Electron Bernstein Waves in Magnetic Fusion

Martin O' Brien
Head of CCFE Theory & Modelling Department
EURATOM/CCFE Fusion Association

6th ITER International School, 2-6 December 2012

with grateful acknowledgements to Vladimir Shevchenko

Aim of this Lecture

- Broad Overview – give a feel for EBWs: what they are, pros and cons, uses
- Not a detailed treatment of a complicated topic, e.g. only a sketch of underlying theory
- Just physics, not technology (which is like ECRH)
- Biased to CCFE's interests & applications !!!

ECRH – Recap.

- Electron cyclotron range of frequencies ~10-200 GHz
- Extraordinary & Ordinary modes (X, O)
- Cold Plasma theory $\Rightarrow$ cut-offs and resonances depending on X or O

ECRH accessibility and Cut-Offs

- Cut-offs ($N^2 = 0$) and Resonances ($N^2 = \infty$) separate regions where propagation is possible – in between it is evanescent ($N^2 < 0$)
- Cut-offs and Resonances depend on X or O mode and (for X) magnetic field. Propagation is usually possible at densities up to $\omega^2 \sim \omega^2_{pe}$

  i.e. $n_e \sim 1.25 \times 10^{16} (f_{GHz})^2\ m^{-3}$ ($1 \times 10^{19} m^{-3}$ for 28GHz)

ECRH pros and cons

- ☺ Easy coupling with no antenna near plasma. Small port. Mirrors can point beam where you want it.
- ☺ Localised heating & current drive – e.g. for instability control
- 😐 Technology not too challenging
- ☹ But off-axis current drive has low efficiency (electron trapping)
- ☹ And can't access high n_e/B^2 plasmas – need EBWs !!
- ☺ **Most promising H&CD system for power station ??**

ECRH no good if n_e/B^2 is high …

- Frequency for resonance and thus heating set by B ($\omega \sim \omega_{ce}$). $f_{GHz} \sim 28\ B_{Tesla}$
- Cut-off density $n_{e19} \sim B^2$ with B in Tesla. So inaccessible in low field machines (spherical tokamaks, stellarators …)
- e.g. MAST has B ~ 0.5T $\Rightarrow$ $2.5\text{x}10^{18}\text{m}^{-3}$ – so ECRH is no use by a factor > ten !!
- It's better for second harmonic but still a problem (for harmonics >2 accessibility is OK but damping usually too weak)

…. but we can use EBWs

- Warm plasma terms in dispersion relation add extra modes – waves can "mode convert" to these Electron Bernstein Waves near resonances
- EBWs have no high density cut-off. It is the only EC wave we can use in low B machines (spherical tokamaks, stellarators …) which have n_e/B^2 too high for conventional ECRH
- But they don't propagate in vacuum so have to be excited by mode coupling from X mode

What are EBWs? – a short overview

Named after Prof. Ira Bernstein, Phys. Rev. 109, 10 (1958)

- Unlike O & X, no high n_e cut-off. But don't propagate in vacuum
- Slow, longitudinal & electrostatic waves.
- Propagate almost perpendicular to field: $N_\perp$~40, $N_{||}$~1.
- Couple to X mode near upper hybrid resonance (UHR) – where X is slow with high $N_\perp$. Can use X-B or O-X-B mode conversion.
- Very strong electron damping near EC resonance
- EBW emission is inverse process to heating. B-X-O emission now a useful diagnostic for plasma pedestal region.

