Core Kinetic & Magnetic Control in Tokamak Reactors
Burn Control, Profile Control, Core-Edge Integration

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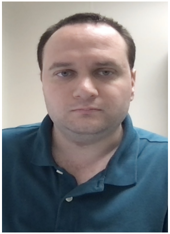

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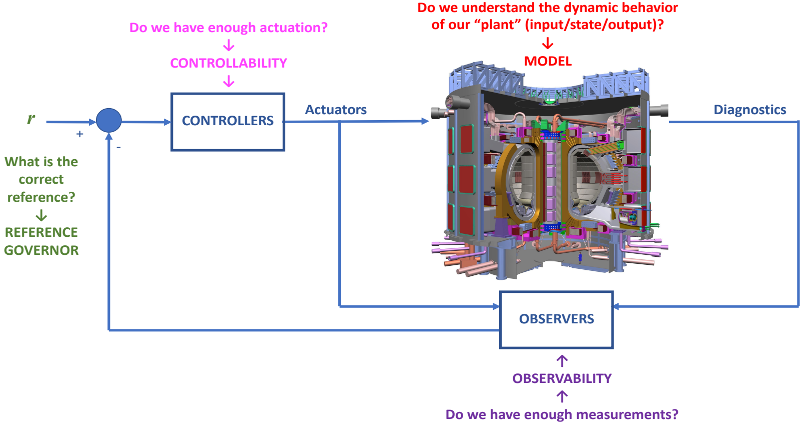
Presentation Outline

Advanced Control Solutions in Fusion Reactors Demands a Model-Based Control Design Approach

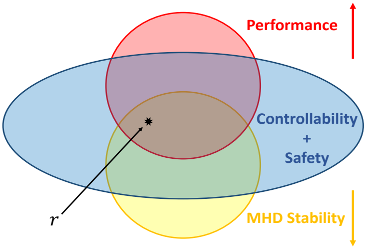


Controllability/Observability are properties of the system (not of controller!)

- Controllability is the curse of present devices → Not incorporated in design!
- Observability: curse of future devices (limited/noisy diagnostics)?

Advanced Control Solutions in Fusion Reactors Demands a Model-Based Control Design Approach

- **MODEL** is absolutely critical to assess **controllability** and **observability**
- **MODEL** is absolutely critical to design **controller + observer (estimator)**
- **MODEL** is absolutely critical to design **reference governor**



- The goal is not just to design controller + observer but to determine $r \rightarrow$ *operating point*
- Operating point: tradeoff between performance and MHD stability within controllability + safety boundaries

Stability is a property of the equilibrium $\rightarrow$ operating point (not of system!)

- **MODEL** is absolutely critical to determine these boundaries
  - Real-time $\rightarrow$ **system supervisor** $\rightarrow$ **reference governor**
- **MODEL** is absolutely critical to design **actuator-management strategies**
- **MODEL** is absolutely critical to assess performance before implementation

Presentation Outline

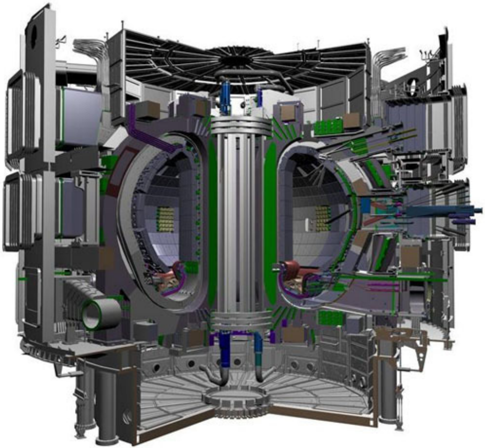
1. Control-oriented Modeling as Enabler of Reactor-level Control Design

2. Kinetic (Burn) Control
   - What Type of Model Do We Need to Use?
   - Nonlinear Control of the Burn Condition
   - How Do We Deal With Model Uncertainties and Unmodeled Dynamics?
   - What is the Correct Reference for the Controller?
   - How Do We Close the Loop if State Is Not Fully Measurable?
   - How Do We Handle Actuator Dynamics?
   - How to Integrate Core Dynamics with SOL/Divertor?

3. Profile Control (Current, Rotation, Temperature, Pressure)
   - What Type of Model Do We Need to Use?
   - Solution Demands Three Components: FF Ctrl + FB Ctrl + Observer
   - Global vs Local Profile Regulation: Fixed vs Moving Targets/Actuators

4. Some Concluding Remarks

The ITER Tokamak Will Explore $Q > 1$ Regimes

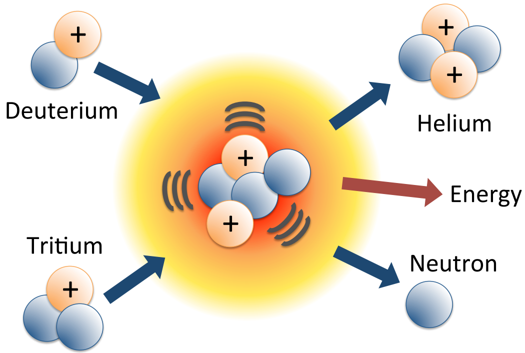


- The first tokamak to explore the **burning plasma regime**
- Designed to achieve
  - $Q > 5$ for $1000s$ long discharges
  - $Q = 10$ for certain operating scenarios
  - $Q \triangleq \frac{P_f}{P_{aux}}$

In order to regulate P_f (and Q), ITER will demand precise control of density and temperature of different plasma species (kinetic control). This is problem is commonly referred to as <u>burn control</u>.

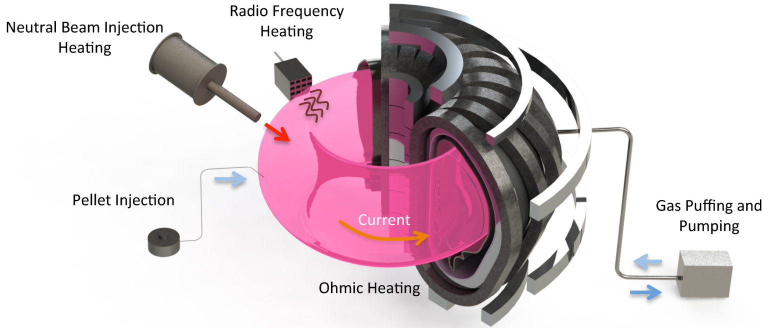
DT Nuclear Fusion Reaction

$$ {}^2_1D + {}^3_1T \rightarrow \underbrace{{}^4_2He}_{3.5MeV} + \underbrace{{}^1_0n}_{14.1MeV}, \qquad Q_{DT} = \Delta mc^2 = 17.6MeV $$



- Neutron escapes to the walls. It cannot be confined magnetically.
  - It does NOT enter the energy balance equation for the plasma
- Energetic **alpha particle** remains in plasma $\rightarrow$ **'self-heating' source**.
  - It does enter the energy/particle balance equations for the plasma

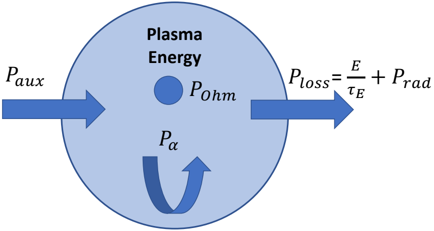
Actuators Used To Control Kinetic Variables

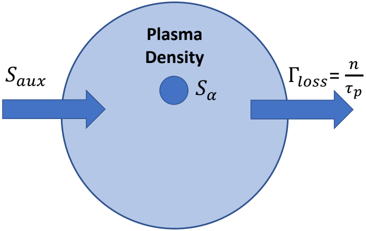


- Current contributes to heating through **Ohmic heating** (small in reactor)
- Magnetic configuration affects burn condition through **confinement time**
- **Neutral beam injectors** and **radio frequency waves** heat the plasma
- Refueling at the plasma boundary is achieved through **gas puffing**
- **Pellet injection** refuels the plasma in the core and/or injects impurities
- **Impurity injection** dilutes the fuel content and increases radiation losses
- **Gas pumping** removes exhausted fuel, alpha particles, and impurities

Presentation Outline

Response Model Based on Balance Equations





$$\frac{dE}{dt} = -P_{loss} + P_\alpha + P_{Ohm} + P_{aux}$$

$$\frac{dn}{dt} = -\Gamma_{loss} \pm B + S$$

- Control goal is 0D $\rightarrow$ Response model is 0D
- 0D response model is based on energy/particle balance equations
- Particle balance equations are needed for all species

Nonlinear Response Model for Kinetic Control Design

$$\text{Energy:} \quad \frac{dE}{dt} = -\frac{E}{\tau_E} + \underbrace{\overbrace{Q_\alpha S_\alpha}^{P_\alpha} - P_{rad} + P_{Ohm} + P_{aux} + P_{aux}^{burn}}_{P}$$

$$\text{Alpha particles:} \quad \frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + S_\alpha$$

$$\text{Deuterium:} \quad \frac{dn_D}{dt} = -\frac{n_D}{\tau_D} - S_\alpha + S_D^{rec} + S_D^{inj}$$

$$\text{Tritium:} \quad \frac{dn_T}{dt} = -\frac{n_T}{\tau_T} - S_\alpha + S_T^{rec} + S_T^{inj}$$

$$\text{Impurities:} \quad \frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I^{inj}$$

(actuators/disturbances in red/blue)

- From the neutrality condition, $n_e = n_D + n_T + 2n_\alpha + Z_I n_I$.
- The density and temperature are

$$n = \overbrace{n_\alpha + n_D + n_T + n_I}^{n_i} + n_e = 2n_D + 2n_T + 3n_\alpha + (Z_I + 1)\, n_I$$

$$T = \frac{2}{3}\frac{E}{n} = \frac{2}{3}\frac{E}{2n_D + 2n_T + 3n_\alpha + (Z_I + 1)\, n_I} \qquad (T_i = T_e = T)$$

Nonlinear Response Model for Kinetic Control Design

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Impurities: $$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I^{inj}$$ (actuators/disturbances in red/blue)

- Reaction rate: $S_\alpha = n_D n_T \langle \sigma \nu \rangle = \gamma\,(1-\gamma)\, n_{DT}^2 \langle \sigma \nu \rangle$, $(\gamma\,(1-\gamma)$ peaks at $\gamma = 0.5)$
- Tritium fraction: $\gamma = n_T/n_{DT}$, DT density: $n_{DT} = n_T + n_D$.
- DT reactivity $\langle \sigma \nu \rangle$ is highly nonlinear function of plasma temperature, i.e.

$$\langle \sigma \nu \rangle = exp\left(\frac{a}{T^r} + a_2 + a_3 T + a_4 T^2 + a_5 T^3 + a_6 T^4\right)$$

Nonlinear Response Model for Kinetic Control Design

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Impurities: $$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I^{inj}$$ (actuators/disturbances in red/blue)

- Confinement is a nonlinear function of states and plasma parameters.
- Confinement scaling: IPB98(y,2) scaling

$$\tau_E = 0.0562 H_H(I_{coil}) I_p^{0.93} B_T^{0.15} P^{-0.69} n_{e19}^{0.41} M^{0.19} R^{1.97} \epsilon^{0.58} \kappa_{95}^{0.78}.$$

- Particle confinement assumed proportional to τ_E, i.e.

$$\tau_\alpha = k_\alpha \tau_E,\ \tau_D = k_D \tau_E,\ \tau_T = k_T \tau_E,\ \tau_I = k_I \tau_E.$$

Nonlinear Response Model for Kinetic Control Design

Energy: $$\frac{dE}{dt} = -\frac{E}{\tau_E} + \underbrace{\overbrace{Q_\alpha S_\alpha}^{P_\alpha} - P_{rad} + P_{Ohm} + P_{aux} + P_{aux}^{burn}}_{P}$$

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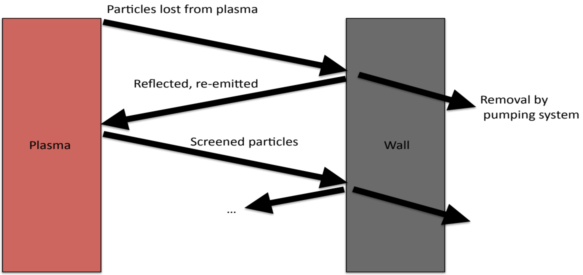
Impurities: $$\frac{dn_I}{dt} = -\frac{n_I}{\tau_I} + S_I^{sp} + S_I^{inj}$$ (actuators/disturbances in red/blue)

- Fuel recycling is included via nonlinear functions S_D^{rec}, S_T^{rec} of the states.
- P_α, P_{rad}, P_{Ohm} are nonlinear functions of states and plasma parameters.

