \propto \nu_i^{-1}$
  - plateau regime, often modeled with simple crossover function (WM Stacey, 1990s)

$$\nu_{NC} = \nu_B \nu_{PS}/(\nu_B + \nu_{PS})$$

- standard treatments work through moment equations
  - conserved quantities $\leftrightarrow F^M$
  - fluxes, relaxation $\leftrightarrow \delta f$

- lots of detail, endless argument ... for basics refer to

FL Hinton and RD Hazeltine, Rev Mod Phys 48 (1976) 239
fluid analog

- write delta-f reduced fluid equations
- do sideband analysis
- keep consequences of neoclassical ordering
  - zonal averages of conserved quantities
  - divergence balance of flux variables ($u_\parallel$, $J_\parallel$, etc., no $A_\parallel$)
  - force balance of state variables ($p_e$, $\varpi \rightarrow \phi$, etc.)
detailed example – divergence balances

• electrons: start with

\[
\frac{\partial p_e}{\partial t} + [\phi, p_e] + B\nabla_{\parallel} \frac{u_{\parallel} - J_{\parallel}}{B} = \mathcal{K} (\phi - p_e)
\]

• state variable sideband component: multiply by \(\sin \theta\) and then take \(\text{fsavg}\)
  ○ note \(\partial/\partial t\) on sideband piece is neglected by the ordering
  ○ integrate \(\partial/\partial \theta\) in \(\nabla_{\parallel}\) by parts under \(\text{fsavg}\)
  ○ in the curvature term approximate \(\text{fsavg}\) of \(\sin^2 \theta\) by \(1/2\)
  ○ for these purposes neglect nonlinear terms

\[
k_{\parallel} \langle (u_{\parallel} - J_{\parallel}) \cos \theta \rangle = -\frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi - p_e \rangle
\]

• considering ions also and separating \(J_{\parallel}\) these are the divergence balances

\[
k_{\parallel} \langle J_{\parallel} \cos \theta \rangle = -\frac{\omega_B}{2} \frac{\partial}{\partial x} \langle p_e + p_i \rangle \quad \quad k_{\parallel} \langle u_{\parallel} \cos \theta \rangle = -\frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi + p_i \rangle
\]
detailed example – parallel forces

• electrons: start with

\[
\frac{\partial}{\partial t} \left( A_\parallel + \mu_e J_\parallel \right) + [\phi, \mu_e J_\parallel] = \nabla_\parallel (p_e - \phi) - 0.51 \mu_e \nu_e J_\parallel
\]

• flux variable sideband component: multiply by \( \cos \theta \) and then take \( \text{fsavg} \)
  ○ note \( \partial / \partial t \) on sideband piece is neglected by the ordering
  ○ integrate \( \partial / \partial \theta \) in \( \nabla_\parallel \) by parts under \( \text{fsavg} \)
  ○ for these purposes neglect nonlinear terms

\[
k_\parallel \langle (p_e - \phi) \sin \theta \rangle = 0.51 \mu_e \nu_e \langle J_\parallel \cos \theta \rangle
\]

• with temperature dynamics there is more to it than this but the ideas remain
detailed example – parallel forces

• ions: start with

$$\mu_i \frac{\partial}{\partial t} u_\parallel + [\phi, \mu_i u_\parallel] = -\nabla_\parallel (p_e + p_i) + \mu_\parallel \nabla_\parallel^2 (u_\parallel + kq_i_\parallel)$$

• flux variable sideband component: multiply by $\cos \theta$ and then take $\text{fsavg}$
  - note $\partial/\partial t$ on sideband piece is neglected by the ordering
  - integrate $\partial/\partial \theta$ in $\nabla_\parallel$ by parts under $\text{fsavg}$
  - for these purposes neglect nonlinear terms

$$k_\parallel \langle (p_e + p_i) \sin \theta \rangle = -\mu_\parallel k_\parallel^2 \langle (u_\parallel + kq_i_\parallel) \cos \theta \rangle$$

• here we’ve kept enough of the temperature dynamics to get rotation effects
  - because these parallel flux variables are considered in divergence balance

$$k_\parallel \langle u_\parallel \cos \theta \rangle = -\frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi + p_i \rangle \quad k_\parallel \langle q_i_\parallel \cos \theta \rangle = -\frac{5 \omega_B}{2} \frac{\partial}{\partial x} \langle T_i \rangle$$
rotation – zonal charge balance