Behaviour in plasma

- Excited by coupling from X mode at UHR
- Then rapidly upshift to high $N_\perp$~40. $N_{||}$ rises more modestly. So a slow wave with ….
- … strong Doppler-shifted damping on weakly suprathermal electrons. $N_{||} \sim 1 \Rightarrow$ damping quite far from cold resonance $\omega = n\omega_{ce}$.

$$\omega - n\omega_{ce} = k_{||}v_{||} = N_{||}v_{||}/c \qquad (\omega_c = eB/\gamma m)$$

EBWs pros and cons

- ☺ Like ECRH, source and launch technology are straightforward
- ☺ High n_e no problem. Strong damping
- ☺ Current drive: more efficient than ECCD (damps on faster electrons, trapping effects weaker).
- ☺ EBW emission can be a useful diagnostic
- 😐 Experimental base not large, but ~ agrees with codes
- ☹ Coupling of free space wave to EBW can be difficult
- 😐 Applications mainly of interest for spherical tokamaks and stellarators, i.e. low B^2/n_e machines

EBW experiments

- EBWs have been used in many experiments for start-up, heating, current drive, though not generally routinely. EBW emission also observed/ exploited on several tokamaks and stellarators

  TJ-I, Vega, W7AS, PLT, TCV, COMPASS-D, MAST, NSTX, TST, LATE, QUEST, etc.

- On MAST we plan 1MW of EBW heating. Already using EBW emission as a diagnostic

ECRH/EBW Theory

Various levels of analysis:

'Cold'	- random thermal motion neglected - plasma treated as a fluid with an effective relative permittivity Gives only electromagnetic modes (X and O)
'Warm'	- fluid approach maintained but with pressure terms Brings out existence of additional modes
'Hot'	- kinetic approach based on Boltzmann/Vlasov equation Brings out wave-particle resonances and wave damping

Wave Equation

- Assume small perturbations ~ exp(i(**k**.**r** - ωt)). Eliminating **B** from Maxwell's equations gives Wave Equation in terms of relative permittivity tensor and Refractive Index vector:

$$\mathbf{N} \times \mathbf{N} \times \mathbf{E} = -\left[\mathbf{E} + i\frac{c^2\mu_0}{\omega}\boldsymbol{\sigma}.\mathbf{E}\right] = -\left[\mathbf{E} + \frac{i}{\omega\varepsilon_0}\boldsymbol{\sigma}.\mathbf{E}\right] = -\boldsymbol{\varepsilon}.\mathbf{E}$$

Recap. Of Cold Plasma Dispersion Relation

- Cold plasma model gives permittivity matrix elements:

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\perp} = 1 - \sum_s \frac{\omega_{ps}^2}{\left(\omega^2 - \omega_{cs}^2\right)}$$

$$\omega_{ps} = \left(\frac{n_s Z_s^2 e^2}{\varepsilon_0 m_s} \right)^{1/2}$$

$$\varepsilon_{zz} = \varepsilon_{\parallel} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

$$\omega_{cs} = \frac{Z_s e B_0}{m_s}$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = \sum_s \frac{i\,\omega_{ps}^2 \omega_{cs}}{\omega\left(\omega_{cs}^2 - \omega^2\right)}$$

- Wave Equation determinant gives Cold Dispersion Relation

$$D = \det \begin{vmatrix} N_{\parallel}^2 - \varepsilon_{\perp} & -\varepsilon_{xy} & -N_{\perp}N_{\parallel} - \varepsilon_{xz} \\ \varepsilon_{xy} & N_{\perp}^2 + N_{\parallel}^2 - \varepsilon_{\perp} & 0 \\ -N_{\perp}N_{\parallel} - \varepsilon_{zx} & 0 & N_{\perp}^2 - \varepsilon_{\parallel} \end{vmatrix} = 0$$

Cold Plasma Dispersion Relation - 2

Set determinant to zero for non-trivial solutions :

$$D = \varepsilon_\perp N_\perp^4 + N_\perp^2\left\{\left(\varepsilon_\perp + \varepsilon_\parallel\right)N_\parallel^2 - \left(\varepsilon_{xy}^2 + \varepsilon_\perp^2 + \varepsilon_\perp\varepsilon_\parallel\right)\right\} + \varepsilon_\parallel\left\{\left(N_\parallel^2 - \varepsilon_\perp\right)^2 + \varepsilon_{xy}^2\right\} = 0$$

Two modes: Ordinary & Extraordinary (O & X) for ECRH

Propagation and how $N_\perp$ and $N_\parallel$ evolve across the plasma can be calculated from the dispersion relation.