Nonlinear Response Model for Kinetic Control Design

$$S_D^R = \frac{1}{1 - f_{ref}\left(1 - f_{eff}\right)} \left\{ f_{ref}\frac{n_D}{\tau_D} + \left(1 - \gamma^{PFC}\right) \times \left[\frac{\left(1 - f_{ref}\left(1 - f_{eff}\right)\right) R^{eff}}{1 - R^{eff}\left(1 - f_{eff}\right)} - f_{ref} \right] \left(\frac{n_D}{\tau_D} + \frac{n_T}{\tau_T} \right) \right\}$$

$$S_T^R = \frac{1}{1 - f_{ref}\left(1 - f_{eff}\right)} \left\{ f_{ref}\frac{n_T}{\tau_T} + \gamma^{PFC} \times \left[\frac{\left(1 - f_{ref}\left(1 - f_{eff}\right)\right) R^{eff}}{1 - R^{eff}\left(1 - f_{eff}\right)} - f_{ref} \right] \left(\frac{n_D}{\tau_D} + \frac{n_T}{\tau_T} \right) \right\}$$



Recycled fluxes S_D^R, S_T^R are functions of$^{\dagger,[1]}$:

- f_{ref} : Reflection Fraction
- f_{eff} : Recycling Efficiency
- R^{eff} : Recycling Coefficient
- γ^{PFC} : PFC Tritium Fraction

$$P_{rad} = \underbrace{A_{brem}\left(n_D + n_T + 4n_\alpha + Z_I^2 n_I\right) n_e \sqrt{T(keV)}}_{\text{Bremmstrahlung}}, \quad P_{Ohm} = 2.8 \times 10^{-9} \frac{Z_{eff} I^2}{a^4 T^{3/2}}$$

† Ehrenberg J. 1996 Physical Processes of the Interaction of Fusion Plasmas with Solids (New York: Academic)
[1] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).

Lawson Criterion for Ignition

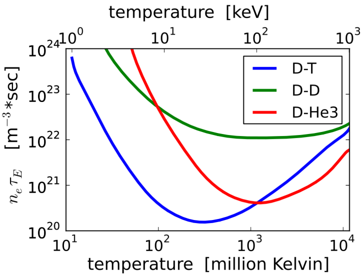
- Steady-state ($dE/dt \equiv 0$) power balance:

$$P_\alpha + P_{aux} = P_L \triangleq E/\tau_E$$

where P_{Ohm}, P_{rad} are neglected.

- At ignition ($P_{aux} \equiv 0, Q = \infty$):

$$P_L = P_\alpha \iff P_{aux} = 0$$



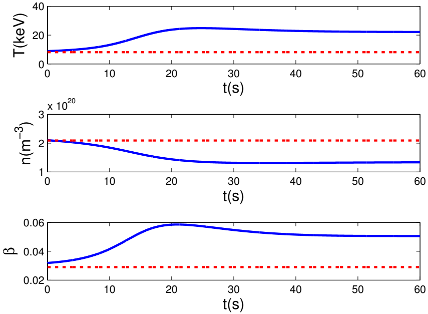
- The ignition condition can be written as

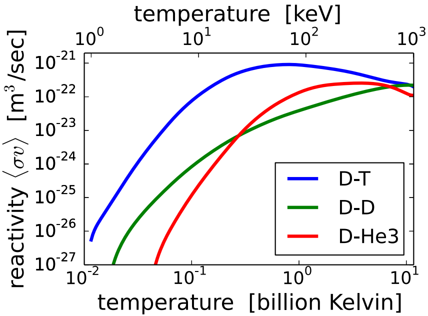
$$\left.\begin{array}{rcl} n_D n_T < \sigma V > E_\alpha & = & \frac{\frac{3}{2}nT}{\tau_E} \\ n_e & = & n_D + n_T = n_i \\ n & = & n_i + n_e \; = \; 2n_e \\ n_D & = & n_T \end{array}\right\} \Rightarrow n_e \tau_E|_{IGN} \propto \frac{T}{< \sigma V > E_\alpha} \equiv g(T)$$

where $n_\alpha = n_I \equiv 0$ is assumed.

– We are interested in operating at the minimizing temperature (lower $n_e \tau_E$).
– The DT reaction appears as the most promising (easiest) reaction.
– Requirements can also be derived for finite Q and non-zero P_{rad}, n_α, n_I.

Burn Control Challenges





- Potential for **thermal instability**:

$$P_f = n_D n_T < \sigma v > Q_{DT} \propto \beta^2 B^4, \quad \beta = \frac{nkT}{\frac{B^2}{2\mu_o}}$$

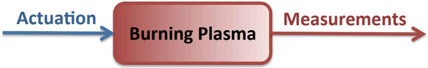
- Even when operating at stable equilibria, system **performance during transients and disturbances** could be **undesirable without control.**

Presentation Outline

Burn Control Needs and Objectives

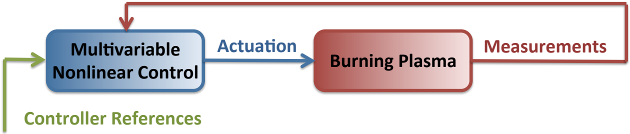
- **Fusion power regulation $\leftrightarrow$ species density/temperature control**
- Burn condition demands effective feedback control scheme to avoid:
  - **Undesirable transient performance** due to **nonlinear/coupled dynamics**
  - **Perturbations** due to **plasma changes (confinement, impurity content)**
  - **Potentially disruptive plasma conditions** due to **thermal instabilities**
- Capability of controller designed based on **linearized model:**
  - ✓ **Regulation around a desired burning equilibrium point**
  - × **Drive plasma from one operating point to another (Modify Q or P_f)**
  - × **Access to and exit from the burning plasma mode**
- Wall heat load tolerance may impose constraints on core burn regulation
  - Requires nonlinear controller that can effectively change operating point
- Coupling with other control problems and objectives is severe
  - Confinement: PF coils (shape, current), Non-axisymmetric coils (RWM/ELM)
  - Heating/Density: Non-inductive current drive (q-profile, NTM)
- Reactor-specific additional challenges for effective burn control:
  - **Limited and noisy set of diagnostics**
  - $P_\alpha >> P_{aux}$**: control by heating may not be effective**
  - **Wall recycling effects may also make control by fueling not effective**

Burn Control Scheme: Nonlinear Feedback Controller



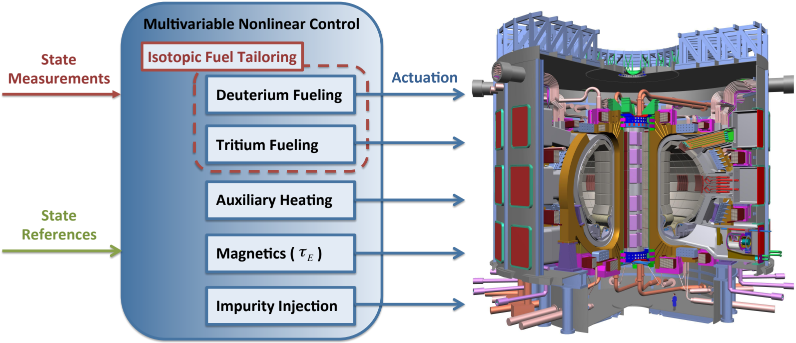
- Nonlinear dynamics
- Multiple inputs and outpus

Burn Control Scheme: Nonlinear Feedback Controller



- Nonlinear dynamics
- Multiple inputs and outpus

Nonlinear (Lyapunov-based) Feedback Controller



- The approach embeds whole nonlinear dynamics of burning plasma in controller by **avoiding linearization of the model** around operating point.
  - Preserving nonlinear dynamics is key to achieve controller's goals.
- The approach uses combination of actuators **(SISO** $\rightarrow$ **MIMO)**.

[1] E. Schuster, M. Krstic and G. Tynan, Fusion Engineering and Design, 63-64, pp. 569-575, 2002.
[2] E. Schuster, M. Krstic and G. Tynan, Fusion Science and Technology, vol. 43, no. 1, 2003.
[3] M.D. Boyer and E. Schuster, Nuclear Fusion 55 (2015) 083021 (24pp).
[4] A. Pajares and E. Schuster, Fusion Engineering and Design 123 (2017) 607–611.

Lyapunov Theory in a Nutshell

Theorem: Let us consider the autonomous nonlinear dynamic system

$$\dot{x} = f(x) \tag{1}$$

where $f : D \to R^n$ is a *continuously differentiable* map from a domain $D \subset R^n$ into R^n. Suppose $\bar{x} = 0 \in D$ is an equilibrium point of (1), i.e.,

$$f(0) = 0. \tag{2}$$

Let $V : D \to R$ be a continuously differentiable function, such that

$$V(0) = 0 \text{ and } V(x) > 0 \text{ in } D - \{0\} \tag{3}$$

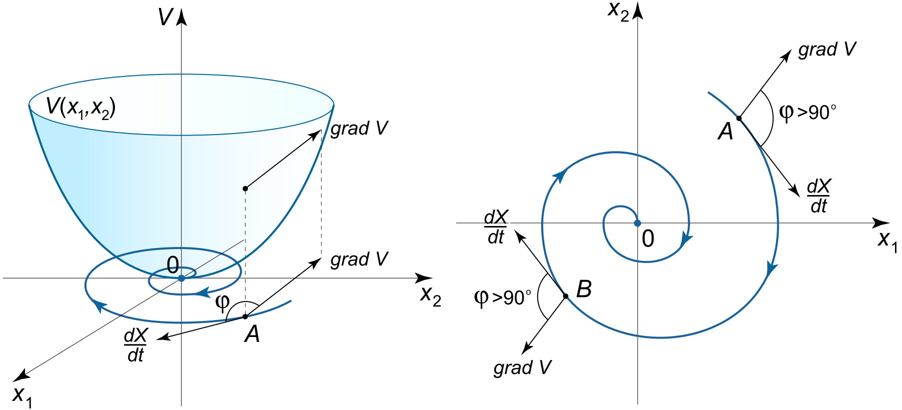
$$\dot{V}(x) = \frac{\partial V}{\partial x}\dot{x} = \frac{\partial V}{\partial x} f(x) \leq 0 \text{ in } D \tag{4}$$

Then, $\bar{x} = 0$ is *stable*. Moreover, if

$$\dot{V}(x) = \frac{\partial V}{\partial x}\dot{x} = \frac{\partial V}{\partial x} f(x) < 0 \text{ in } D - \{0\} \tag{5}$$

then $\bar{x} = 0$ is *asymptotically stable* ($\lim_{t\to\infty} x(t) = 0$).

Lyapunov Theory in a Nutshell



If the derivative $\frac{dV}{dt} = \frac{\partial V}{\partial x} f(x)$ along a phase trajectory is everywhere negative, then the trajectory tends to the origin, i.e. the system is asymptotically stable.

$\dot{V} \equiv \frac{dV}{dt}$ will be negative as long as the angle ϕ between grad $V \equiv \frac{\partial V}{\partial x}$ and $\dot{x} \equiv \frac{dx}{dt} = f(x)$ is higher than 90°.

Lyapunov Theory in a Nutshell

We are interested in an extension of the Lyapunov function concept, called a *control Lyapunov function* (CLF). Let us consider the following system:

$$\dot{x} = f(x, u), \quad x \in R^n, \quad u \in R, \quad f(0,0) = 0,$$

Task: Find feedback control $u = \alpha(x)$, Lyapunov function candidate $V(x)$ s.t.

$$\dot{V} = \frac{\partial V}{\partial x}(x) f(x, \alpha(x)) \leq -W(x), \quad W(x) \text{ positive definite}$$

A system for which good choices of $V(x)$ and $W(x)$ exist is said to have a CLF.

Nonlinear Feedback Controller Design

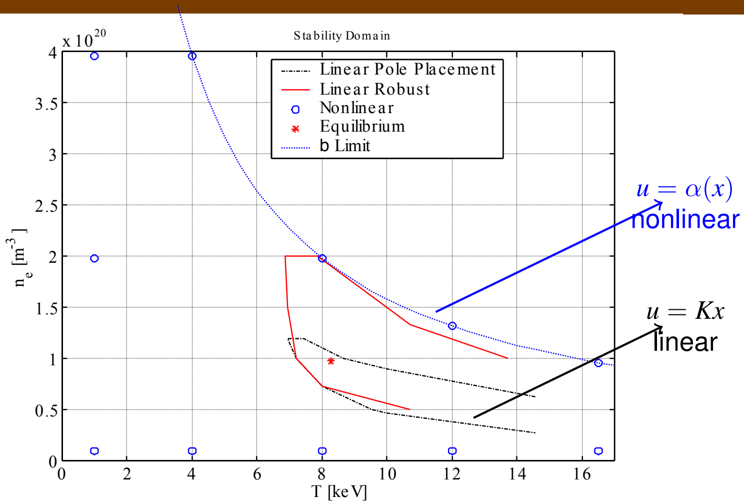
1. Define operating point: $\bar{E}$, $\bar{n}_D$, $\bar{n}_T$, $\bar{n}_\alpha$, $\bar{n}_I \equiv 0$.
2. Write **dynamics of deviations** of states from desired operating point:

$$\tilde{E} \triangleq E - \bar{E}, \tilde{n}_D \triangleq n_D - \bar{n}_D, \tilde{n}_T \triangleq n_T - \bar{n}_T, \tilde{n}_\alpha \triangleq n_\alpha - \bar{n}_\alpha, \tilde{n}_I \triangleq n_I$$

3. Choose a **Lyapunov function candidate** $V = V(\tilde{E}, \tilde{n}_D, \tilde{n}_T, \tilde{n}_\alpha, \tilde{n}_I)$ and calculate its derivative $\dot{V} = \frac{dV}{d\tilde{E}}\dot{\tilde{E}} + \frac{dV}{d\tilde{n}_D}\dot{\tilde{n}}_D + \frac{dV}{d\tilde{n}_T}\dot{\tilde{n}}_T + + \frac{dV}{d\tilde{n}_\alpha}\dot{\tilde{n}}_\alpha + \frac{dV}{d\tilde{n}_I}\dot{\tilde{n}}_I$.
4. Determine **control laws** for available actuators P_{aux}^{burn}, S_D^{inj}, S_T^{inj}, S_I^{inj}, I_{coil} that make $\dot{V}$ **negative everywhere except at equilibrium**, where it is zero.
   - Actuators are used to cancel nonlinear and possibly destabilizing terms, and to add in stabilizing terms with **design parameters** that can be chosen to adjust response time, robustness to uncertainties, and sensitivity to noise.

