# Plasma Rotation in Tokamaks

C. C. Hegna
Departments of Engineering Physics and Physics
University of Wisconsin
Madison, WI

Co-authors: J. D. Callen, A. J. Cole

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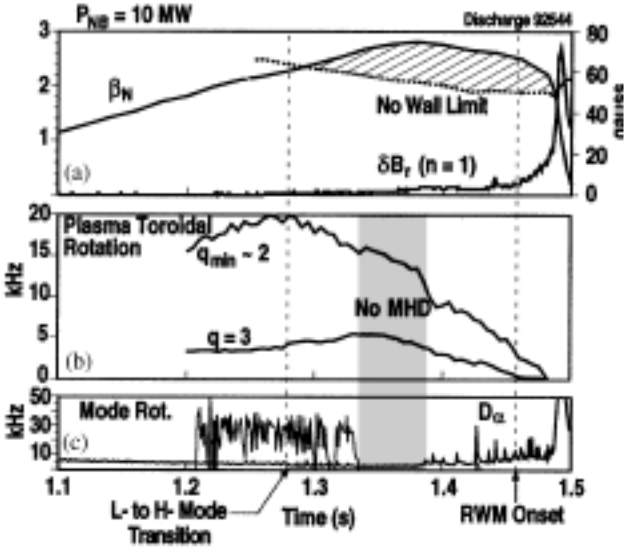
# Toroidal rotation impacts a variety of important plasmas physics topics

- Determining the magnitude, profile and evolution of toroidal flow is important in a number of topical areas
  - **E** X **B** flow shear of anomalous transport
  - Prevention of locked modes --- penetration of resonant field errors
  - Control of edge localized modes via resonant magnetic perturbations
  - Resistive wall mode physics



Garofalo et al, PRL ‘99

# Mechanisms have been developed to control/affect plasma rotation

- Toroidal rotation is influenced by:
  - External sources --- neutral beams
  - Intrinsic rotation --- topic of considerable research
    - A number of mechanisms have been proposed for intrinsic rotation --- turbulence, etc.
  - 3-D Magnetic fields
    - Field errors
    - Due to MHD instabilities
    - Applied 3-D fields

    -- both resonant and non-resonant

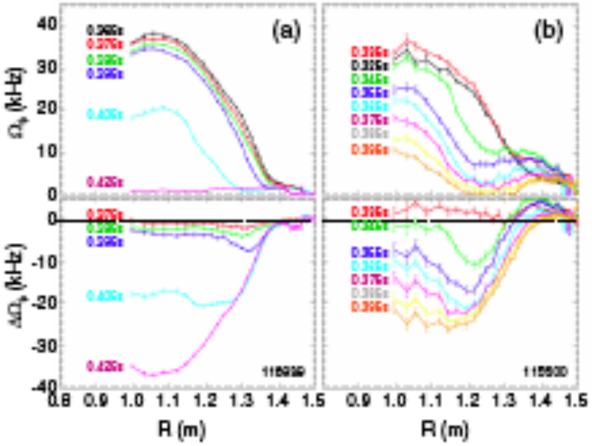


FIG. 2 (color online). Toroidal plasma rotation profile vs major radius, and the difference between initial profile and subsequent profiles for rotation damping (a) during application of nonaxisymmetric field, and (b) during excitation of rotating tearing instability.

W. Zhu et al, PRL, 2006

# Describing toroidal rotation in tokamaks is a transport problem

- To date, most treatments describing toroidal rotation evolution rely on:
  - Braginskii formulation (collisional plasma $\nu > v_{th}/qR$) --- in practice, never rigorously applicable to modern tokamaks
  - Additional physics added in ad hoc manner
    - Sources
    - Collisional transport
    - Radial plasma transport due to neoclassical effects
    - Turbulent transport --- often modeled as anomalous diffusion/pinch coefficients
    - 3-D fields
    - Magnetic field transients
    - etc.

# Thesis

- A new approach to construct transport equations for tokamak plasmas is derived. [Callen et al, NF **49**, 085021 (2009), Callen et al PoP **16**, 082504 (2009)].
  - Starting point is kinetic equation, not Braginskii
  - Multiple timescale equilibration processes
  - Toroidal momentum balance (or $E_r$) derived
  - Ambipolar particle transport
- Momentum balance equation is derived that accounts for classical, neoclassical collisional transport, anomalous transport, sources and sinks, magnetic field transients, and the effects of 3-D fields (neoclassical toroidal viscosity --- NTV).

# Outline

- Fluid moment equations
- Expansion procedure
- Multiple timescale approach
  - Radial momentum balance
  - Parallel momentum balance
  - Toroidal momentum balance
- Physics of neoclassical toroidal viscosity (NTV)
  - Theory
  - Experimental validation
- Toroidal rotation equation
- Radial electric field and ambipolarity

# Starting point for the calculation is the plasma kinetic equation

- Plasma kinetic equation for $f_s(\mathbf{x},\mathbf{v},t)$.