Cut-offs ($N_\perp = 0$) and resonances ($N_\perp = \infty$) from D = 0 relation.

Upper Hybrid resonance for X mode $\omega^2 = \omega_{UH}^2 = \omega_{pe}^2 + \omega_{ce}^2$

Near UHR $N_\perp$ gets high enough to couple X mode to EBW

Electric field polarisations are eigenmodes of wave equation

More General Dispersion Relation

- For EBWs, same approach as for cold plasma, but terms more complicated - need warm terms in dielectric tensor ε.
- But for damping (of EBW or O or X) we must keep hot terms: complicated expressions involving sums over Bessel functions which come from electron Larmor motion – $J_n(N_\perp v_\perp/c)$
- Real and imaginary parts of dispersion relation ⇒ propagation and damping.
- Propagation – ideally full wave solution but time-consuming. But ray-tracing is generally good enough – keep *D*=0 along rays, ray equations are analogues of Hamilton's equations

Sketch derivation of EBW dispersion relation

Consider longitudinal, electrostatic waves with $\tilde{E}$ and $\tilde{\varphi}$ varying as $\exp\left(i(\underline{k}.\underline{r} - \omega t)\right)$

Linearised Vlasov equation

$$\frac{d\tilde{f}}{dt} = \frac{e}{m}\underline{\tilde{E}}.\frac{\partial f_{\max}}{\partial \underline{v}} = \frac{e}{k_B T}\underline{v}.\frac{\partial \tilde{\varphi}}{\partial \underline{r}} f_{\max} = \frac{e}{k_B T} f_{\max}\left(\frac{d\tilde{\varphi}}{dt} - \frac{\partial \tilde{\varphi}}{\partial t}\right)$$

Poisson's equation

$$\nabla^2 \tilde{\varphi} = \frac{e}{\varepsilon_o}\int d\underline{v}\tilde{f} = \frac{ne^2\tilde{\varphi}}{\varepsilon_o \kappa_B T} - \frac{e^2}{\varepsilon_o \kappa_B T}\int d\underline{v} f_{\max}\int_{-\infty}^{t} dt' \frac{\partial \tilde{\varphi}}{\partial t'}$$

Put Larmor motion in eikonal in t' integral $r = \left[\frac{v_\perp}{\omega_{ce}}\sin(\omega_{ce}t + \vartheta), -\frac{v_\perp}{\omega_{ce}}\cos(\omega_{ce}t + \vartheta), v_{||}t\right]$

Dispersion relation

$$D = 0 = 1 + k^2\left(\frac{\varepsilon_o \kappa_B T}{ne^2}\right) - \omega e^{-\lambda}\sum_{n=-\infty}^{\infty}\frac{I_n(\lambda)}{\omega - n\omega_{ce}} \quad \text{with } \lambda = \left(k^2 \kappa_B T\right)/ m\omega_{ce}^2$$

I_n is modified Bessel function. Comes from integral over J_n.

Small λ , n=0 get UHR solution $\omega^2 = \omega^2_{pe} + \omega^2_{ce}$

EBW Damping

- Heating is mainly cyclotron damping ⇒ diffusive damping in $v_\perp$. All components of Electric field can contribute to damping as wavelength is comparable to Larmor radius so arguments of Bessel functions $J_n(N_\perp v_\perp/c)$ aren't small. For harmonic n (n=1 is fundamental)