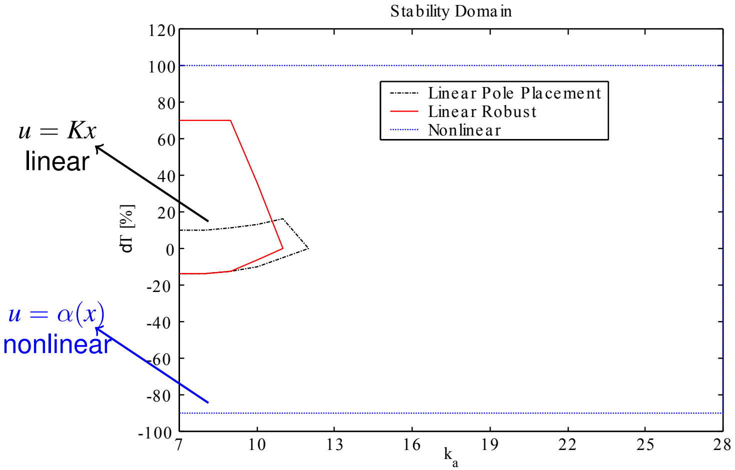
- This technique results in a nonlinear control law and **avoids the need for linearization** around a particular operating point, which satisfies goals:
  - ✓ **Regulation around a desired burning equilibrium point**
  - ✓ **Drive plasma from one operating point to another (Modify Q or P_f)**
  - ✓ **Access to and exit from the burning plasma mode**

Potential of Nonlinear Control: Burn Performance



- Comparative study is carried out generating initial perturbations around the equilibrium for T and n_e keeping $f_\alpha = n_\alpha / n_e$ constant.
- While the boundaries shown for the linear controllers are absolute, for the nonlinear controller they only indicate the test limits.

Potential of Nonlinear Control: Burn Robustness



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Presentation Outline

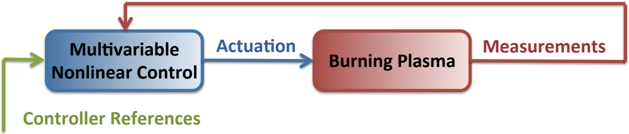
Adding Robustness by Specific Design Techniques

- **Embedding nonlinear dynamics of burning plasma in the control synthesis allows for higher levels of performance and robustness.**

How Do We Handle Uncertainties/Time-variation In Control-oriented Models?

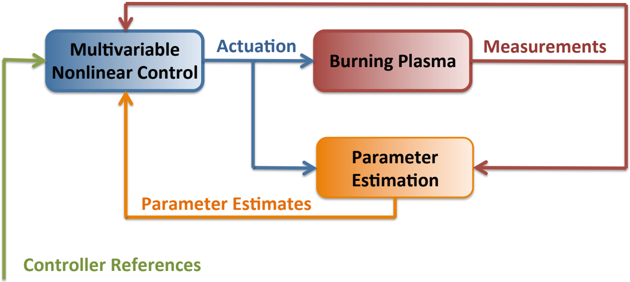
- The main characteristic of feedback is its ability to deal with model uncertainties (unmodeled dynamics in approximate response models).
- Moreover, there are specific tools within the body of mathematical theory of control to specifically deal with model uncertainties:
  - **Adaptive Control**
  - **Robust Control**

Burn Control Scheme: Adaptation by Estimation



- Many of the burning plasma model parameters may be uncertain.
- The control algorithm must make use of estimated model parameters.
- Adaptive control is proposed to ensure tracking despite uncertainty.

Burn Control Scheme: Adaptation by Estimation



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- The control algorithm must make use of estimated model parameters.
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Adaptive Estimator for Unknown Model Parameters

We define a system observer as

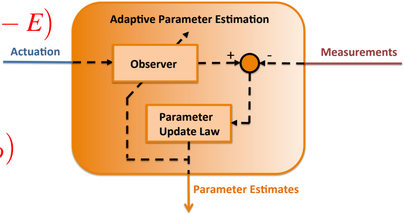
$$\dot{E}^{ob} = -\hat{\theta}_1 \frac{E}{\tau_E} + P_\alpha - P_{rad} + P_{aux} + P_{Ohm} - K_E^{ob}\left(E^{ob} - E\right)$$

$$\dot{n}_\alpha^{ob} = -\hat{\theta}_2 \frac{n_\alpha}{\tau_E} + S_\alpha - K_\alpha^{ob}\left(n_\alpha^{ob} - n_\alpha\right)$$

$$\dot{n}_D^{ob} = -\hat{\theta}_3 \frac{n_D}{\tau_E} + \hat{\theta}_4 \frac{n_T}{\tau_E} - S_\alpha + S_D^{inj} - K_D^{ob}\left(n_D^{ob} - n_D\right)$$

$$\dot{n}_T^{ob} = \hat{\theta}_5 \frac{n_D}{\tau_E} - \hat{\theta}_6 \frac{n_T}{\tau_E} - S_\alpha + S_T^{inj} - K_T^{ob}\left(n_T^{ob} - n_T\right)$$

$$\dot{n}_I^{ob} = -\hat{\theta}_7 \frac{n_I}{\tau_E} + S_I^{inj} + S_I^{sp} - K_I^{ob}\left(n_I^{ob} - n_I\right)$$



The dynamics of the error $\tilde{\theta} = \theta - \hat{\theta}$ can be asymptotically stabilized by taking

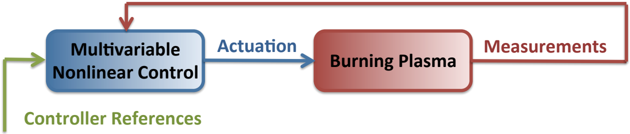
$$\dot{\hat{\theta}} = -\frac{1}{\tau_E}\Gamma \left[\begin{array}{ccccccc} \tilde{n}_\alpha^{ob} n_\alpha & \tilde{E}^{ob} E & \tilde{n}_D^{ob} n_D & -\tilde{n}_D^{ob} n_T & -\tilde{n}_T^{ob} n_D & \tilde{n}_T^{ob} n_T & \tilde{n}_I^{ob} n_I \end{array}\right]^T, \Gamma > 0$$

where

$$\tilde{n}_\alpha^{ob} = n_\alpha^{ob} - n_\alpha, \tilde{E}^{ob} = E^{ob} - E, \tilde{n}_I^{ob} = n_I^{ob} - n_I, \tilde{n}_D^{ob} = n_D^{ob} - n_D, \tilde{n}_T^{ob} = n_T^{ob} - n_T.$$

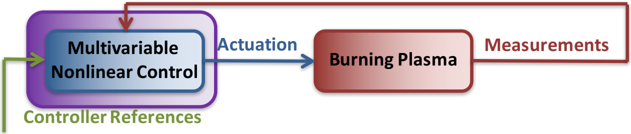
[1] M.D. Boyer and E. Schuster, Plasma Physics and Controlled Fusion 56 104004 (2014).

Burn Control Scheme: Robustness by Augmentation



- Many of the burning plasma model parameters may be uncertain.
- The control algorithm must make use of uncertainty bounds.
- Robust control is proposed to ensure tracking despite uncertainty.

Burn Control Scheme: Robustness by Augmentation



- Many of the burning plasma model parameters may be uncertain.
- The control algorithm must make use of uncertainty bounds.
- Robust control is proposed to ensure tracking despite uncertainty.

Robustness Against Drifts in Fueling Concentrations

- Injection rates for D and T can be written as

$$S_D^{inj} = (1 - \gamma_{DT}) S_{DT}^{line} + (1 - \gamma_D) S_D^{line}$$
$$S_T^{inj} = \gamma_{DT} S_{DT}^{line} + \gamma_D S_D^{line}$$

where the tritium fractions $\gamma_{DT} \in [0, 1]$, $\gamma_D \in [0, 1]$ characterize the tritium concentration in the DT and D fueling lines.

- In the nominal case, $\gamma_{DT} = \gamma_{DT}^{nom} = 0.9$ and $\gamma_D = \gamma_D^{nom} = 0$.
- Unknown variations over time in the tritium fractions are modeled as

$$\gamma_{DT} = \gamma_{DT}^{nom} + \delta_{DT}, \quad \gamma_D = \gamma_D^{nom} + \delta_D, \qquad (6)$$

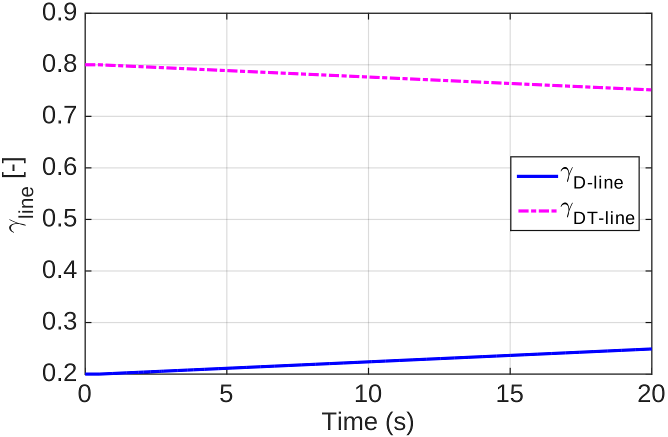
where δ_{DT} and δ_D are “model uncertainties” in the tritium fractions.

- From definition, $\delta_{DT} \in [-0.9, 0.1]$, $\delta_D \in [0, 1] \Rightarrow$ bounded uncertainties.

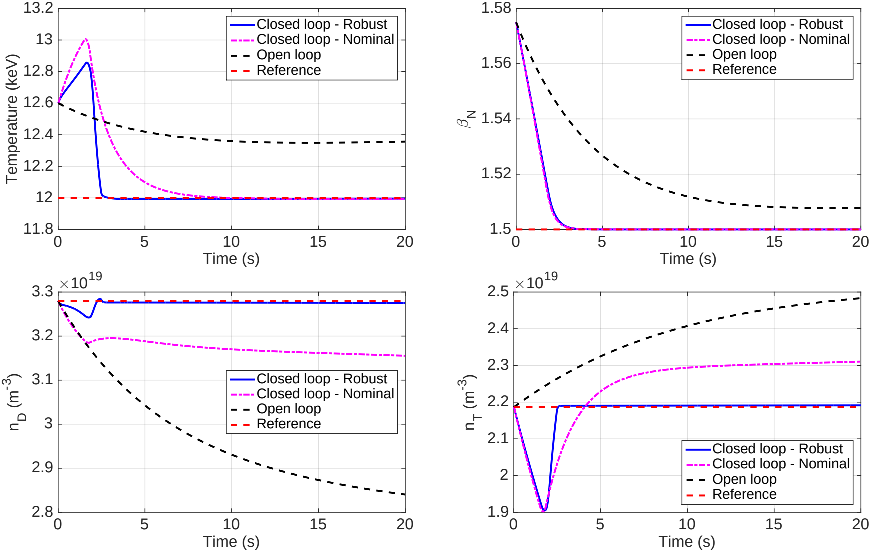
[1] A. Pajares, E. Schuster, Nuclear Fusion 59 (2019) 096023 (18pp).