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla f_s + \frac{q_s}{m_s}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}} = C(f_s) + S(f_s)$$

  - $C(f_s)$ = Fokker-Planck collision operator
  - $S(f_s)$ = Kinetic source --- applied RF fields, neutral beams, etc.
- Fluid moment equations are obtained from velocity-space moments of the plasma kinetic equation

$$\int d^3\vec{v}(1, m_s\vec{v}, \frac{m_s v^2}{2})[\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla f_s + \frac{q_s}{m_s}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}} = C(f_s) + S(f_s)]$$

  - Evolution equations for low order velocity space moments $(n_s, \mathbf{V_s}, p_s)$

# Fluid equations require kinetically determined closure moments

- Exact fluid equations
  - Density

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{V}_s) = S_n$$

  - Momentum

$$\frac{\partial}{\partial t}(m_s n_s \vec{V}_s) + \nabla \cdot (m_s n_s \vec{V}_s \vec{V}_s) = n_s q_s (\vec{E} + \vec{V}_s \times \vec{B}) - \nabla p_s - \nabla \cdot \overleftrightarrow{\pi}_s + \vec{R}_s + \vec{S}_m$$

  - Energy

$$\frac{3}{2}\frac{\partial p_s}{\partial t} + \nabla \cdot (\frac{5}{2} p_s \vec{V}_s + \vec{q}_s) = Q_s + \vec{V}_s \cdot \nabla p_s - \overleftrightarrow{\pi}_s : \nabla \vec{V}_s + S_E$$

$$< Q > = \frac{\iint \frac{d\zeta d\theta}{\vec{B} \cdot \nabla \theta}}{\iint \frac{d}{\vec{B} \cdot}}$$

- Closure moments are required for a closed set of equations
  - Heat flux $\mathbf{q}_s$, viscous stress tensor $\boldsymbol{\pi}_s$ determined kinetically
  - Collision operator physics determines $Q_s$, $\mathbf{F}_s$

# A number of assumptions are made to make analytic progress

- Small gyroradius expansion $\frac{\rho}{L} << 1$
  - Consequences for how we describe flows
    - Lowest order --- MHD force balance
    - First order flows are within flux surfaces
    - Second order “transport” fluxes across surfaces
- Lowest order axisymmetric magnetic fields --- nested toroidal flux surfaces
- Small 3-D non-axisymmetric magnetic fields --- no magnetic islands
  - In practice, many 3-D fields of interest

$$\frac{\tilde{B}}{B_o} \sim 10^{-3} - 10^{-4}$$

- Small plasma fluctuations $\frac{\tilde{n}_s}{n_o} \sim \frac{\tilde{T}_s}{T_o} << 1$
- Slow magnetic field transients

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# Multi-stage strategy is used to determine transport equations

- Average the density, momentum and energy equations over fluctuations (average over toroidal angle) and then average over the flux surfaces

$$< Q >= \frac{\iint \frac{d\zeta d\theta}{\vec{B}\cdot\nabla\theta} Q(\psi,\theta,\zeta)}{\iint \frac{d\zeta d\theta}{\vec{B}\cdot\nabla\theta}}$$

- Sequentially consider specific components of the equilibrium force balance equation and their consequences
  - Radial --- zeroth order radial force balance enforced by compressional Alfven waves to obtain relation between flows, electric field and pressure gradients
  - Parallel --- first order poloidal flows, heat fluxes within magneitc surface
  - Toroidal --- require net radial current from all particle fluxes to vanish --> establishes flux surface averaged toroidal momentum balance equation
    - Use results of the net second order ambipolar fluxes back into flux surface averaged transport equations to obtain comprehensive “radial” transport equations for $n_s$, $T_s$ and toroidal rotation

# Small gyroradius expansion is used

- Gyroradius expansion: order terms and physical processes such as equilibrium, Pfirsch-Schluter flows, non-axisymmetries and fluctuations

$$p = p(\psi) + \delta[\bar{p}_1(\psi,\theta) + \tilde{p}(\psi,\theta,\zeta)] + O(\delta^2) \qquad \delta \sim \rho_i / a << 1$$

Equilibrium

Pfirsch-Schluter variations

3-D fluctuations

  - Magnetic field is the sum of an axisymmetric magnetic field and small 3-D fluctuations

$$\vec{B} = \vec{B}_o + \delta\vec{\tilde{\tilde{B}}} = q(\psi)\nabla\psi \times \nabla\theta + \nabla\zeta \times \nabla\psi + \delta\vec{\tilde{\tilde{B}}}$$

$$| B |= B_o(\psi,\theta) + \delta B_{\|}(\psi,\theta,\zeta)$$

  - Fluctuation derivatives are large perpendicular to **B** --- ballooning-like ordering

$$\nabla_{\perp}\tilde{p} \sim \frac{1}{\delta}\delta \sim 1, \quad \nabla_{\|}\tilde{p} \sim \delta^0\delta \sim \delta$$

# Transport equations for density and pressure are obtained by flux surface averaging

- Flux surface averaging density and energy equations with V′ = dV/dψ

Density

$$\frac{\partial n_{0s}}{\partial t}+\frac{1}{V'}\frac{\partial}{\partial\psi}(V'\Gamma_s)=<S_n>\qquad \Gamma_s=<(n_{0s}\vec{V}_2+\tilde{n}_{s1}\tilde{\vec{V}}_{s1})\cdot\nabla\psi>$$

Particle flux

Energy

$$\frac{3}{2}\frac{\partial p_{s0}}{\partial t}+\frac{1}{V'}\frac{\partial}{\partial\psi}(V'<[\vec{q}_{2s}+\frac{5}{2}(p_{s0}\vec{V}_{2s}+\tilde{p}_{1s}\tilde{\vec{V}}_{1s})]\cdot\nabla\psi>$$

$$=<Q_s>-<\vec{R}_{1s}\cdot\vec{V}_{1s}+\tilde{\vec{R}}_{1s}\cdot\tilde{\vec{V}}_{1s}>+<\vec{V}_{2s}\cdot\nabla p_{0s}+\tilde{\vec{V}}_{1s}\cdot\nabla\tilde{p}_{1s}>-<\overleftrightarrow{\pi}_s:\nabla\vec{V}_{1s}>+<S_E>$$