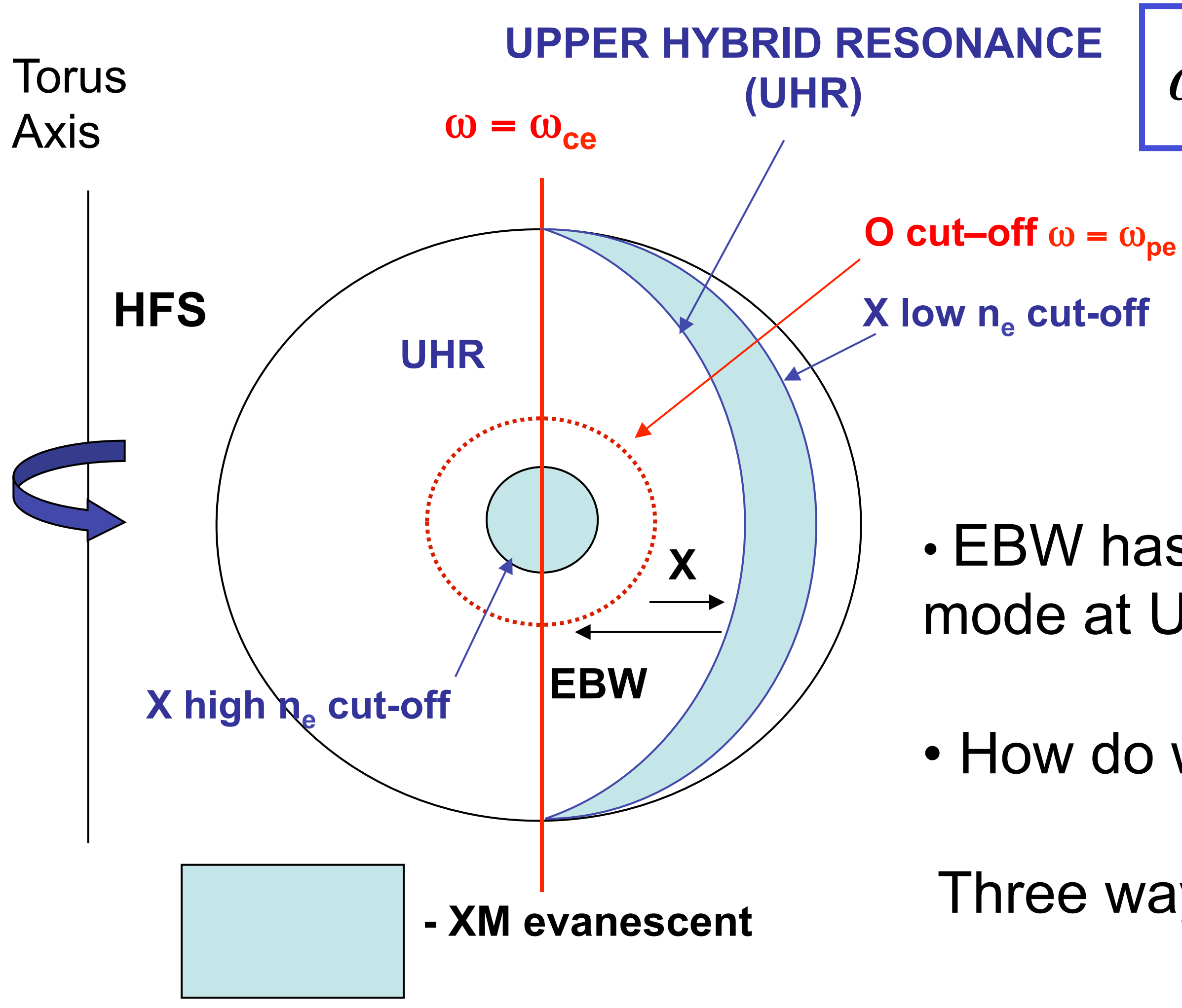
$$\left.\frac{\partial f}{\partial t}\right)_{\mathrm{cyc}} = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(v_\perp D_{\mathrm{cyc}}\frac{\partial f}{\partial v_\perp}\right)$$

$$D_{cyc} \sim \left| E_- J_{n-1} + \frac{N_{||} v_{||}}{c} E_z J_n + \frac{v^2}{c^2} E_+ J_{n+1}\right|^2$$