Robustness Against Drifts in Fueling Concentrations

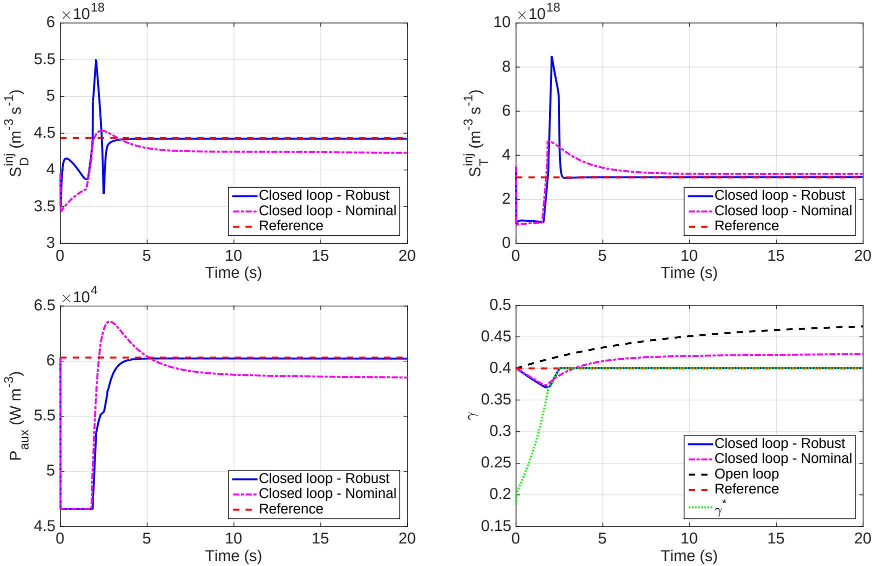
- Controller tries to regulate the system around a nominal equilibrium point defined by $\bar{T} = 12keV$, $\bar{\gamma} = 0.4$ and $\bar{\beta}_N = 1.5$
- The system starts from a perturbed initial condition of +5% in E.
- Time variations in γ_{DT} and γ_D are introduced to the system.



Robustness Against Drifts in Fueling Concentrations



Robustness Against Drifts in Fueling Concentrations



Presentation Outline

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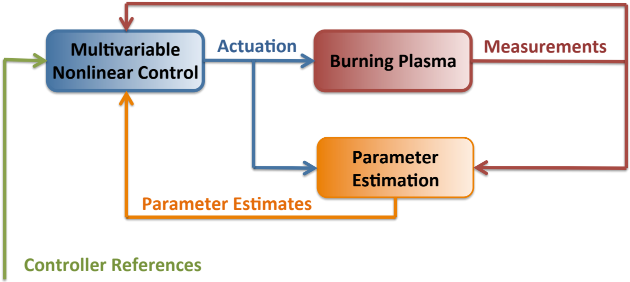
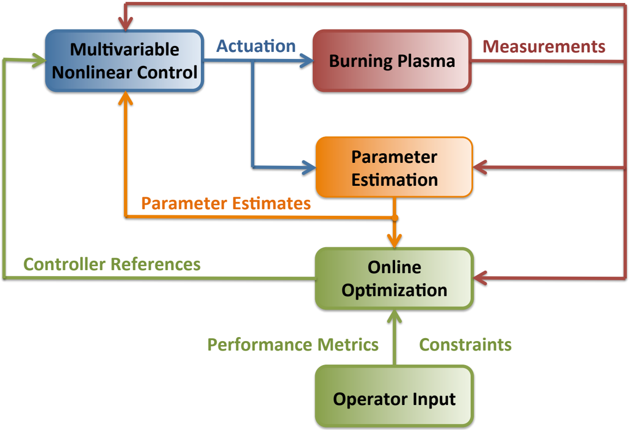
Burn Control Scheme: Optimal Reference Governor



- Part of burn control problem is selection of controller references.
- References must be chosen to optimize figure of merit for performance.
- Convex optimization is proposed to ensure optimal reference selection.

Burn Control Scheme: Optimal Reference Governor

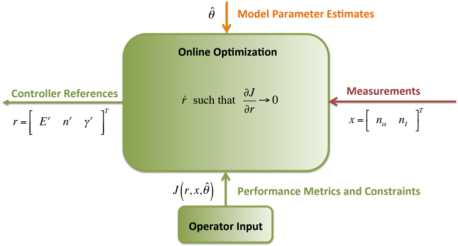


Real-time Optimal Reference Governor

$$J = \overbrace{\frac{w_T}{2}}^{\text{weight}}\Big(T - \underbrace{T^{des}}_{\text{target}}\Big)^2 + \overbrace{\frac{w_{P_\alpha}}{2}}^{\text{weight}}\Big(P_\alpha - \underbrace{P_\alpha^{des}}_{\text{target}}\Big)^2 + \overbrace{\frac{w_\gamma}{2}}^{\text{weight}}\Big(\gamma - \underbrace{\gamma^{des}}_{\text{target}}\Big)^2 \overbrace{-\frac{1}{\eta_c}\sum_{i=1}^{K}\ln(-g_i)}^{\text{constraints}}.$$

- A **reference for the controlled states** $r = [E^r, n^r, \gamma^r]^T$ determines the burn condition.
- T^{des}, P_α^{des}, γ^{des} are desired targets.
- w_T, w_{P_α}, w_γ are tracking weights.
- Constraints by **barrier function**

$$g_i(E, n, \gamma, n_\alpha, n_I) < 0$$

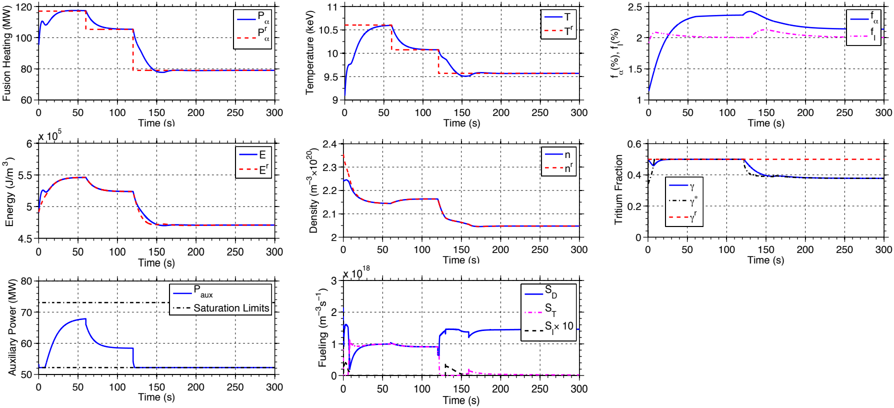


- The optimization is achieved by defining $V_r = \frac{1}{2}\left(\frac{\partial J}{\partial r}\right)^T \frac{\partial J}{\partial r}$ and choosing

$$\dot{r} = -\left(\frac{\partial^2 J}{\partial r^2}\right)^{-1}\left[K_{RTO}\frac{\partial J}{\partial r} + \frac{\partial^2 J}{\partial r \partial x}\dot{x} + \frac{\partial^2 J}{\partial r \partial \hat{\theta}}\dot{\hat{\theta}}\right] \Rightarrow \dot{V}_r \leq 0 \Rightarrow \frac{\partial J}{\partial r} \to 0 \Rightarrow r \to r^*$$

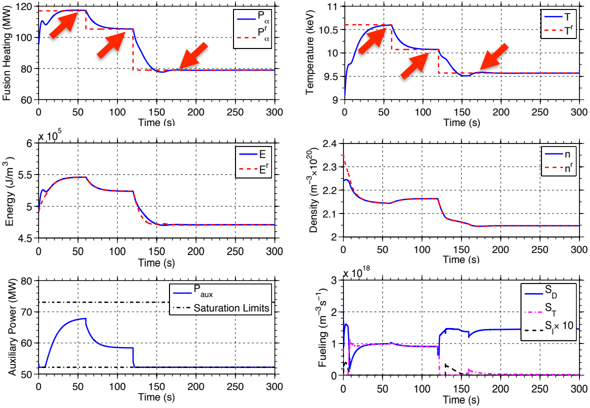
- The cost function is user-defined!
- More sophisticated optimization problems are feasible!

Changing Operating Points via Online Optimization



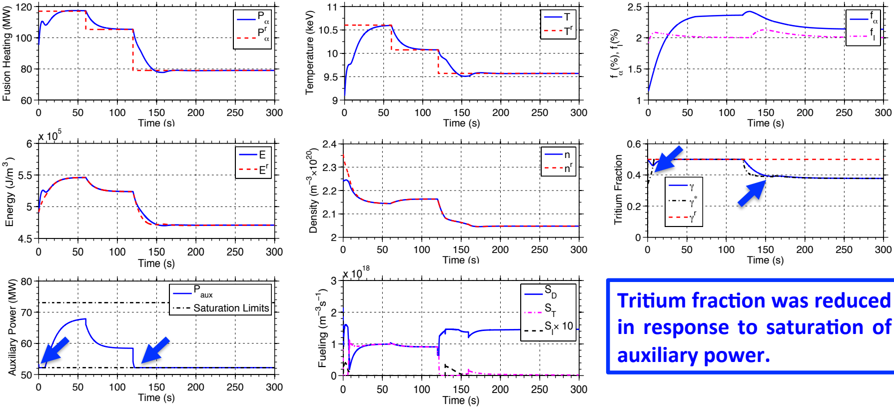
- Conditions: γ^r is kept constant in this simulation and $w_\gamma \equiv 0$
- Constraints: 53MW $< P_{aux} <$ 73MW
- Recycling: $\gamma^{PFC} = 0.5, f_{eff} = 0.3, f_{ref} = 0.5, R_{eff} = 0.95 \Rightarrow$ poor γ control
- Simulation conditions chosen to ensure impurity injection is needed

Changing Operating Points via Online Optimization



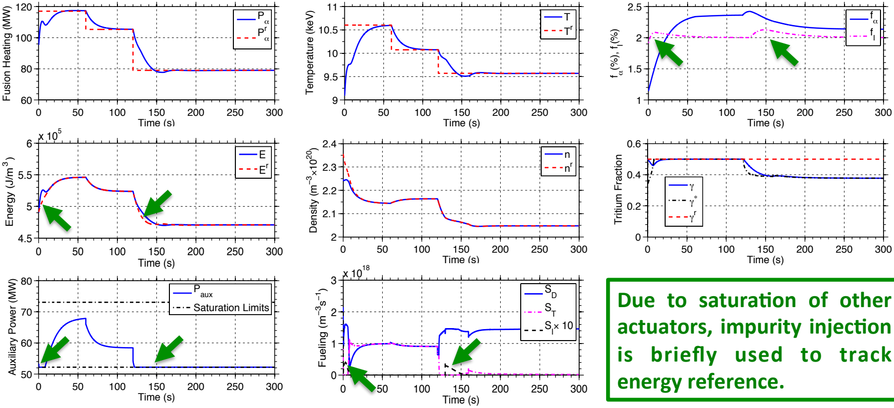
Nonlinear controller and online optimization scheme were able to switch between desired operating points.

Changing Operating Points via Online Optimization



Tritium fraction was reduced in response to saturation of auxiliary power.

Changing Operating Points via Online Optimization



Due to saturation of other actuators, impurity injection is briefly used to track energy reference.

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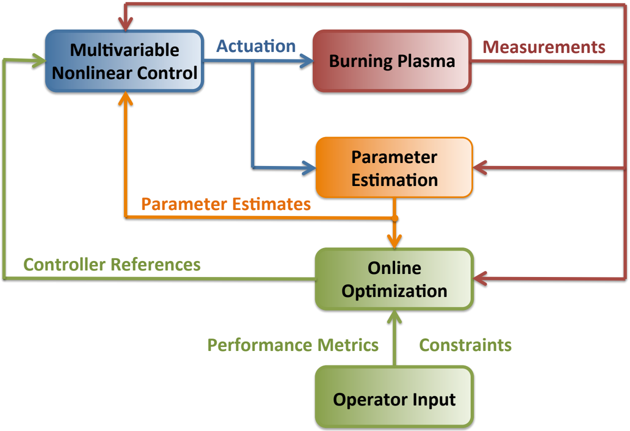
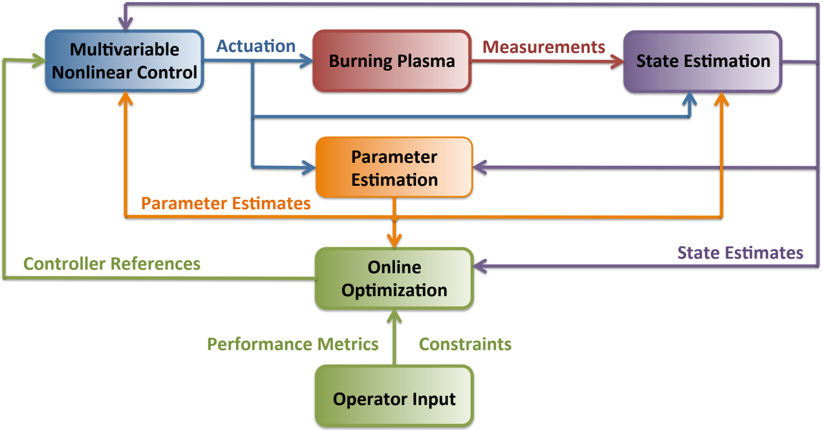
Burn Control Scheme: State Estimation via Observer



- The plasma state needed for feedback control may not be fully measurable.
- This will be a critical issue in future fusion reactors ...
- Limited number and lower quality (e.g., noise) of diagnostics.

Burn Control Scheme: State Estimation via Observer



- The plasma state needed for feedback control may not be fully measurable.
- This will be a critical issue in future fusion reactors ...
- Limited number and lower quality (e.g., noise) of diagnostics.