• start with (warm-ion) vorticity equation, keep polarisation, neglect nonlinearities

\[ \frac{\partial}{\partial t} \rho_s^2 \nabla^2 \perp (\phi + p_i) = B \nabla \frac{J_\parallel}{B} - K (p_e + p_i) \]

• zonal component

\[ \frac{\partial}{\partial t} \rho_s^2 \frac{\partial^2}{\partial x^2} \langle \phi + p_i \rangle = -\omega_B \frac{\partial}{\partial x} \langle (p_e + p_i) \sin \theta \rangle \]

plug in from last page, evaluate coefficients

\[ \frac{\partial}{\partial t} \rho_s^2 \frac{\partial^2}{\partial x^2} \langle \phi + p_i \rangle = -\frac{\mu_\parallel \omega_B^2}{2} \frac{\partial^2}{\partial x^2} \left\langle \left( \phi + p_i + \frac{5}{2} kT_i \right) \right\rangle \]

• this says \( \phi \)-profile relaxes into neoclassical balance with rate \( \mu_\parallel \omega_B^2 / 2 \)
  ○ nonzero fluid rotation given by \( kT_i \) piece, same (relation, damping model) as in

MHD and flow equilibration processes

- the way we did it for flow energetics, *i.e.*, don’t order $\partial / \partial t$
  - allow it to cover from $v_A/qR$ to $c_s/R$ to $nu_i$ to transport

- displayed: isothermal version for clarity, except keeping $kq_i ||$ in viscosity
  - in the computations carry the entire system (12 gyrofluid equations)

- acoustic branch: zonal vorticity $\varpi$, sidebands for $n_e$ and $u ||$

- MHD branch: zonal $n_e$, sidebands for $J ||$ and $\varpi$

- **main point:** these branches are coupled only by two-fluid processes
  - adiabatic compression among sidebands for $n_e$ and $J ||$ and $\phi$
  - diamagnetic compression between zonal and sideband $n_e$
  - in the thermal version, especially important for $T_i$
  - why? because $T_i$ is not constrained by Alfvén dynamics

- these processes strongly alter zonal flow relaxation
  - they are absent in the “resistive ballooning” and “2D interchange” models

- details: *New J Phys* 7 (2005) 92
Sideband Dynamics for Warm-Ion Flows

- zonal vorticity, pressure sideband, sound wave sideband

\[
\frac{\partial}{\partial t} \langle \omega \rangle = -\omega_B \frac{\partial}{\partial x} \langle (p_e + p_i) \sin \theta \rangle
\]

\[
\frac{\partial}{\partial t} \langle n_e \sin \theta \rangle = \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle \phi - p_e \rangle + k_\parallel \langle u_\parallel \cos \theta \rangle - k_\parallel \langle J_\parallel \cos \theta \rangle
\]

\[
\frac{\partial}{\partial t} \mu_i \langle u_\parallel \cos \theta \rangle = -k_\parallel \langle (p_e + p_i) \sin \theta \rangle - \mu_i k_\parallel^2 \langle (u_\parallel + kq_i_\parallel) \cos \theta \rangle
\]

- Alfvén sideband, flow sideband, zonal pressure

\[
\frac{\partial}{\partial t} \left( \langle A_\parallel \cos \theta \rangle + \mu_e \langle J_\parallel \cos \theta \rangle \right) = k_\parallel \langle p_e \sin \theta \rangle - k_\parallel \langle \phi \sin \theta \rangle - 0.51 \mu_e \nu_e \langle J_\parallel \cos \theta \rangle
\]

\[
\frac{\partial}{\partial t} \langle \omega \sin \theta \rangle = -k_\parallel \langle J_\parallel \cos \theta \rangle - \frac{\omega_B}{2} \frac{\partial}{\partial x} \langle p_e + p_i \rangle
\]

\[
\frac{\partial}{\partial t} \langle n_e \rangle = \omega_B \frac{\partial}{\partial x} \langle \phi \sin \theta \rangle - \omega_B \frac{\partial}{\partial x} \langle p_e \sin \theta \rangle
\]
flow equilibration in the fluid model