- Fokker-Planck code (e.g. BANDIT-3D) can calculate damping and current drive profiles using wave data ($N_\perp$, $N_{||}$, polarisations, etc.) from full wave or ray tracing code

Mode Conversion: EBW from X at UHR

$$\omega^2 = \omega_{UH}^2 = \omega_{pe}^2 + \omega_{ce}^2$$

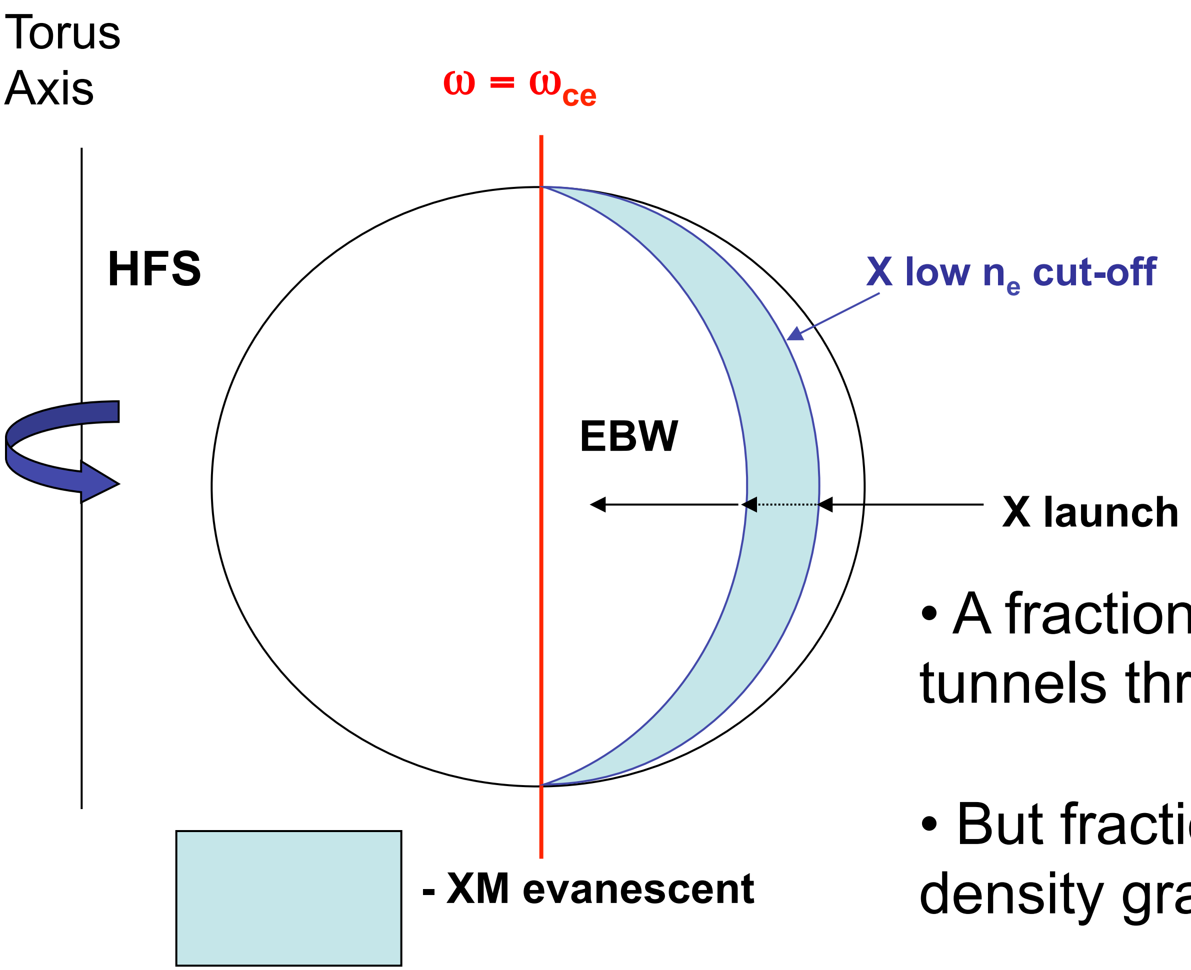


Resonances and cut-offs in tokamak

- EBW has to be excited from X mode at Upper Hybrid Resonance
- How do we get X mode to UHR?