Estimator for State Variables Needed by Control Law

We define an observer as

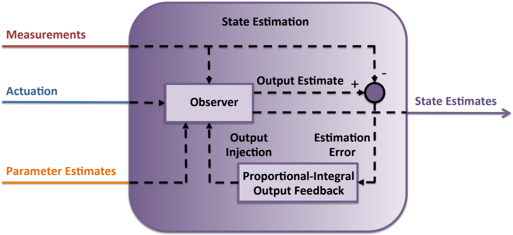
$$\dot{\mathring{E}} = -\hat{\theta}_1 \frac{\mathring{E}}{\tau_E^{sc}} + P_\alpha - P_{rad} + P_{Ohm} + P_{aux} + L_E,$$

$$\dot{\mathring{n}}_\alpha = -\hat{\theta}_2 \frac{\mathring{n}_\alpha}{\tau_E^{sc}} + S_\alpha + L_\alpha,$$

$$\dot{\mathring{n}}_D = -\hat{\theta}_3 \frac{\mathring{n}_D}{\tau_E^{sc}} + \hat{\theta}_4 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_D^{inj} + L_D,$$

$$\dot{\mathring{n}}_T = \hat{\theta}_5 \frac{\mathring{n}_D}{\tau_E^{sc}} - \hat{\theta}_6 \frac{\mathring{n}_T}{\tau_E^{sc}} - S_\alpha + S_T^{inj} + L_T,$$

$$\dot{\mathring{n}}_I = -\hat{\theta}_7 \frac{\mathring{n}_I}{\tau_E^{sc}} + S_I^{sp} + S_I^{inj} + L_I,$$



- We consider a general nonlinear output map $y = h(n_\alpha, E, n_I, n_D, n_T)$.
- The system is augmented with an additional state, $\check{z}$, governed by

$$\dot{\check{z}} = \mathring{y} - y = \check{y}.$$

- Based on Lyapunov analysis, the injection terms L_E, L_α, L_D, L_T, L_I adopt a proportional-integral output feedback form.

[1] M. D. Boyer, E. Schuster, International Federation of Automatic Control World Congress (2014).

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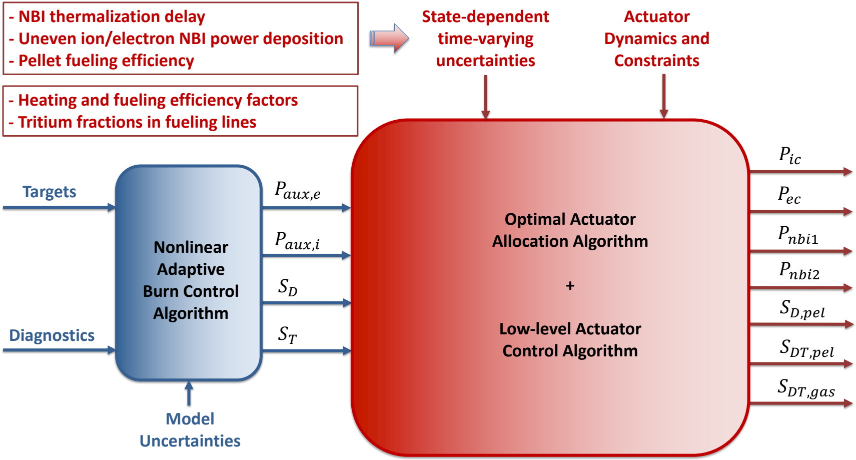
Optimal Allocation of Actuators with Dynamics

- Two-temperature model ($T_i \neq T_e$). Heating and fueling as actuation.
- Virtual control inputs $\leftrightarrow$ Effector System $\leftrightarrow$ Physical control inputs.



[1] V. Graber and E. Schuster, Nuclear Fusion 62 (2022) 026016 (18pp).
[2] V. Graber and E. Schuster, “Nonlinear Burn Control and Optimal Actuator Allocation of ITER Plasmas with Uncertain Parameters and Actuator Dynamics,” Thursday Poster Session (#14).

Effector System: Heating and Fueling From Actuators

The Effector System maps the control efforts v to the actuator efforts u:

$$v = [P_{aux,i}\ P_{aux,e}\ S_D\ S_T]^T \quad \longleftrightarrow \quad u = [P_{ic}\ P_{ec}\ P_{nbi_1}\ P_{nbi_2}\ S_{D_{pel}}\ S_{DT_{pel}}\ S_{DT_{gas}}]^T$$

$$P_{aux,i} = \eta_{ic} P_{ic} + \eta_{nbi_1} \phi_{nbi} P_{nbi_1} + \eta_{nbi_2} \phi_{nbi} P_{nbi_2}$$

$$P_{aux,e} = \eta_{ec} P_{ec} + \eta_{nbi_1} \bar{\phi}_{nbi} P_{nbi_1} + \eta_{nbi_2} \bar{\phi}_{nbi} P_{nbi_2} \qquad (\text{where } \bar{\phi}_{nbi} = 1 - \phi_{nbi})$$

$$S_D = \eta_{nbi_1} \frac{P_{nbi_1}}{\varepsilon_{nbi_0}} + \eta_{nbi_2} \frac{P_{nbi_2}}{\varepsilon_{nbi_0}} + \eta_{pel_1} S_{D_{pel}} + \eta_{pel_2}(1-\gamma_{pel}) S_{DT_{pel}} + \eta_{gas}(1-\gamma_{gas}) S_{DT_{gas}}$$

$$S_T = \eta_{pel_2} \gamma_{pel} S_{DT_{pel}} + \eta_{gas} \gamma_{gas} S_{DT_{gas}}$$

Uncertain Parameters

- Ion cyclotron, electron cyclotron & NBI heating: P_{ic}, P_{ec}, P_{nbi_1}, P_{nbi_2}
- DT pellet & gas injection with Tritium fractions γ_{pel} & γ_{gas}: $S_{D_{pel}}$, $S_{DT_{pel}}$, $S_{DT_{gas}}$
- Efficiency factors: η_{ic}, η_{ec}, η_{nbi_1}, η_{nbi_2}, η_{pel_1}, η_{pel_2}, η_{gas}
- The pellet fueling efficiency decreases with increasing plasma energy:

$$\eta_{pel_1} = \rho_{pel_1}(1 - E/E_0) \qquad \eta_{pel_2} = \rho_{pel_2}(1 - E/E_0)$$

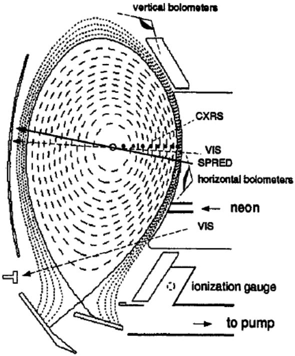
- The NBI ion-heating fraction $\phi_{nbi} = \rho_{nbi} \phi^\star_{nbi}$ [1] contains uncertainty (ρ_{nbi}).
- NBI thermalization delay contains uncertainty: ρ_{th}

$$\tau^{lag}_{nbi} = \rho_{th} \tau^\star_{nbi} = -\rho_{th} \frac{2}{3B} \ln \left[\frac{\left(\frac{\varepsilon_{nbi_{th}}}{\varepsilon_{nbi_0}}\right)^{3/2} + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}}{1 + \left(\frac{\varepsilon_c}{\varepsilon_{nbi_0}}\right)^{3/2}} \right] \qquad (\varepsilon_{nbi_{th}} = T_i)$$

Presentation Outline

Control-Oriented Model of Core-SOL-Divertor

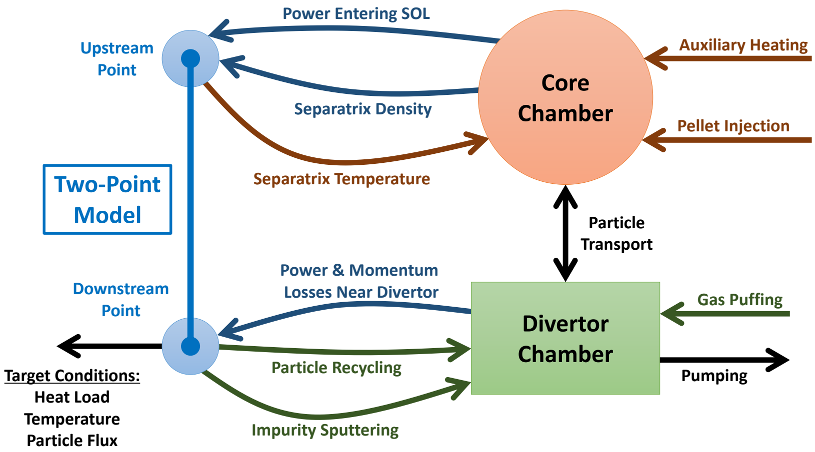
- **Core Chamber**
  - Core plasma energy and density balance equations
  - Particles outflow to Divertor Chamber
  - External Actuators: Pellet Injection & Auxiliary Heating
- **Divertor Chamber**
  - Divertor neutral-particle balance equations
  - Particle outflow to Core Chamber
  - External Actuators: Gas Puffing & Pumping



- **Two-Point Model**†
  - Connects upstream SOL conditions with downstream target conditions
  - Particle recycling and sputtering to Divertor Chamber
  - Control Knobs: Core Plasma Density & Power Entering SOL

† P.C. Stangeby, "The Plasma Boundary of Magnetic Fusion Devices," IoP Publishing, 2000.

Coupling Two-Chamber Model and Two-Point Model



Alternatives to Two-Point model:

- Parameterized SOLPS model
- NN-based surrogate models of 1D/2D models (UEDGE, Ben Zhu (LLNL))

POPCON Analysis of ITER Plasmas

Integrated CORE-SOL-Divertor (CSD) Model:

- Incorporate divertor operation constraints in burn control
- Study reactor operation space fulfilling divertor constraints $\rightarrow$ POPCON

Plasma Operation CONtour (POPCON) Plots

- Steady-state points with high fusion power output in ITER are found
- Results are presented in POPCONs that span density-temperature space

Constraints to the ITER Operable Space

- Auxiliary power saturates at 73 MW
- Pellet injection line with 90%T$-$10%D saturates at 111 Pa m^3/s
- Pellet injection line with 100%D saturates at 120 Pa m^3/s
- Maintenance of H-mode confinement:

$$P_{total} = P_{SOL} > P_{thres} = 4.3M^{-1}n_{e20}^{0.782}B_T^{0.772}a^{0.975}R^{0.999}$$

- Maximum heat load on divertor target plates: $q_{dep} < 10$ MW/m^2
- Maintenance of divertor detachment: $T_t < 7$ eV

POPCONs for the Core-SOL-Divertor (CSD) Model

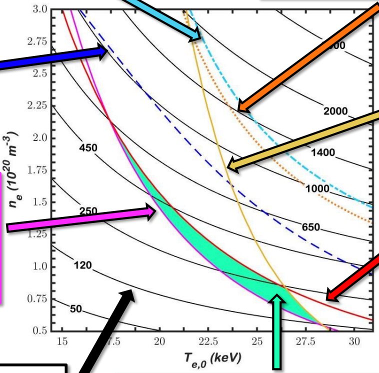
POPCON Plots Show Operational Space for ITER Plasmas

D fueling line is saturated at 120 Pa m³/s.

DT fueling line is saturated at 111 Pa m³/s with 90% tritium concentration.

Auxiliary Power saturated at 73 MW.

To the left of this line, divertor is in the desirable detached regime. To the right of this line, the divertor is in the undesirable attached regime. To achieve detachment, having $T_t < 7$ eV is required.



Above this line, the total plasma power exceeds the Threshold Power and plasma is in H-mode. Below this line, the plasma is in L-mode and the IPB98(y,2) scaling is invalid.

Below this line, the Heat Load on the target plates is below the 10 MW/m² maximum. Above this line, the targets are in danger of melting.

Fusion Power in MW mapped over $T_e - n_e$ space.

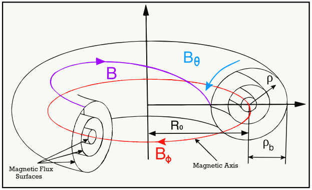
Green zone is operable space. Actuators not saturated, plasma is in H-mode, the heat load on the targets is safe, and divertor is in the detached regime.

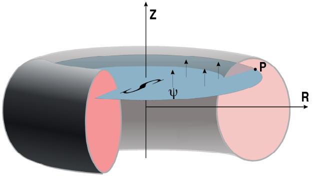
[1] V. Graber and E. Schuster, Fusion Engineering and Design, 171 (2021) 112516.
[2] V. Graber and E. Schuster, “Nonlinear Burn Control and Optimal Actuator Allocation of ITER Plasmas with Uncertain Parameters and Actuator Dynamics,” Thursday Poster Session (#14).