  - Cross-field particle/heat fluxes due to collisional and fluctuation processes

# Flux surface average of the momentum balance equation has three components

- Convenient to consider the "radial" <$\mathbf{e}_\psi$·(momentum balance)>, parallel <$\mathbf{B}_O$·(momentum balance)>, and toroidal <$\mathbf{e}_\zeta$·(momentum balance)> projections

Radial O($\delta^0$) $m_s n_{0s} \dfrac{\partial \vec{V}_s}{\partial t} = n_{so} q_s (\vec{E} + \vec{V}_s \times \vec{B}) - \nabla p_s$

Parallel O($\delta$) $m_s n_{0s} \dfrac{\partial < \vec{B}_0 \cdot \vec{V}_s >}{\partial t} = n_{0s} q_s < \vec{B}_0 \cdot \vec{E} > - < \vec{B}_0 \cdot \nabla \cdot \overleftrightarrow{\pi}_s > + < \vec{B}_0 \cdot \vec{R}_s > - m_s n_s < \vec{B}_0 \cdot \tilde{\vec{V}}_{s1} \cdot \nabla \tilde{\vec{V}}_{s1} > + \ldots$

Toroidal O($\delta^2$) $\dfrac{\partial}{\partial t} < \vec{e}_\zeta \cdot m_s n_{s0} \vec{V}_s >= q_s \Gamma_s - < \vec{e}_{\zeta} . \nabla \cdot \overleftrightarrow{\pi}_s > - < \nabla \cdot (m_s n_{s0} (\vec{e}_\zeta \cdot \tilde{\vec{V}}_{1s}) \tilde{\vec{V}}_{1s} > + \ldots$

Radial particle flux enters toroidal momentum balance equation

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# The different orders of the momentum balance equation refer to different timescales

- To leading order in δ, MHD force balance
  - Summing radial momentum balance $\vec{J}_0 \times \vec{B}_0 = \nabla p_0$
  - Radial force balance produces relationship between toroidal, poloidal flows, $E_r$ and pressure gradient

$$0 = \vec{e}_\psi \cdot [n_{i0} q_i (\vec{E} + \vec{V}_i \times \vec{B}) - \nabla p_i]$$

$$\Omega_t \equiv \vec{V}_i \cdot \nabla \zeta \cong \vec{V} \cdot \nabla \zeta = -(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} - q \vec{V} \cdot \nabla \theta)$$

$$V_t \cong \frac{E_r}{B_p} - \frac{1}{n_i q_i} \frac{dp_i}{dr} + \frac{B_t}{B_p} V_P$$

- First order flows are on magnetic surfaces $\mathbf{V}_1 \sim \delta$

$$\vec{V}_1 = \vec{e}_\theta \vec{V} \cdot \nabla \theta + \vec{e}_\zeta \vec{V} \cdot \nabla \zeta = V_\parallel \frac{\vec{B}_0}{B_0} + \vec{V}_\perp \qquad \vec{V}_\perp = \frac{\vec{B}_0 \times \nabla \psi}{B_0^2} (\frac{d\Phi}{d\psi} + \frac{1}{n_s q_s} \frac{dp_{s0}}{d\psi})$$

- Radial flows perpendicular to flux surfaces are second order

$$\vec{V}_{1s} \cdot \nabla \psi = 0, \quad \vec{V}_{2s} \cdot \nabla \psi \neq 0$$

# Poloidal flow is obtained from parallel force balance

- Summing the parallel force balance over species

$$m_i n_{i0} \frac{\partial < \vec{B}_0 \cdot \vec{V}_i >}{\partial t} = -\sum_s < \vec{B}_0 \cdot \nabla \cdot \overleftrightarrow{\pi}_s > - m_i n_{i0} < \vec{B}_0 \cdot \tilde{\tilde{\vec{V}}}_i \cdot \nabla \tilde{\tilde{\vec{V}}}_i > + < \vec{B}_0 \cdot \tilde{\tilde{\vec{J}}} \times \tilde{\tilde{\vec{B}}} > + \sum_s < \vec{B}_0 \cdot \vec{S}_s >$$

  - The poloidal flow is determined mainly by the parallel viscous force
  - Parallel viscosity is calculated from kinetic theory --- collisional process, accounts for the speed dependence of the Fokker-Planck collision operator

$$< \vec{B}_0 \cdot \nabla \cdot \overleftrightarrow{\pi}_{i\|} > \cong m_i n_{i0} [\mu_{i00} U_{i\theta} + \mu_{i01} \frac{-2}{5 p_i} Q_{i\theta} + \ldots] < B^2 > \quad \mu_{i00}, \mu_{i01} \sim \sqrt{\varepsilon} \nu_i$$

  - On times longer than the ion collision time

$$U_{i\theta}^0 \equiv \frac{\vec{V} \cdot \nabla \theta}{\vec{B} \cdot \nabla \theta} = -\frac{\mu_{i01}}{\mu_{i00}} \frac{-2}{5 p_i} Q_{i\theta} \cong \frac{c_p I}{q_i < B^2 >} \frac{dT_i}{d\psi} \quad \Rightarrow V_p \cong \frac{1.17}{q_i B} \frac{dT_i}{dr}$$