Three ways

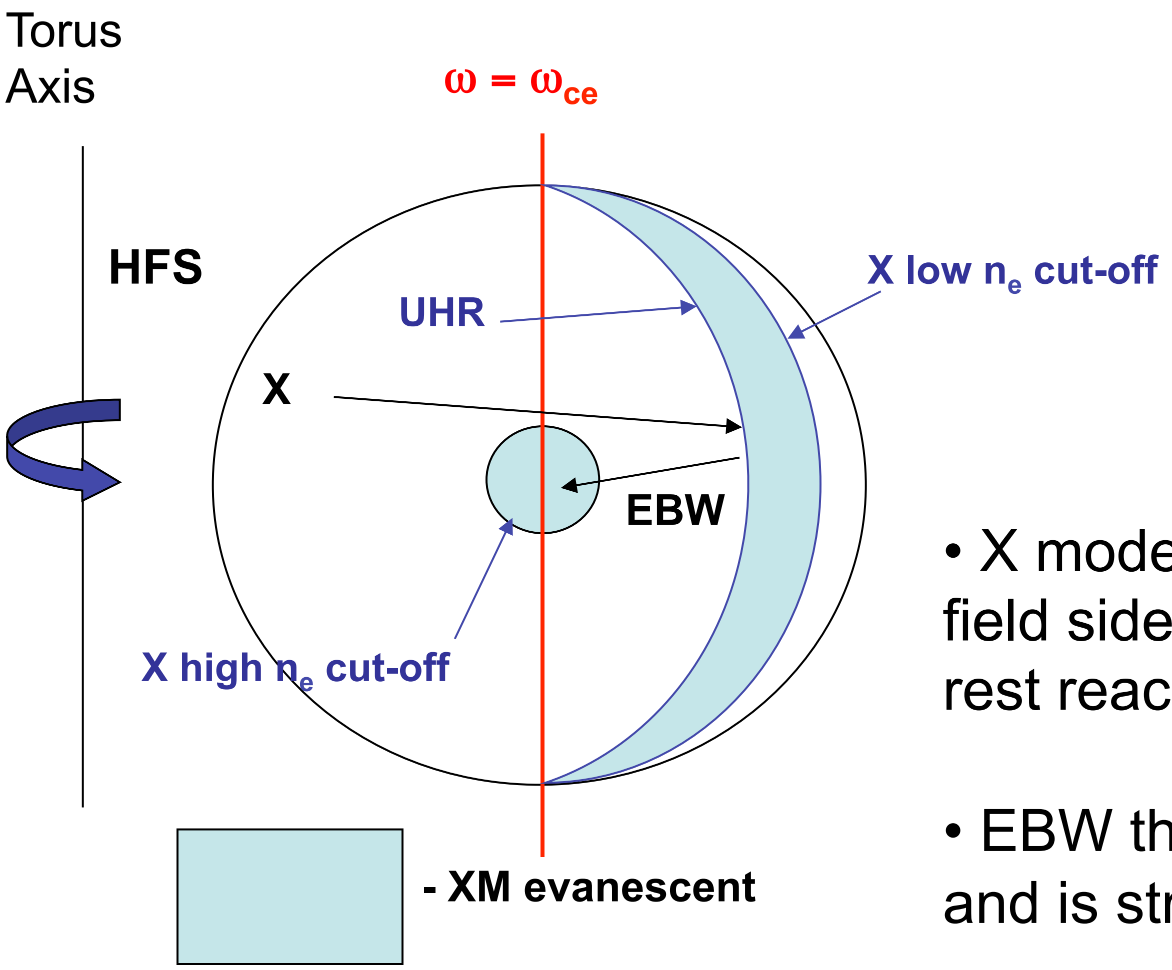
1. EBW from X mode tunnelling



Resonances and cut-offs in tokamak

- A fraction of X mode power tunnels through low density cut-off
- But fraction is low unless edge density gradient very high
- B-X heating observed on TST-2 - and B-X emission on COMPASS-D