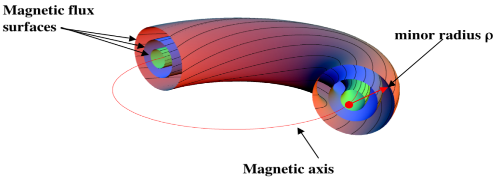
Presentation Outline

Magnetic Flux Surfaces as Spatial Coordinates

- Helical magnetic field lines generate nested surfaces due to ergodic motions
- Assume axisymmetric plasma (same properties at all toroidal angles ϕ)
- P = point in the poloidal cross-section; $\Psi(P)$ = magnetic flux through surface S bounded by ring through P
- Define poloidal flux function map $\Psi(R, Z)$ $\rightarrow$ Points of equal flux define surfaces
- Poloidal stream function: $\psi \triangleq \frac{1}{2\pi} \underbrace{\int_S \vec{B}_\theta \cdot d\vec{S}}_{\Psi}$



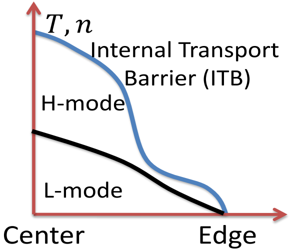




One possible index for the surfaces is the mean effective minor radius:
$\rho \triangleq \sqrt{\Phi/\pi B_{\phi,0}}$, $\Phi \triangleq \int_{S_\phi} \vec{B}_\phi \cdot d\vec{S}_\phi$
Φ: toroidal magnetic flux
$B_{\phi,0}$: toroidal magnetic field at R_0
ρ_b: ρ at boundary $\rightarrow \hat{\rho} \triangleq \rho/\rho_b$

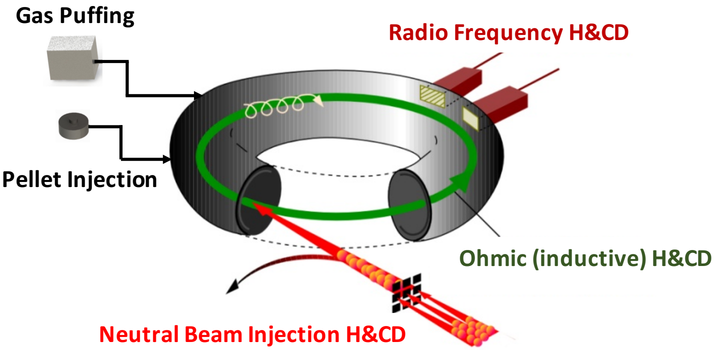
Profile Control: Why & How to Shape Profiles?

- Some plasma variables (pressure, magnetic field, etc.) are constant on magnetic surfaces
- Any variable indexing surfaces works as spatial coordinate $\rightarrow$ mean effective minor radius
- Spatial coordinate + toroidal symmetry: 3D $\rightarrow$ 1D
- Spatial variation of plasma variables $\rightarrow$ **"profiles."**



- **Current, pressure, rotation profile control capabilities may play a key role in achieving/sustaining desired scenarios in fusion reactors**
  - Maintain plasma in high-performance, MHD-stable, (steady) state.
- Reactor-specific additional challenges for effective plasma profile control:
  - Profile diagnostics will most likely not survive during power plant phase
    - + Profile control will rely on **model-based estimators (observers)**
  - Current profile may become too stiff after burn phase is initiated on flattop
    - + **Current profile optimization** may need to be carried out during ramp-up
  - Pressure profile shaping may be limited since $P_\alpha >> P_{heating}$
- Dimensionality + nonlinear kinetic/magnetic coupling $\rightarrow$ model-based control

Actuators Used for Plasma Profile Control



- **Ohmic coils** drive current into the plasma by induction
- **Non-axisymmetric coils** modify **confinement** and generate torque
- **Neutral beam injectors** heat plasma, drive current and generate torque
- **Radio frequency waves** heat the plasma and drive current
- **Plasma density** affects actuator efficiency and bootstrap current

Presentation Outline

Current & Rotation Evolution Model for Control Design

- **Magnetic Flux (ψ) Dynamics Modeled by 1D Diffusion Equation**

$$\frac{\partial\psi}{\partial t}=\eta(T_e)\left[\frac{1}{\mu_0\rho_b^2\hat{F}^2}\frac{1}{\hat{\rho}}\frac{\partial}{\partial\hat{\rho}}\left(\hat{\rho}\hat{F}\hat{G}\hat{H}\frac{\partial\psi}{\partial\hat{\rho}}\right)+R_0\hat{H}\frac{<\bar{j}_{NI}\cdot\bar{B}>}{B_{\phi,0}}\right],\quad \left.\frac{\partial\psi}{\partial\hat{\rho}}\right|_{\hat{\rho}=0}=0,\quad \left.\frac{\partial\psi}{\partial\hat{\rho}}\right|_{\hat{\rho}=1}=-\frac{\mu_0 R_0}{2\pi\hat{G}\hat{H}}I_p(t)$$

Resistivity (Spitzer) — Geometric Parameters — Non-inductive CD

$$\frac{\langle\bar{j}_{NI}\cdot\bar{B}\rangle}{B_{\phi,0}}=\frac{\langle\bar{j}_{BS}\cdot\bar{B}\rangle}{B_{\phi,0}}+\sum_{i=1}^{n_{lh}}\frac{\langle\bar{j}_{EC_i}\cdot\bar{B}\rangle}{B_{\phi,0}}+\sum_{i=1}^{n_{nbi}}\frac{\langle\bar{j}_{NBI_i}\cdot\bar{B}\rangle}{B_{\phi,0}},$$

Bootstrap (Sauter) — Auxiliary CD Sources

$$q=\frac{d\Phi}{d\Psi}=-\frac{B_{\phi,0}\rho_b^2\hat{\rho}}{\partial\psi/\partial\hat{\rho}}$$

- **Angular Momentum (P_ϕ) Modeled by 1D Transport Equation**

$$\frac{\partial P_\phi}{\partial t}=\frac{1}{\hat{\rho}\hat{H}}\frac{\partial}{\partial\hat{\rho}}\left[\hat{\rho}\hat{H}n_{\mathrm{i}}m_{\mathrm{i}}\langle R^2\left(\nabla\hat{\rho}\right)^2\rangle\left(\chi_\phi\frac{\partial\Omega_\phi}{\partial\hat{\rho}}+V_p\Omega_\phi\right)\right]+\tau,\quad \left.\frac{\partial\Omega_\phi}{\partial\hat{\rho}}\right|_{\hat{\rho}=0}=0,\quad \Omega_\phi|_{\hat{\rho}=1}=0,$$

$$P_\phi=n_{\mathrm{i}}m_{\mathrm{i}}\langle R^2\rangle\Omega_\phi$$

Momentum Diffusivity — Torque Sources

$\tau=\tau_{NBI}+\tau_{NRMF}+\tau_{eff}$ $\longmapsto$ NBI, Non-resonant magnetic field, Intrinsic Torque

$F(\hat{\rho})$, $G(\hat{\rho})$, $H(\hat{\rho})$, $\langle R^2\rangle(\hat{\rho})$, $\langle R^2\left(\nabla\hat{\rho}\right)^2\rangle(\hat{\rho})$, ρ_b $\leftarrow$ magnetic equilibrium

Kinetic Profile Evolution Model for Control Design

- **Fast Evolving Kinetic Profiles Modeled by Singular Perturbation**

$$T_e(\hat{\rho},t) = T_e^{prof}(\hat{\rho}) \frac{I_p(t)^{\alpha} P_{tot}(t)^{\beta}}{\bar{n}_e(t)^{\gamma}} \qquad n_e(\hat{\rho},t) = n_e^{prof}(\hat{\rho}) \bar{n}_e(t)$$

Stored Energy (W) Dynamics Modeled by 0D Power Balance

$$\frac{dW}{dt} = -\frac{W}{\tau_W} + P_{tot} (P_{tot} = P_{aux} + P_{ohm} - P_{rad}) \Rightarrow \beta_N = \frac{a(2W/3)}{I_p B_{\phi,0}/(2\mu_0)}, \tau_W \propto I_p^{\alpha_s} P_{tot}^{-\beta_s} \bar{n}_e^{\gamma_s}$$

- **Temperature (T_e) Modeled by 1D Heat Transport Equation**

$$\frac{3}{2}\frac{\partial}{\partial t}[n_e T_e] = \frac{1}{\rho_b^2 \hat{H}} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left[\hat{\rho} \frac{\hat{G}\hat{H}^2}{\hat{F}} \left(\chi_e(\cdot) n_e \frac{\partial T_e}{\partial \hat{\rho}} \right) \right] + Q_e^{ohm} - Q_e^{rad} + Q_e^{aux}, \quad \left. \frac{\partial T_e}{\partial \hat{\rho}} \right|_{\hat{\rho}=0} = 0, \quad T_e|_{\hat{\rho}=1} = T_e^{bdry}$$

Thermal Diffusivity

Auxiliary Heating

- Models needed for sources $\left(\frac{\langle \bar{j}_{EC} \cdot \bar{B} \rangle}{B_{\phi,0}}, \frac{\langle \bar{j}_{NBI} \cdot \bar{B} \rangle}{B_{\phi,0}}, \tau_{NBI}, \tau_{NRMF}, \tau_{eff}, Q_e^{aux}, \ldots \right)$
- Models needed for momentum, thermal, particle diffusivities (χ_ϕ, χ_e, ...)
- Coupling needed between equilibrium and transport
- **Control-oriented models needed to make control-design tractable!**

Possible Approaches Towards Modeling of Sources, Transport, and Transport/Equilibrium Coupling

- **Approaches to control-oriented modeling of sources:**
  - Fixed deposition multiplied by source power → Extremely reduced physics
  - Empirical scaling laws → May only be valid for specific scenarios
  - Simplified analytical models
  - Machine learning:
    - NUBEAM (NBI Monte-Carlo model) → DIII-D NubeamNet [1]
- **Approaches to control-oriented modeling of transport:**
  - Fixed profiles → Extremely reduced physics
  - Semi-empirical models (Bohm/gyro-Bohm, Coppi-Tang) → Limited accuracy
  - Machine learning:
    - MMM (Multi-Mode anomalous transport Model) → DIII-D MMMNet [2]
- **Approaches to control-oriented transport/equilibrium coupling:**
  - Fixed equilibrium → Extremely reduced physics
  - Analytical fixed-boundary solvers → Limited control applications
  - Numerical free-boundary solvers → Computationally expensive
  - Machine learning:
    - Numerical free-boundary solver → Machine-specific EquiNet [ongoing]
- **Neural networks replicate physics codes with faster calculation times**

[1] S. Morosohk, M.D. Boyer, E. Schuster, Fusion Engineering and Design, 163 (2021) 112125.
[2] S. Morosohk. A. Pajares, T. Rafiq, E. Schuster, Nuclear Fusion 61 (2021) 106040 (10pp).

Neural Network Models Are Integrated into COTSIM to Enable Fast Accurate Prediction in Control Design

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_0 \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} \hat{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \hat{\rho}} \right) + R_0 \hat{H} \eta(T_e) \left[j_{nbi} + j_{ec} + j_{bs} \right]$$

$$\frac{3}{2} \frac{\partial}{\partial t} [n_e T_e] = \frac{1}{\rho_b^2 \hat{H}} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left[\hat{\rho} \frac{\hat{G} \hat{H}^2}{\hat{F}} \left(\chi_e n_e \frac{\partial T_e}{\partial \hat{\rho}} \right) \right] + \left[Q_{ohm} + Q_{nbi} + Q_{ec} - Q_{rad} \right]$$

$$n_i m_i \langle R^2 \rangle \frac{\partial \Omega_\phi}{\partial t} + m_i \langle R^2 \rangle \Omega_\phi \frac{\partial n_i}{\partial t} = \tau_{nbi} + \tau_{ec} + \frac{1}{\hat{\rho} \hat{H}} \frac{\partial}{\partial \hat{\rho}} \left[\hat{\rho} \hat{H} n_i m_i \chi_\phi \langle R^2 (\nabla \hat{\rho})^2 \rangle \frac{\partial \Omega_\phi}{\partial \hat{\rho}} \right]$$

Can be calculated by NubeamNet

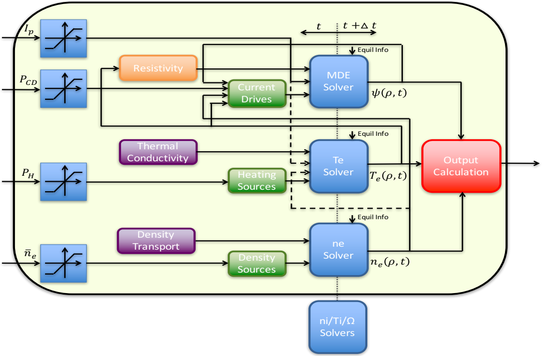
Can be calculated by MMMNet

Can be calculated by EquiNet

- NN-based surrogate models can play critical role in both off-line and real-time control applications demanding fast but accurate prediction:
  - Off-line: Closed-loop-capable testbed simulator
    - *Accurate+fast* integrated predictive capability
  - Off-line: Model-based optimal scenario planning
  - Real-time: State estimator and forecaster

PCG's Tokamak Transport (Control) Predictive Code

Control-Oriented Transport SIMulator (COTSIM)