$U_{i\theta} = U_{i\theta}^0 + S_\theta$ $S_\theta$= Sum of sources and turbulent Maxwell/Reynolds stress

# Viscous damping occurs in the direction of asymmetry

- Viscsous stress tensor can be written in CGL form

$$\overleftrightarrow{\Pi}_{s\|} = \frac{1}{3}(p_\| - p_\perp)(\hat{b}\hat{b} - \overleftrightarrow{I})$$

- Pressure anisotropies are driven by flows/heat fluxes in the direction of Grad | B |

$$p_\| - p_\perp \sim \mu\vec{V}\cdot\nabla B, \quad \mu\vec{Q}\cdot\nabla B$$

  - To leading order, axisymmetric magnetic fields
  - Lowest order flows are within magnetic flux surfaces

---> Damping in the poloidal direction

$$B = B(\psi,\theta)$$

$$\vec{V}_{s1}\cdot\nabla\psi = 0 \rightarrow \vec{V}_{s1}\cdot\nabla B \sim \vec{V}_{s1}\cdot\nabla\theta\frac{\partial B}{\partial\theta}$$

# After determining the poloidal flow, there is a unique relationship between $E_r$ and the toroidal rotation

- Recalling the radial force balance relationship

$$\Omega_t \equiv \vec{V} \cdot \nabla\zeta = -(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i}\frac{dp_i}{d\psi} - q\vec{V} \cdot \nabla\theta)$$

  - Using parallel momentum balance result $< \vec{B}_0 \cdot \nabla \cdot \overset{\leftrightarrow}{\pi}_i > \cong 0$
  - Relationship between radial electric field and toroidal flow

$$V_t \cong \frac{E_r}{B_p} - \frac{1}{n_i q_i B_P}\frac{dp_i}{dr} + \frac{1.17}{q_i B_p}\frac{dT_i}{dr}$$

Pressure/temperature profiles determined by transport processes

  - Poloidal flow damping produces parallel plasma flow

$$\vec{V}_1 = \vec{e}_\theta \vec{V} \cdot \nabla\theta + \vec{e}_\zeta \vec{V} \cdot \nabla\zeta = V_\parallel \frac{\vec{B}_0}{B_0} + \vec{V}_\perp \qquad \vec{V}_\perp = \frac{\vec{B}_0 \times \nabla\psi}{B_0^2}(\frac{d\Phi}{d\psi} + \frac{1}{n_s q_s}\frac{dp_{s0}}{d\psi})$$

$$V_\parallel \cong -R(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i}\frac{dp_{i0}}{d\psi}) + \frac{1.17 R_0^2}{q_i R}\frac{dT_i}{d\psi}$$

# Electron parallel momentum balance produces parallel Ohm’s law

- Following the same logic for the parallel electron momentum balance equation

$$0 = -n_e e < \vec{E}\cdot \vec{B} > - < \vec{B}_0 \cdot \nabla \cdot \overleftrightarrow{\pi}_e > + < \vec{B}\cdot \vec{R}_e > + < \vec{B}_0 \cdot \vec{S}_{em} >$$
$$- m_e n_{e0} < \vec{B}_0 \cdot \tilde{\vec{V}}_e \cdot \nabla \tilde{\vec{V}}_e > - n_{e0} e < \vec{B}_0 \cdot \tilde{\vec{V}}_e \times \tilde{\vec{B}} >$$

  - Using collision friction and neoclassical closure from kinetic theory

$$\vec{B}_0 \cdot \vec{R}_e \cong \eta_{||} n_e e \vec{J}\cdot \vec{B}_0$$
$$< \vec{B}_0 \cdot \nabla \cdot \overleftrightarrow{\pi}_{e||} >= m_e n_{e0} < B^2 > (\mu_{e00} U_{e\theta} + \mu_{e01} Q_{e\theta})$$

  - Parallel Ohm’s law

$$< \vec{E}\cdot \vec{B}_0 >= \eta_{||}^{nc} < \vec{J}\cdot \vec{B}_0 > -\eta_{||}[< \vec{J}\cdot \vec{B}_{BS} > + < \vec{J}\cdot \vec{B}_{CD} > + < \vec{J}\cdot \vec{B}_{dyn} >]$$

Boostrap current, curent drive from external sources, dynamo due to fluctuations

$$\eta_{||}^{nc} = \eta_{||}(1 + \frac{\eta_{\perp}\mu_{e00}}{\eta_{||}\nu_e}) \qquad < \vec{J}\cdot \vec{B}_{BS} >= -\frac{\eta_{\perp}\mu_{e00}}{\eta_{||}\nu_e}(I\frac{dp}{d\psi} - n_e e U_{i\theta} < B^2 >)$$

$$< \vec{J}\cdot \vec{B}_{CD} >= -\frac{< \vec{B}\cdot \vec{S}_{em} >}{n_e e \eta_{||}} \qquad < \vec{J}\cdot \vec{B}_{dyn} >= \frac{1}{\eta_{||}}(< \vec{B}_0 \cdot \tilde{\vec{V}}_e \times \tilde{\vec{B}} > + \frac{m_e}{e} < \vec{B}_0 \cdot \tilde{\vec{V}}_e \cdot \nabla \tilde{\vec{V}}_e >)$$

# Toroidal torque from force balance gives radial flows

- Toroidal force balance produces an equation for radial particle flux

$$\vec{e}_\zeta \cdot \vec{V}_s \times \vec{B} = -\vec{V}_s \cdot \vec{e}_\zeta \times \vec{B} = \vec{V}_s \cdot \nabla\psi$$