2. EBW from X mode HFS launch



Resonances and cut-offs in tokamak

- X mode launched from high field side. Some X damping but rest reaches UHR
- EBW then goes back in plasma and is strongly damped
- Narrow "window of opportunity"

X-B ray tracing & damping in COMPASS-D

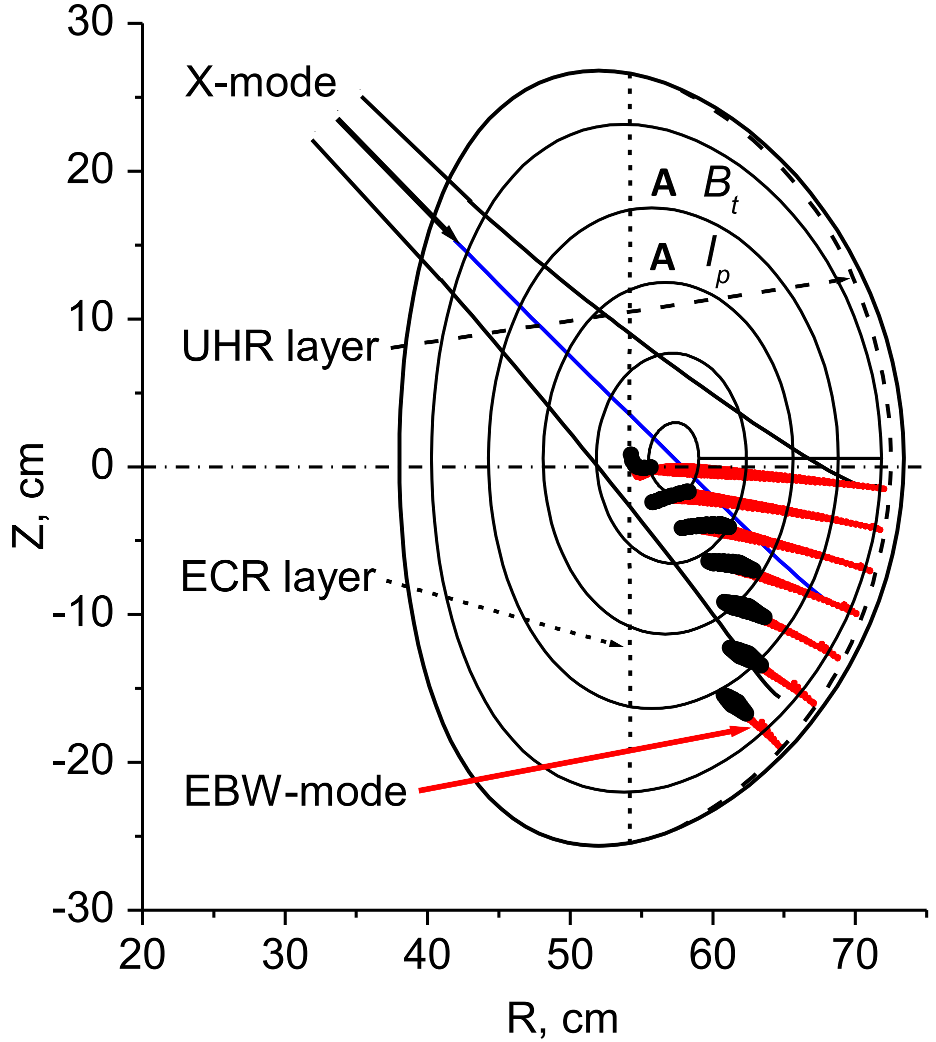
- In COMPASS-D waves were injected from HFS to avoid X-mode low density cut-off