Ongoing Development Efforts

- Numerical GS solver: free boundary simulation (magnetic-control integration)
- NN modeling: transport (TGLF, MMM) and H&CD sources (NUBEAM, GENRAY)
- Core-edge integration

- 1D transport code
- MHD Equilibrium:
  - Prescribed
  - Analytical (fixed bdry)
  - Numerical (free bdry)
- Modular configuration for physics complexity
- Matlab/Simulink-based
- Control-design friendly
- Closed-loop capable
- Optimizer wrappable
- Fast (full shot → few min) Effective Iterative Design
- Reduction → real-time & faster-than-real-time

Models are Reduced to Enable Use in Control Design

$$\frac{\partial y}{\partial t} = f\left(y, \frac{\partial y}{\partial x}, \frac{\partial^2 y}{\partial x^2}, u, t\right)$$

$$\downarrow$$

$$\dot{z} = g(z, u, t)$$

$$\downarrow$$

$$\dot{z} = g_z(z, t) g_u(u, t)$$

$$\downarrow$$

$$\dot{e} = A(t)e + B(e, t)h(u)$$

$$\downarrow$$

$$\dot{e} = A(t)e + B(t)u$$

$$\downarrow$$

$$\dot{e} = Ae + Bu$$

1. A first-principles-driven (FPD) plasma transport model is written as a parabolic PDE where $y(x, t)$ denotes the infinite-dimensional state, e.g., q, T_e.
2. A reduced-order solution $y(x, t) \approx \sum_{i=1}^{l} \alpha_i(t)\psi_i(x)$ can be obtained by applying the Galerkin projection method, which leads to a finite-dimensional ODE approximation where $z(t) = [\alpha_1(t), \ldots, \alpha_l(t)]^T$.
3. By modeling the actuators through an explicit separation of temporal and spatial variables, the ODE approximation can be further simplified.
4. By defining state deviation $e(t) = z(t) - r(t)$, a new bilinear representation can be obtained.
5. Further simplification leading to a LTV model is possible by linearizing the system dynamics, where $A(t)$ and $B(t)$ are time-varying matrices.
6. When the reference state is constant over time, i.e., $r(t) = r$, further simplification leads to a LTI model.

Models are Reduced to Enable Use in Control Design

$$\frac{\partial y}{\partial t} = f\left(y, \frac{\partial y}{\partial x}, \frac{\partial^2 y}{\partial x^2}, u, t\right)$$

$$\downarrow$$

$$\dot{z} = g(z, u, t)$$

$$\downarrow$$

$$\dot{z} = g_z(z, t) g_u(u, t)$$

$$\downarrow$$

$$\dot{e} = A(t)e + B(e, t)h(u)$$

$$\downarrow$$

$$\dot{e} = A(t)e + B(t)u$$

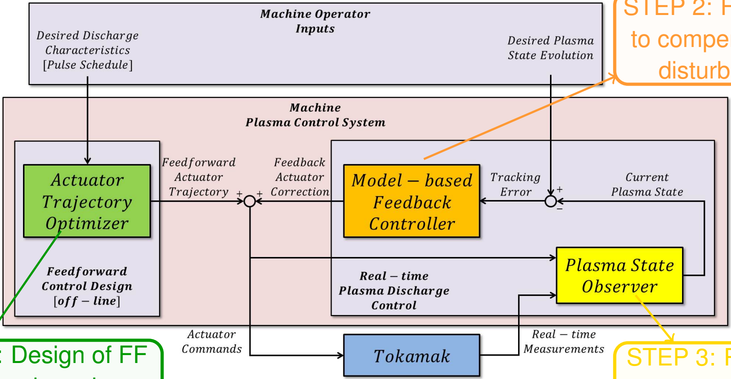
$$\downarrow$$

$$\dot{e} = Ae + Bu$$

- **Control Simulation:** A high-order nonlinear finite-dimensional model is required for closed-loop numerical simulations. FDP model enables analysis.
- **Feedforward Control Synthesis:** A medium-order (fast optimization) nonlinear finite-dimensional model is the best candidate for this task.
- **State Observer Synthesis:** A low-order (real-time operation) nonlinear finite-dimensional model, or at least a low-order bilinear finite-dimensional model, are required for this task.
- **Feedback Control Synthesis:** A LTI model may suffice for feedback control synthesis in some applications, but synthesis based on LTV, bilinear, or low-order nonlinear models is tractable and may be necessary in some applications.
- **Control Implementation Debugging:** A low-order nonlinear finite-dimensional model is required for PCS-in-the-loop simulations.

Presentation Outline

Scenario Control in Reactors Need Key Components: Feedforward (FF)+Feedback (FB) Controller, Observer



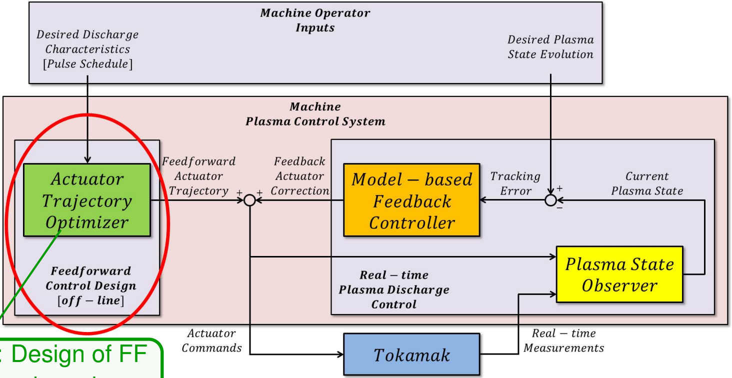
STEP 2: FB control to compensate for disturbances

STEP 1: Design of FF actuator trajectories to drive plasma close to desired target state

Dynamic (nonlinear) models are required for design of components of control architecture

STEP 3: Real-time profile estimation when diagnostics not available

Scenario Control in Reactors Need Key Components: Feedforward (FF)+Feedback (FB) Controller, Observer



STEP 1: Design of FF actuator trajectories to drive plasma close to desired target state

Step 1: FF Control as Nonlinear Programming

- **Objective:** Reach **target plasma state** at some time t_{targ} by designing **actuator waveforms** subject to **plasma dynamics** and **constraints**.
- Open-loop (feedforward) control policy u_{FF} (+ target trajectory ψ_{FF} (q_{FF}), W_{FF}) obtained via parameterization + constrained nonlinear optimization.

$$\min_{\alpha} \quad J(\psi_{FF}, \dot{\psi}_{FF}, \beta_N)$$ } Cost Function (Optimization Objective)

s.t.

$$\dot{\psi}_{FF} = f_{\psi}(\psi_{FF}, u_{\text{FF}}), \psi_{FF}(t_0) = \psi_0$$ } MDE

$$\dot{W}_{FF} = f_W(W_{FF}, u_{\text{FF}}), W_{FF}(t_0) = W_0$$ } Energy Balance

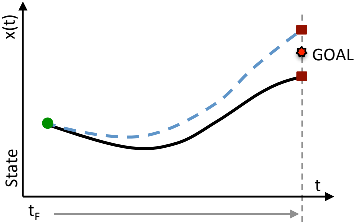
$$\beta_N(t) \leq \beta_{N_{\text{max}}}, q \geq 1$$ } State Constraint, MHD Stability Limit

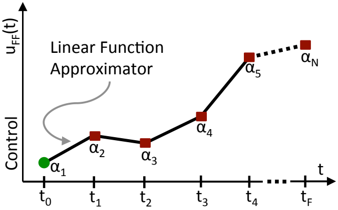
$$\bar{n}_e|_{20} \leq I_{\text{p}}/\pi a^2$$ } State Constraint, Density Limit

$$P_{\text{tot}}(t) \geq P_{\text{tot}_{\text{min}}}$$ } State Constraint, Prevent H→L Transition

$$u_{\text{FF}}(t) \in \mathcal{U}$$ } Input Constraint, Saturation / Rate Limit

$$u_{\text{FF}}(t) = u_{\text{FF}}(\alpha)$$ } Linear Function Approximator

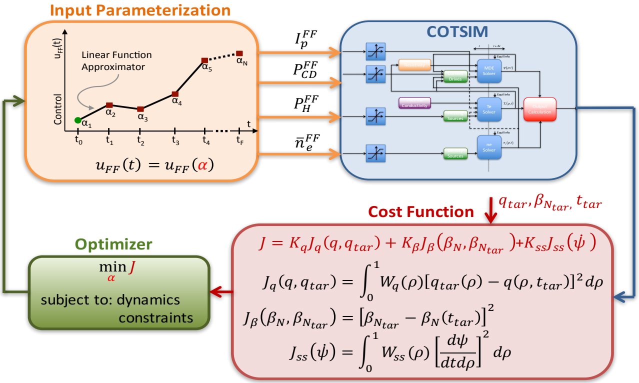




[1] C. Xu, J. Dalessio, Y. Ou, E. Schuster, et al. IEEE Trans Plasma Sci, vol. 38, no. 2, pp. 163-173, Feb 2010.

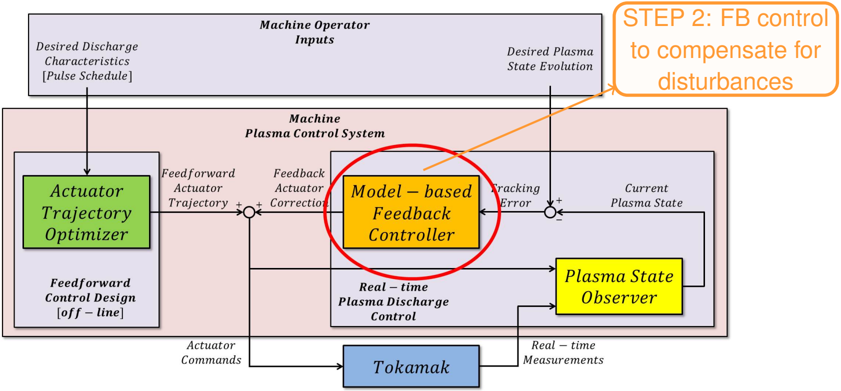
Step 1: FF Control as Nonlinear Programming

- **COTSIM enables systematic model-based scenario planning based on nonlinear constrained optimization with arbitrary cost function.**



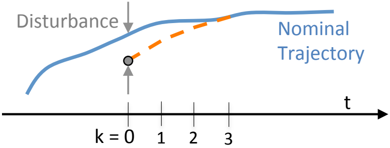
- **FF Design (open loop): Very sensitive to model accuracy**
- **FF Design (offline): Arbitrary model complexity**
  - NN surrogate models of physics-oriented codes → Accurate scenario design
- **COTSIM + Optimizer → Pulse Design Simulator (PDS)**

Scenario Control in Reactors Need Key Components: Feedforward (FF)+Feedback (FB) Controller, Observer



Step 2: FB Control as Quadratic Programming (QP)

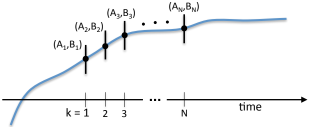
- Feedback controller is designed to reject deviations from desired (nominal) trajectory arising from disturbances/unmodeled dynamics (**trajectory tracking problem**).



- The dynamics of the (small) deviations can be well approximated linearly. The **trajectory tracking problem** over receding finite horizon (N) arises as **Quadratic Programming (QP)** to be solved in **real time** at each sampling time (new IC for optimization $\rightarrow$ FB mechanism $\rightarrow$ MPC/RHC).

$$\min_{\{u_t\}_{t=0}^{N-1}} \underbrace{\left\{ x_N^T P x_N + \sum_{t=0}^{N-1} \left(x_t^T Q x_t + \Delta u_t^T R \Delta u_t \right) \right\}}_{\textbf{Quadratic Objective}}$$

$$\text{s.t.} \quad \underbrace{\begin{aligned} x_{t+1} &= A_t x_t + B_t u_t, \quad x_t|_{t=0} = x_k \\ \Delta u_t &= u_{t+1} - u_t \\ (x_t, u_t) &\in \mathcal{X}_t \times \mathcal{U}_t \end{aligned}}_{\textbf{Linear Equality / Inequality Constraints}}$$



$$\dot{\psi} = f_\psi(\psi, u), \psi(t_0) = \psi_0$$
$$\dot{W} = f_W(W, u), W(0) = W_0$$

$\downarrow$ *Discretization*

$$\bar{x}_{k+1} = f(\bar{x}_k, u_k) \xrightarrow[\textit{Linearization}]{} x_{k+1} = A_k x_k + B_k u_k$$

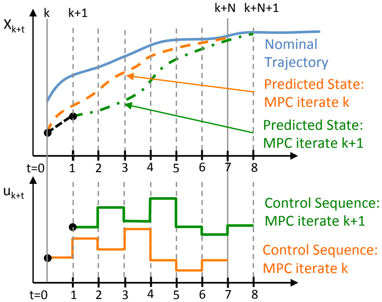
$$x = \left[\begin{array}{cccc} \psi|_{\hat{\rho}_1} & \ldots & \psi|_{\hat{\rho}_N} & W \end{array} \right]$$

[1] Y. Ou, C. Xu, E. Schuster et al., Control Engineering Practice, 19 (2011) 22-31.