  - Particle flux is induced by toroidal torques on the plasma

$$\vec{e}_\zeta \cdot (n_s q_s \vec{V}_s \times \vec{B} + \sum_j \vec{F}_j) = 0 \quad q_s\Gamma_s = -\sum_j \vec{e}_j \cdot \vec{F}_j = -\sum_j T_{j\zeta}$$

  - Flux surface averaged toroidal momentum balance equation produces equation for particle flux

$$q_s\Gamma_s = -<\vec{e}_\zeta \cdot \vec{R}> + <\vec{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\pi}_s> - n_o <\vec{e}_\zeta \cdot \vec{E}> - <\vec{e}_\zeta \cdot \vec{S}_m>$$

$$+\frac{\partial}{\partial t}(m_s n_s <\vec{e}_\zeta \cdot \vec{V}_s>) - <\vec{e}_\zeta \cdot n_o \vec{\tilde{V}} \times \vec{\tilde{B}}> + <\nabla \cdot m_s n_s (\vec{e}_\zeta \cdot \vec{\tilde{V}})\vec{\tilde{V}}>$$

Collisions, sources,
Inertia, fluctuations

# Parallel Ohm's law is used to describe collisional ambipolar particle flux

- Consider the particle fluxes from collisional friction

$$\Gamma_s^a = -\frac{1}{q_s} < \vec{e}_\zeta \cdot \vec{R}_s > - n_0 < \vec{e}_\zeta \cdot \vec{E} > \qquad \vec{R}_e \cong n_e e(\eta_\| \vec{J}_\| + \eta_\perp \vec{J}_\perp) = -\vec{R}_i$$

  - Vector identity used to facilitate analysis

$$\vec{e}_\zeta = \frac{I}{B_0^2}\vec{B}_0 - \frac{\vec{B}_0 \times \nabla\psi}{B_0^2}$$

  - Collisional-friction can be decomposed into parallel and perpendicular contributions

$$\frac{1}{q_s} < \vec{e}_\zeta \cdot \vec{R}_s > = -n_e I \eta_\| < \frac{\vec{J}\cdot\vec{B}_0}{B_0^2} > + n_e \eta_\perp < \frac{|\nabla\psi|^2}{B_0^2} > \frac{dp_0}{d\psi}$$

Classical transport

$$< \frac{\vec{J}\cdot\vec{B}_0}{B_0^2} > = \frac{< \vec{J}\cdot\vec{B}_0 >}{< B_0^2 >} + < \vec{J}\cdot\vec{B}_0 \left(\frac{1}{B_0^2} - \frac{1}{< B_0^2 >}\right) >$$

Pfirsch-Schluter

Calculated from parallel Ohm's law

# Particle flux has many contributions --- six ambipolar components

- The intrinsically ambipolar contributions to particle fluxes can be identified

$$\Gamma^a = \Gamma_{cl} + \Gamma_{PS} + \Gamma_{bp} + \Gamma_{CD} + \Gamma_{dyn} + \Gamma_E$$

$$\Gamma_{cl} = -n_{e0}\eta_\perp < \frac{|\nabla\psi|^2}{B_0^2} > \frac{dp_0}{d\psi} \qquad D_{cl} \cong \nu_e \rho_e^2$$

$$\Gamma_{PS} = -n_{e0} I^2 \eta_\| < \frac{1}{B_0^2}(1 - \frac{B_0^2}{<B_0^2>})^2 > \frac{dp_0}{d\psi} \qquad D_{PS} \sim q^2 D_{cl}$$

$$\Gamma_{bp} = \frac{I}{e<B_0^2>} < \vec{B}_0 \cdot \nabla \cdot \vec{\pi}_e > \qquad D_{bp} \cong \mu_e \rho_{ep}^2 \sim \frac{q^2}{\varepsilon^{3/2}} D_{cl}$$

$$\Gamma_{CD} + \Gamma_{dyn} = \frac{n_{e0} I \eta_\|}{<B_0^2>} < \vec{J} \cdot \vec{B}_{CD} + \vec{J} \cdot \vec{B}_{dyn} >$$

$$\Gamma_E = -n_{e0}[< \vec{e}_\zeta \cdot \vec{E} > (1 - \frac{I^2 < R^{-2} >}{<B_0^2>}) - \frac{I < \vec{B}_p \cdot \vec{E} >}{<B_0^2>})]$$

Collisional transport
from classical,
Pfirsch-Schluter,
banana-plateau.
current drive
dynamo and
electric field pinch
contributions

# Plasma fluctuations influence particle flux/toroidal momentum balance

- At O($\delta^2$), plasma fluctuation effects enter into the toroidal momentum balance
  - Microturbulence effects --- turbulent Reynolds/Maxwell stresses
  - 3-D magnetic fields --- error fields, applied 3-D coils
    - Resonant magnetic perturbations ---> localized electromagnetic torques
    - Non-resonant magnetic perturbations ---> Neoclassical toroidal viscosity
- In general, these effects are not intrinsically ambipolar and hence will affect toroidal momentum balance.