- logarithmic time axis, shows all successive phases

- rotation balances \((c_s/R)\), relaxes \((\nu_i)\), then decays with transport
  - radial electric field \((A_P\) is axis value of \(\phi\)), poloidal flow \((E_u\) is energy in \(u_\parallel\))
relaxation in a gyrokinetic model

- large-scale “shear-Alfvén gyrokinetics” with fields $\phi, A_\parallel$ and no gyroaveraging
  - axisymmetric geometry and dynamics – a 4D ($2X\times2V$) model


- for each species (ions, electrons, $Z = \pm 1$)

$$B^*_\parallel \frac{\partial f}{\partial t} + \nabla H \cdot \frac{c}{Ze} b \times \nabla f + \left( B + p_z \frac{c}{Ze} \nabla \times b \right) \cdot \left( \frac{\partial H}{\partial p_z} \nabla f - \frac{\partial f}{\partial p_z} \nabla H \right) = C(f)$$

- self-consistent fields

$$\sum_{sp} \int dW \left[ Ze f + \frac{1}{B^*_\parallel} \nabla \cdot B^*_\parallel \frac{f mc^2}{B^2} \nabla \perp \phi \right] = 0$$

$$\nabla^2 \perp A_\parallel + \frac{4\pi}{c} \sum_{sp} \int dW \left[ \frac{Ze}{m} \left( p_z - \frac{Ze}{c} A_\parallel \right) f \right] = 0$$
what Lagrangian did we get this from

- gyrocenter Lagrangian and Hamiltonian in coordinates \( \{ \mathbf{R}, p_z, \mu \} \)

\[
L_p = \left( \frac{Z_e}{c} \mathbf{A} + p_z \mathbf{b} \right) \cdot \dot{\mathbf{R}} + \frac{mc}{Z_e} \mu \dot{\varphi} - H
\]

\[
H = \frac{1}{2m} \left( p_z - \frac{Z_e}{c} A_\parallel \right)^2 + \mu B + Z e \phi - \frac{m c^2}{2 B^2} |\nabla_\perp \phi|^2
\]

- full system Lagrangian as a field theoretical model

\[
L = \int d\mathcal{V} \mathcal{L} \quad \mathcal{L} = \sum_{\mathbf{sp}} \int d\mathcal{W} f \, L_p - \frac{1}{8\pi R^2} |\nabla_\perp (\psi + A_\parallel R)|^2
\]

- integration over space, \( d\mathcal{V} \), and over velocity space, \( d\mathcal{W} \)
  - in this case \( \mathbf{b} = R \nabla \varphi \) and \( B_\parallel^* = B \)

- vary coordinates (eqn for \( f \)), and field variables (eqs for \( \phi, A_\parallel \))
bootstrap current equilibration (collisions)

FEFI 4D, Edge Base Case, nominal $\nu_e a/c_s = 1.88$

\[ bootstrap\ current \quad J_b/\eta_e e c_s \]
bootstrap current equilibration (rho-star)

FEFI 4D, Edge Base Case, nominal $\rho_s/a = 1.83 \times 10^{-3}$

$J_b/n_e e c_s$ vs $c_s t/\alpha$ for different values of $\delta$.
bootstrap current equilibration (rho-star)

FEFI 4D, Edge Base Case, more ITER-relevant, to $\rho_s/a = 6 \times 10^{-4}$
squeezed orbit regime in ITER

• KC Shaing’s squeezed-orbit regime requires $L_{\perp} < \rho_B$


• orbit squeezed by $\partial^2 \phi / \partial x^2$ defined by width of constant-$H$ curve in $RZ$-plane
  ○ orbit is a 4D curve with 3 coordinates fixed $(\epsilon, \mu, \psi_c)$

• in equilibrium state, scale of $\phi$ tied to $L_{\perp}$
  ○ but $L_{\perp}$ is limited by MHD stability (given fixed $qR/a$ etc.)