Solution of Feedback (FB) Receding Horizon Control Problem Requires Optimization in Real Time

Receding Horizon Control (RHC) Solves a Series of QP Problems

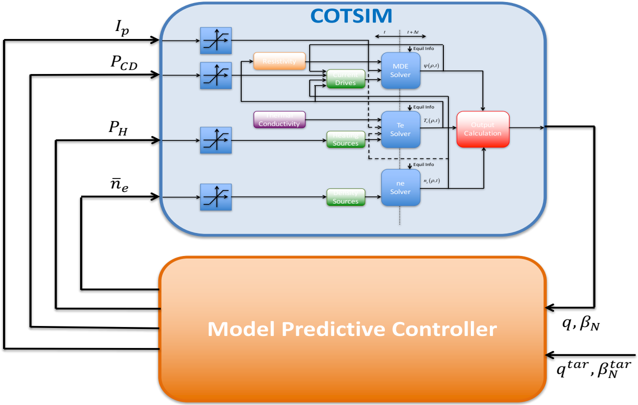
- Defines size N of prediction horizon
- At time k, samples current state of plasma x_k (FB)
- rtEFIT+MSE $\rightarrow (q, W) \rightarrow x_k$
- Makes $x_t|_{t=0} = x_k$ (QP Initial Condition)
- Solves QP to obtain control sequence $\{u_t\}_{t=0}^{N-1}$ (FF)
- Applies first step of control sequence $\Rightarrow u_k = u_t|_{t=0}$
- Discards the rest of control sequence
- Holds control until next sampling time $k+1$
- Repeats optimization with horizon receded 1 step



- Computation time in DIII-D for RHC is ~ 1 ms (sampling time: 20 ms)
- Warm Start: Previous solution $\rightarrow$ initial guess for next solution
- There is room for added complexity: Incorporation of nonlinearities, increase of horizon window, addition of state constraints for both stability/performance (e.g., MHD instability avoidance, minimum q, etc.)

Step 2: FB Control as Quadratic Programming (QP)

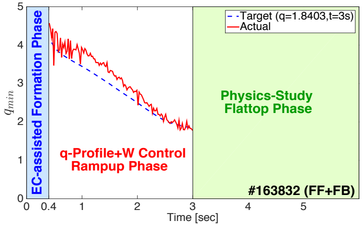
- **COTSIM enables assessment of <u>any type</u> of feedforward+feedback control algorithms in fast closed-loop simulations.**

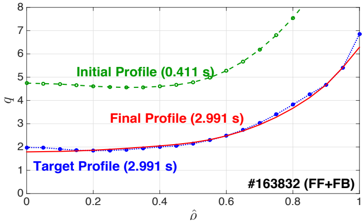


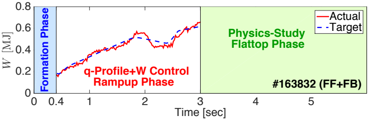
- **COTSIM + (FF+FB) Controllers → PCS Simulation Platform (PCSSP)**
- **PCS architecture is built around control-oriented models**
- **Matlab/Simulink PCS → Code generation → Fast PCS deployment**

Step 2: Discharge Reproducibility Is Enhanced by Feedback Controlling q-Profile and W in DIII-D

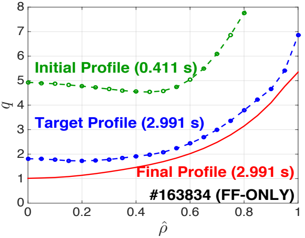
- **Control Objective**
  - Achieve desired q profile and W at given time regardless of initial condition by using individual NB power, total EC power, and I_p regulation in H-mode DIII-D discharges
- **Model-based Control Approach Required**
  - First-principles-driven (FPD) PDE model:
    1D magnetic flux diffusion equation
    0.5D density/temperature equations
    Scenario/control-oriented source models
- **Novel Control Approach**
  - Feedforward (FF) Control Design (Offline):
    Model-based scenario planning by solving nonlinear constrained optimization
  - Feedback (FB) Control Design (Online):
    Real-time optimization based on linearized dynamics for faster-than-real-time prediction

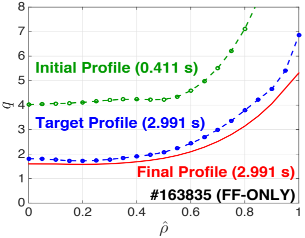


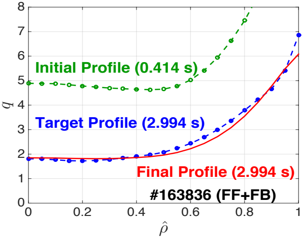




Step 2: FB-based Experiments in DIII-D Improves FF-only Profile Matching by Real-time Optimization





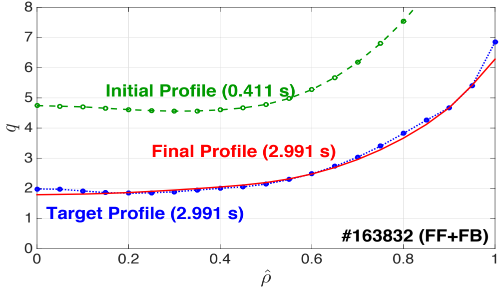


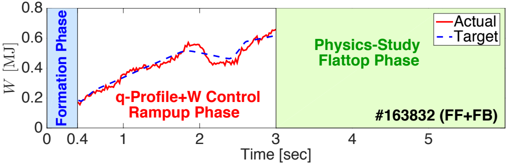
- Beams (330R, 150R) do not reproduce optimal FF input trajectories
- Actual initial q profile not close to shape assumed for FF optimization
- Matching of desired profile at target time clearly far from desired

- Actuators are able to reproduce optimal FF input trajectories
- Actual initial q profile close to assumed shape for FF optimization
- Matching of desired profile at target time still not as good as desired

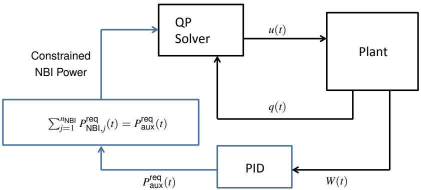
- Addition of FB control improves desired-profile matching at target time
- Actual initial q profile not close to assumed shape (similar to shot 163834)
- Real-time optimization + FB add robustness (disturbances/uncertainties)

Experiments: Simultaneous $q + W$ Control Possible





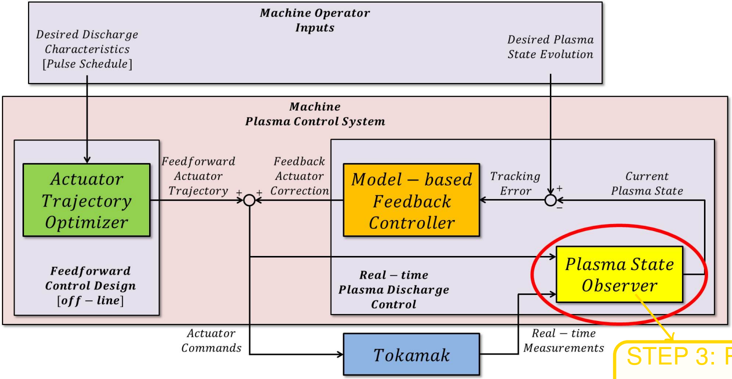
- W controller determines P_{aux}^{req} to track W_{targ}
- P_{aux}^{req} is passed to MPC as constraint
- $\sum_j P_{NBI_j}^{req} = P_{aux}^{req}$ for tight W regulation
- $\sum_j P_{NBI_j}^{req} \in [P_{aux}^{req} - \Delta P, P_{aux}^{req} + \Delta P]$ for loose W regulation and tighter q control



[1] W.P. Wehner, M. Lauret, E. Schuster et al. IEEE Multi-conference on Systems and Control, 2016.

[2] W.P. Wehner, J.E. Barton, M.D. Boyer, E. Schuster et al. IEEE Conference on Decision and Control, 2015.

Scenario Control in Reactors Need Key Components: Feedforward (FF)+Feedback (FB) Controller, Observer



STEP 3: Real-time profile estimation when diagnostics not available

Step 3: Observer Design as Extended Kalman Filter

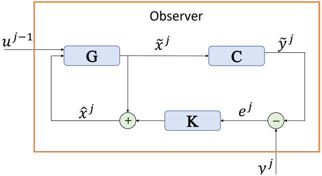
- T_e observer filters in real time measurements not consistent with physics
  - Prediction step:

$$\tilde{x}^j = G(\hat{x}^{j-1}, u^{j-1}), \qquad \tilde{y}^j = C\tilde{x}^j$$
$$\tilde{P}^j = F^{j-1}\hat{P}^{j-1}{F^{j-1}}^T + Q^{j-1}$$

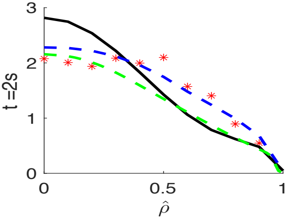
  - Correction step:

$$e^j = y^j - \tilde{y}^j, \quad \hat{x}^j = \tilde{x}^j + K^j e^j, \quad \hat{P}^j = (I - K^j H^j)\tilde{P}^j$$
$$K^j = \tilde{P}^j {H^j}^T (H^j \tilde{P}^j {H^j}^T + R^j)^{-1}, \quad K_j \in \mathbb{R}^{n\times m}$$

  - P is covariance of x; F, H are Jacobians of G, C; and Q, R are covariances of internal/measurement noise



- Prediction nonlinear model G is derived from Heat Transport Equation
- This is another control application that can benefit from NN surrogate models: MMMNet, NUBEAMNet in real time



Red: Thomson Scattering measurements, Black: TRANSP (off-line), Green: Fitting (real-time), Blue: Observer (real-time)

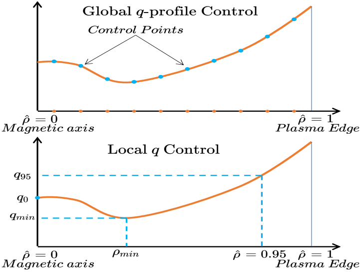
- Gain K regulates tradeoff between model prediction and measurement
- Observer:Fault detection/isolation (analytical redundancy)→fault-tolerant control!

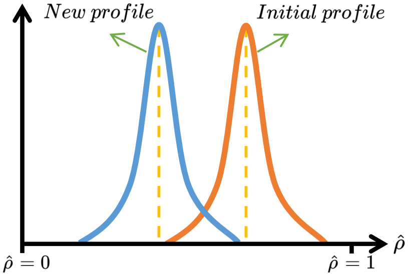
[1] H. Wang, J.E. Barton, E. Schuster, IEEE MSC (2015). [2] S. Morosohk, A. Pajares, E. Schuster, ACC (2022). [3] S. Morosohk, S.-T. Paruchuri, A. Pajares, E. Schuster "Estimation of the Electron Temperature Profile in DIII-D using Neural Network Models," Thursday Poster Session (#31).

Presentation Outline

Global vs Local + Fixed vs Moving Profile Regulation

- In some cases only local profile control is desired or possible (controllability)
  - Fixed location: q at $\hat{\rho} = 0$, q at $\hat{\rho} = 0.95$
  - Moving location: q at $\hat{\rho}_{min}$ (q_{min}), gradient of q at given rational surface [1, 2]
- Moving properties can drift to locations where control authority is low
- Potential solution to handle such drift is to incorporate moving actuators





- Using moving RF H&CD [3] can improve controllability of moving properties!
- Moving target → Significantly more challenging control-design problem

[1] S.-T. Paruchuri, A. Pajares and E. Schuster, IEEE CDC, Austin, TX, USA, 2021.
[2] S. T. Paruchuri, E. Schuster, A. Pajares, "Leveraging EC H&CD spatial variation for Enhanced Regulation of Current Profile in Tokamaks," Thursday Poster Session (#35).

Presentation Outline

Some Concluding Remarks

Response Modeling & Control:

- Control objectives → Model characteristics → Model reduction
  - Burn control: Nonlinearities. Profile control: Spatial dependence.
- **ML-based surrogate models** may close the **accuracy vs speed gap**
- Multiple control problems → Multiple models → Multiple controllers
  - Competition for actuators → **Actuator manager** in **control architecture**
  - Need for adaptation in real time → **Supervisor** in **control architecture**

Control Sciences:

- Control science offers methods to incorporate nonlinearities in the design
- Control science offers methods to deal with model uncertainties
  - Feedback provides robustness against model uncertainties
  - Robust and adaptive control theory provide additional methods
- Control sciences are mature: actuator + diagnostics + model → controller
- Physics: Keep improving abundant set of models + operating point (reference)

Scenario Control in Burning Plasmas with SOL/Divertor Integration:

- Integrated burn and profile control → 1D core + SOL/Divertor model
- Fueling control (part of burn control) offers unique (more urgent) challenges

Thank You for Your Attention! Questions?