# Resonant magnetic fields produce localized electromagnetic torques at rational surfaces

- Inherent magnetic field errors or applied 3-D magnetic coils may have components that are resonant in the plasma

$$\vec{B} \sim e^{im\theta - in\zeta} \Rightarrow \frac{m}{n} = q(\psi)$$

  - Two asymptotic limits
    - Fully penetrated - radial magnetic field produces a magnetic island at the rational surface
    - Fully shielded - eddy currents flow in a resistive layer at q = m/n, magnetic perturbation does not penetrate, current sheet
  - Sufficient plasma rotation relative to the 3-D field source provides effective shielding --- rotation sustains current sheet
    - Produces localized electromagnetic perturbation (Fitzpatrick and Hender ‘91)

$$< \tilde{\vec{J}} \times \tilde{\vec{B}} > \sim \delta(r - r_{mn}) \frac{B_{rmn}^2}{B_0^2} \frac{\omega}{\Delta^2 + (\omega\tau_L)^2} \hat{b} \times \hat{n}$$

# 3-D magnetic fields produce neoclassical toroidal viscous forces (NTV) throughout the plasma

- In an axisymmetric magnetic field, the toroidal component of the parallel viscous stress tensor is zero ($\mu dB/d\zeta = 0$)

$$< \vec{e}_\zeta \cdot \nabla \cdot \pi_{\|s}^{A} >= 0$$

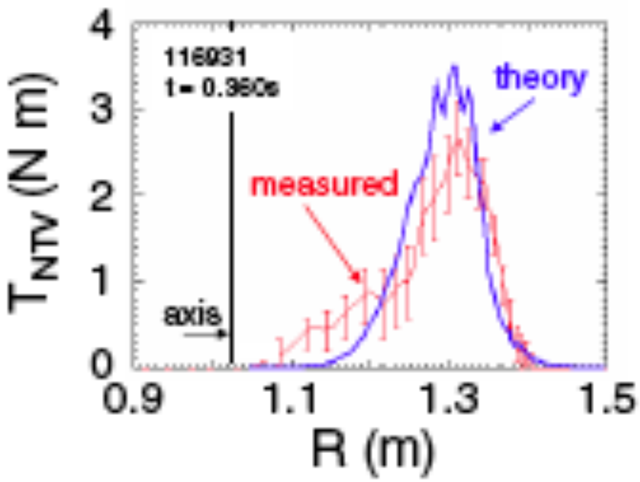
  - However, in the presence of 3-D magnetic fields, toroidal torques on toroidally flowing plasmas are generated.

$$< \vec{e}_\zeta \cdot \nabla \cdot \pi_{\|s} >= m_i n_i \mu_{ti} \frac{\tilde{B}_{eff}^2}{B_0^2}[< R^2\Omega_t > - < R^2\Omega_* >]$$

    - Physics --- transit-time magnetic pumping, banana-drift, ripple-trapping effects
    - Generally, the ion component dominates (the ion root of stellarator physics)
    - Ion viscous damping coefficient $\mu_{it}$ depends on collisionality, $E_r$
    - $B_{eff}^2$ is a weighted average sum over all m and n.

# The NTV force is felt throughout the plasma

- Unlike torques due to resonant 3-D magnetic fields, the NTV force is global
  - Applied 3-D fields on NSTX demonstrated the damping effect of toroidal flow (Zhu et al, PRL ‘06)



- Favorable comparison to analytic predictions

- NTV physics has been seen on NSTX, DIII-D, MAST, JET

# Another unusual aspect of NTV is the appearance of an off-set rotation frequency

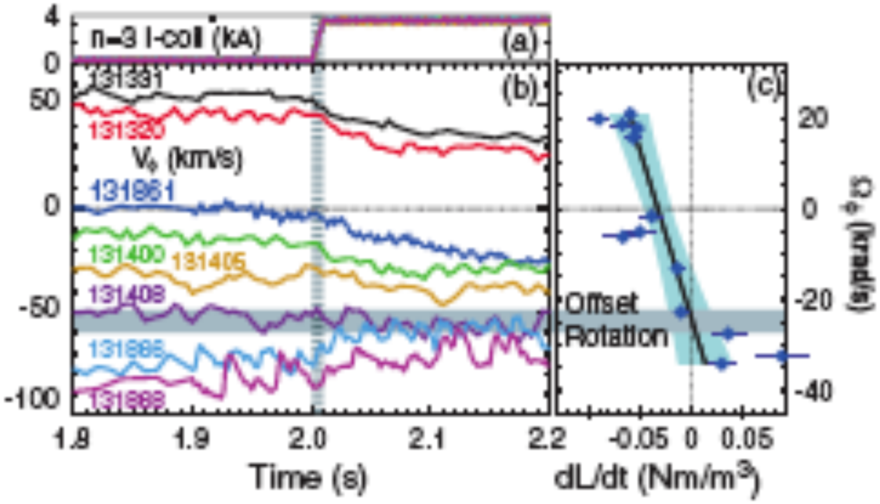
- NTV force ~ $\mu_i B_{eff}^2(\Omega - \Omega_*)$
  - Offset velocity $\Omega_*$ is a diamagnetic-type toroidal rotation frequency proportional to ion temperature gradient

$$\Omega_* = \frac{c_p + c_t}{q_i} \frac{dT_i}{d\psi}$$

    - Physics of the offset is due to ions of different energy having different radial drift speeds --- produces $c_t$. Poloidal flow damping coefficient $c_p$ due to parallel viscosity.