• by definition, $\rho_s / L_{\perp} \lesssim a/qR$ should then be enough $\rightarrow$ neoclassical transport
  ○ previous slides: only very small $\delta \lesssim 10^{-3}$ reached relaxed neoclassical state

• this was my experience running these cases: for a squeezed-regime start, transport reduced gradients before the finite $\nu_i$ could establish a steady neoclassical state

• it is hard to envisage this regime in the face of transport simultaneously occurring

• nevertheless definitive results not yet in, and it remains an interesting topic
pedestal width – models

  - simple version sets gradient to MHD boundary and width via KBM stability
  - KBM (kinetic ballooning) is a core mode which limits the pedestal top
  - EPED represents current data sets very well
  - let’s say it works for JET-size and smaller, at least

- alternative: use experience that local fluxtube models do not show H-mode transport
  - local requirement: small $\rho_s/L_\perp$
  - hence conjecture that $L_\perp/\rho_s$ has to be below some limit in the pedestal
  - evaluate parameters at pedestal halfway point, with $L_\perp = |\nabla \log T_e|^{-1}$

- the values for AUG #17151 are $L_\perp = 3$ cm and $T = 360$ eV and $B = 2$ T
  - this gives $L_\perp = 24\rho_s$ (if you want sharper $L_\perp$ then it’s even fewer $\rho_s$)

- suppose we say $L_\perp$ must be below $32\rho_s$ to achieve H-mode
  - result: optimistic for AUG, approximate for JET, but very pessimistic for ITER

- suppose we (EPED or other) say in ITER you’ll have $> 64\rho_s$
  - then: local conditions are felt by the turbulence within the pedestal
  - then: $\rightarrow$ no H-mode
local models don’t make H mode

• all the above physics is present
  ◦ it is a 3D model in tokamak geometry, correct boundary conditions
    (global consistency, periodicity constraints, no radial periodicity or ballooning)
  ◦ 12-moment \((e, i)\) electromagnetic gyrofluid model
  ◦ two species \((e, i)\) electromagnetic gyrokinetic model
  ◦ flow effects included (both zonal flow and equilibrium flow, neoclassics)

• always find smooth monotonic rise of flux with parameters, \(e.g., \ldots\)
  ◦ gradient of \(\beta\)
  ◦ collisionality
  ◦ rho-star \((\rho_s/L_\perp) \leftrightarrow \) system size
Edge Core Transition Power Ramp

- model: GEM, 8 flux tubes, spaced at normalised volume radius values
  \[ r_a = \{0.55, 0.61, 0.67, 0.73, 0.79, 0.85, 0.91, 0.97\} \]

- \( T \) and \( \nabla T \) for \( T_e = T_i \) adjusted to get flux times sfc \( \approx \) given input power
  ○ it is an optimisation scheme, not a transport model

- GEM is formulated for all parameters, but lacks trapped electrons
  ○ physics is found to stay in EM/NL ITG plus MHD regime anyway

- model is AUG-sized, profiles for \( q, n_e \), and \( T \) given with LCFS values fixed

- time traces and profiles at several times
  ○ sweep: \( P = 1 \text{ MW} \) ramped after \( t = 1000\tau_{GB} \) to \( 20 \text{ MW} \) at \( t = 20 \times 10^3 \)

power sweep to 20 MW

transport power (MW)

GB time
power sweep to 20 MW
Two Regimes of Edge Pedestal Width Limitation

limited by MHD and KBM (EPED)

limited by DWT (→ local limit/ftubes)

it is possible that new nonlinear physics enters to additionally limit the ITER pedestal
Edge/Pedestal – several physical processes

- we didn’t cover them all, just the basics present in any reasonable model
  - energetic coupling between ExB and thermal energy defines the dynamics
  - not only turbulence, but also ELMs (which saturate on turbulence they generate)
  - not only that but also flow energetics, coupling to resistivity through currents
  - not only that but also equilibrium relaxation which involves diamagnetic compression

- gyrokinetic study of neoclassical flows is very relevant
  - remember: neoclassical process, not “neoclassical theory”

- several external processes which may turn out to be decisive
  - coupling across the LCFS (separatrix) to the SOL
  - sources, e.g., penetration of neutrals into the edge
  - turbulence/MHD energy avalanching down from the core

- despite PR claims, no reasonable L-H transition model exists (process not known)

lots of opportunity for new people to make a mark
think outside the box, stay grounded but independent
you don’t have to follow self-styled gurus