# Experiments on DIII-D have demonstrated the presence of the NTV offset velocity

- Off-set rotation velocity observed on DIII-D (Garofalo et al '08)



Initially, slowly rotating Plasmas sped up to the Offset NTV velocity when 3-D fields are applied

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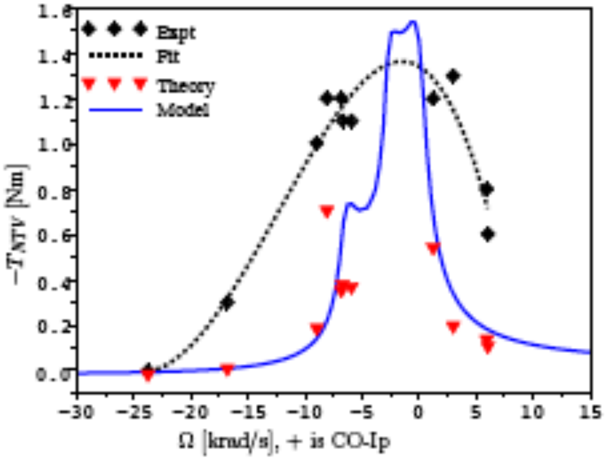
# Recent experiments on DIII-D have demonstrated a peak in the NTV force at zero radial electric field

- The toroidal damping rate ($\mu_{ti}$) is sensitive to the value of the radial electric field
  - Damping rate corresponds to different collisionality regimes of stellarator neoclassical transport
  - Smoothed formula constructed to model different collisionality regimes (Cole et al, '10)

$$\mu_{ti} \sim \frac{n v_{ti}^2 \sqrt{\varepsilon \hat{\nu}}}{< R^2 > [0.3 |\omega_{|\nabla B|}| \sqrt{\hat{\nu}} + 0.04 \hat{\nu}^{3/2} + |\omega_E|^{3/2}]}$$

Peaks at $\omega_E \sim 0$

Recent experiment on DIII-D Demonstrates peak NTV at $\omega_E \sim 0$



# Particle flux has 8 non-ambipolar contributions

- Not intrinsically ambipolar

$$\Gamma^{na} = \Gamma_{\pi\|}^{NA} + \Gamma_{\pi\perp} + \Gamma_{pol} + \Gamma_{\mathrm{Re}\,y} + \Gamma_{Max} + \Gamma_{J\times B} + \Gamma_{\psi_p} + \Gamma_S$$

$$\Gamma_{\pi\|}^{NA} = \frac{1}{q_s} < \vec{e}_\zeta \cdot \nabla \cdot \vec{\pi}_{s\|}^{NA} > \cong \frac{m_i n_i \mu_{it}}{q_i} \left(\frac{\tilde{B}_{eff}^2}{B_0^2}\right)(< R^2\Omega_t > - < R^2\Omega_* >) \quad \Omega_* \cong \frac{c_p + c_t}{q_i}\frac{dT_i}{d\psi}$$

$$\Gamma_{\pi\perp} = \frac{1}{q_s} < \vec{e}_\zeta \cdot \nabla \cdot \pi_{s\perp} > \sim -\chi_t \nabla^2 \Omega_t \quad \chi_t \sim (1+q^2)\nu_i \rho_i^2$$

$$\Gamma_{pol} = \frac{\partial}{\partial t}\left(\frac{m_s n_{so}}{q_s} < e_\zeta \cdot \vec{V}_s >\right)$$

$$\Gamma_{\mathrm{Re}\,y} = \frac{1}{q_s V'}\frac{\partial}{\partial \psi}(V'\Pi_{s\psi\zeta}) \quad \Pi_{s\psi\zeta} = m_s n_s < \vec{\tilde{V}}_s \cdot \nabla\psi\, \vec{\tilde{V}} \cdot \vec{e}_\zeta > + < \nabla\psi \cdot \overleftrightarrow{\pi}_{s\wedge} \cdot \vec{e}_\zeta >$$

$$\Gamma_{Max} = - < \vec{e}_\zeta \cdot n_s \vec{\tilde{V}}_{s1} \times \vec{\tilde{B}} >$$

$$\Gamma_{J\times B} = \frac{1}{e} < \vec{e}_\zeta \cdot \vec{\tilde{J}}_{\|} \times \vec{\tilde{B}} > \sim \delta(r - r_{mn}) \frac{c_{A\theta} m_i n_{io} R}{e} \frac{\omega}{\Delta^2 + (\omega\tau_L)^2} \frac{\tilde{B}_{rmn}}{B_0^2}$$

$$\Gamma_{\psi_p} = \frac{\dot{\psi}_p}{q_s}\frac{\partial}{\partial\psi}(m_s n_{s0} < \vec{e}_\zeta \cdot \vec{V}_s >) \qquad \Gamma_S = -\frac{1}{q_s} < \vec{e}_\zeta \cdot \vec{S}_{sm} >$$

Additional particle fluxes from neoclassical toroidal viscosity (NTV), cross field viscosity, polarization flow, microturbulence induced Reynolds and Maxwell stresses, field-error induced localized EM torques, poloidal field transients and momentum sources.

# Zero radial current produces torque balance relation

- Summing radial species currents to obtain net radial plasma current

$$< \vec{J} \cdot \nabla \psi >= \sum_s q_s \Gamma_s = \sum_s q_s \Gamma_s^{NA}$$

  - Charge continuity requires no net radial current

$$\frac{\partial}{\partial t} < \rho_q >= -\frac{1}{V'} \frac{d}{d\psi} V' < \vec{J} \cdot \nabla \psi >= 0$$

  - Setting radial current equal to zero produces a comprehensive toroidal torque balance relation

$$L_t \equiv m_i n_{i0} < R^2 \Omega_t >$$

$$\frac{1}{V'} \frac{\partial}{\partial t}(V' L_t) = - < \vec{e}_\zeta \cdot \nabla \cdot \pi_\parallel^{NA} > - < \vec{e}_\zeta \cdot \nabla \cdot \pi_{i\perp} > - \frac{1}{V'} \frac{\partial}{\partial \psi}(V' \Pi_{i\psi\zeta}) + < \vec{e}_\zeta \cdot \tilde{\vec{J}} \times \tilde{\vec{B}} > - \dot{\psi}_p \frac{\partial L}{\partial \psi} + < \vec{e}_\zeta \cdot \sum_s \vec{S}_{sm} >$$

# Toroidal rotation equation includes many different effects

- Equation for toroidal angular momentum density $L_t = m_i n_{i0} \langle R^2 \Omega_t \rangle$

$$\frac{1}{V'}\frac{\partial}{\partial t}(V' L_t) = -<\vec{e}_\zeta \cdot \nabla \cdot \pi_{\parallel}^{NA}> - <\vec{e}_\zeta \cdot \nabla \cdot \pi_{i\perp}> - \frac{1}{V'}\frac{\partial}{\partial \psi}(V' \Pi_{i\psi\zeta}) + <\vec{e}_\zeta \cdot \tilde{\vec{J}} \times \tilde{\vec{B}}> - \dot{\psi}_p \frac{\partial L}{\partial \psi} + <\vec{e}_\zeta \cdot \sum_s \vec{S}_{sm}>$$

  - NTV damping by 3-D magnetic fields

$$-<\vec{e}_\zeta \cdot \nabla \cdot \pi_{\parallel s}> = -m_i n_i \mu_{ti} \frac{\tilde{B}_{eff}^2}{B_0^2}[<R^2\Omega_t> - <R^2\Omega_*>] \qquad \Omega_* = \frac{c_p + c_t}{q_i}\frac{dT_i}{d\psi}$$

  - Collision damping

$$<\vec{e}_\zeta \cdot \nabla \cdot \pi_{s\perp}> \sim -\chi_t \nabla^2 \Omega_t \qquad \chi_t \sim (1+q^2)\nu_i \rho_i^2$$

  - Microturbulence-induced ion Reynolds stresses causes radial transport of $L_t$ (diffusion, pinch, residual stress)

$$\Pi_{s\psi\zeta} = m_s n_s < \tilde{\vec{V}}_s \cdot \nabla\psi \, \tilde{\vec{V}} \cdot \vec{e}_\zeta > + < \nabla\psi \cdot \overleftrightarrow{\pi}_{s\wedge} \cdot \vec{e}_\zeta >$$

$$\sim -\chi_t \nabla L_t + L_t V_{pinch} + \Pi^{RS}$$

# Toroidal rotation determines radial electric field required for net ambipolar particle flux

- From toroidal rotation equation, radial electric field is determined

$$E_r = -\nabla\psi \frac{d\Phi}{d\psi} = -\nabla\psi(<\Omega_t> + \frac{1}{n_{i0}q_i}\frac{dp_i}{d\psi} - \frac{c_p}{q_i}\frac{dT_i}{d\psi})$$

- The resultant electric field causes the electron and ion non-ambipolar radial particle fluxes to be equal (ambipolar)
  - Hence, the net ambipolar particle flux is the sum of $\Gamma^a + \Gamma^{NA}(E_r)$, which is easiest to evaluate in ion root $[\Gamma_\iota^{NA}(E_r) \sim 0]$

$$\Gamma_e^{net} = \Gamma_e^A + \Gamma_e^{NA}(E_r) = \Gamma_i^{net}$$

  - Procedure is familiar to stellarator researchers
    - Nonlinear dependences of $\Gamma_s$ on $E_r$ can lead to different 'roots', transition barriers, etc.

# This is approach is different than Braginskii-like approach and has some consequences

- Key differences in this new approach for plasma transport equations
  - First solve for flows of electrons, ions in flux surfaces --- Ohm's law, poloidal ion flow
  - Derivation of particle flux and toroidal flow are naturally joined
  - Simultaneously solve transport equations for n, T, $\Omega_t$
  - Effects of micro-turbulence are all included self-consistently
  - Fluctuation induced particle flux is determined from Reynolds/Maxwell stresses
  - Source effects, poloidal field transients are included
  - Net transport equations follow naturally from extended two-fluid moment equations --- consistent with formulations of extended MHD code frameworks (NIMROD, M3D)

# Summary

- Comprehensive transport equations for n, T, $\Omega_t$ have been derived
- Radial, parallel and toroidal components of force balance are considered
  - Radial force balance --- relationship between $V_t$, $V_p$, $E_r$ and $dp_i/d\psi$
  - Parallel viscous damping determines neoclassical Ohm's law and poloidal ion flow
  - Radial particle fluxes arise from average toroidal torques on the plasma
- Radial particle flux has many contributions --- ambipolar and non-ambipolar
- Requiring ambipolar particle flux yields evolution equation for toroidal angular momentum density

# Summary

- 3-D magnetic fields have an important effect of flow evolution
  - Localized EM torques from resonant magnetic fields
  - Neoclassical toroidal viscosity (NTV) from variations in $|B|$
- Many aspects of NTV theory are being tested against experiments
  - Global damping of toroidal flow profile
  - Appearance of an offset rotation ~ $dT_i/d\psi$
  - Peak of NTV torque near $E_r \sim 0$