- Some X-mode damping – but most power reached Upper Hybrid Resonance

- X-mode converted to EBW which propagated back to damp near cyclotron resonance

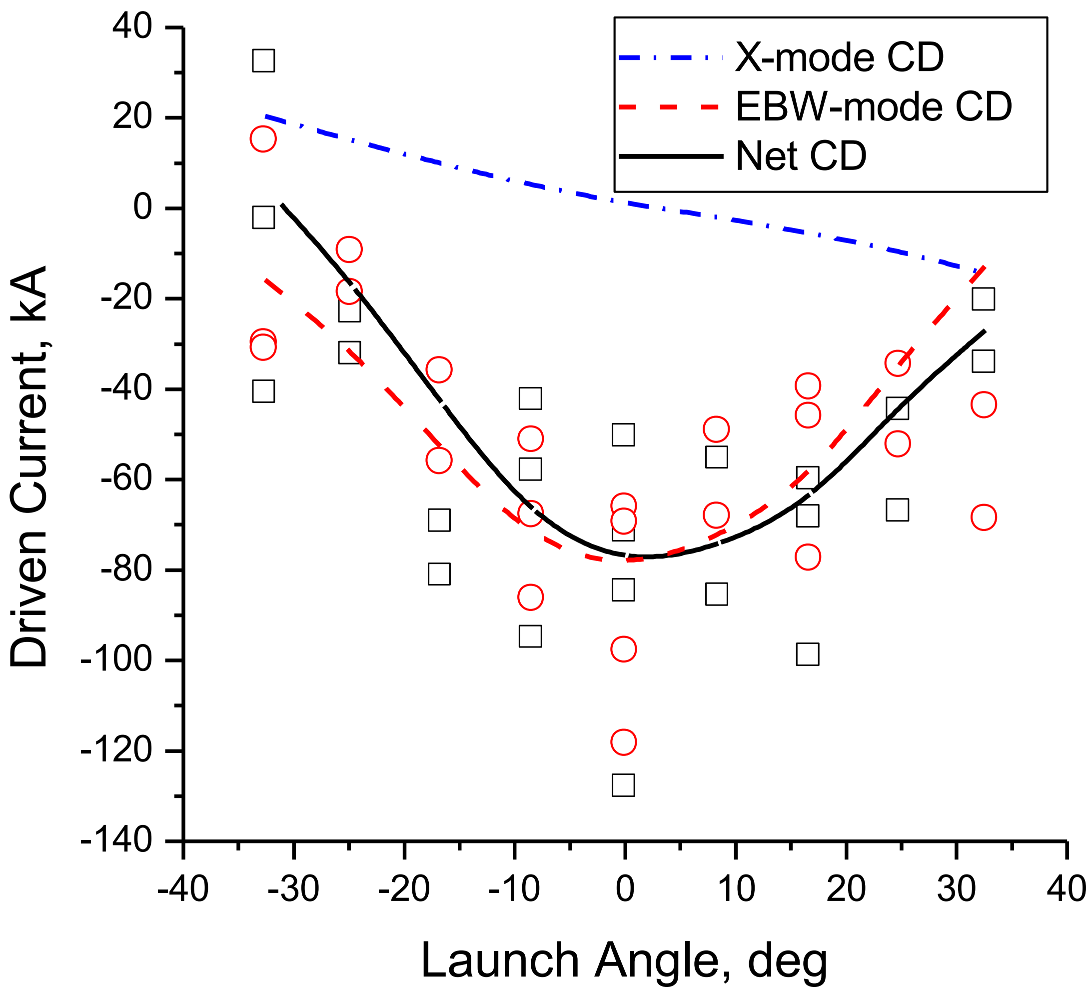
- Good test of theory and codes – agreed with experiment !!!

V. Shevchenko et al., Phys. Rev. Lett. ***89*** *(2002) 265005.*



Poloidal projection of beam tracing for X-mode and ray-tracing for EBW H&CD in COMPASS-D

X-B Current Drive on COMPASS-D



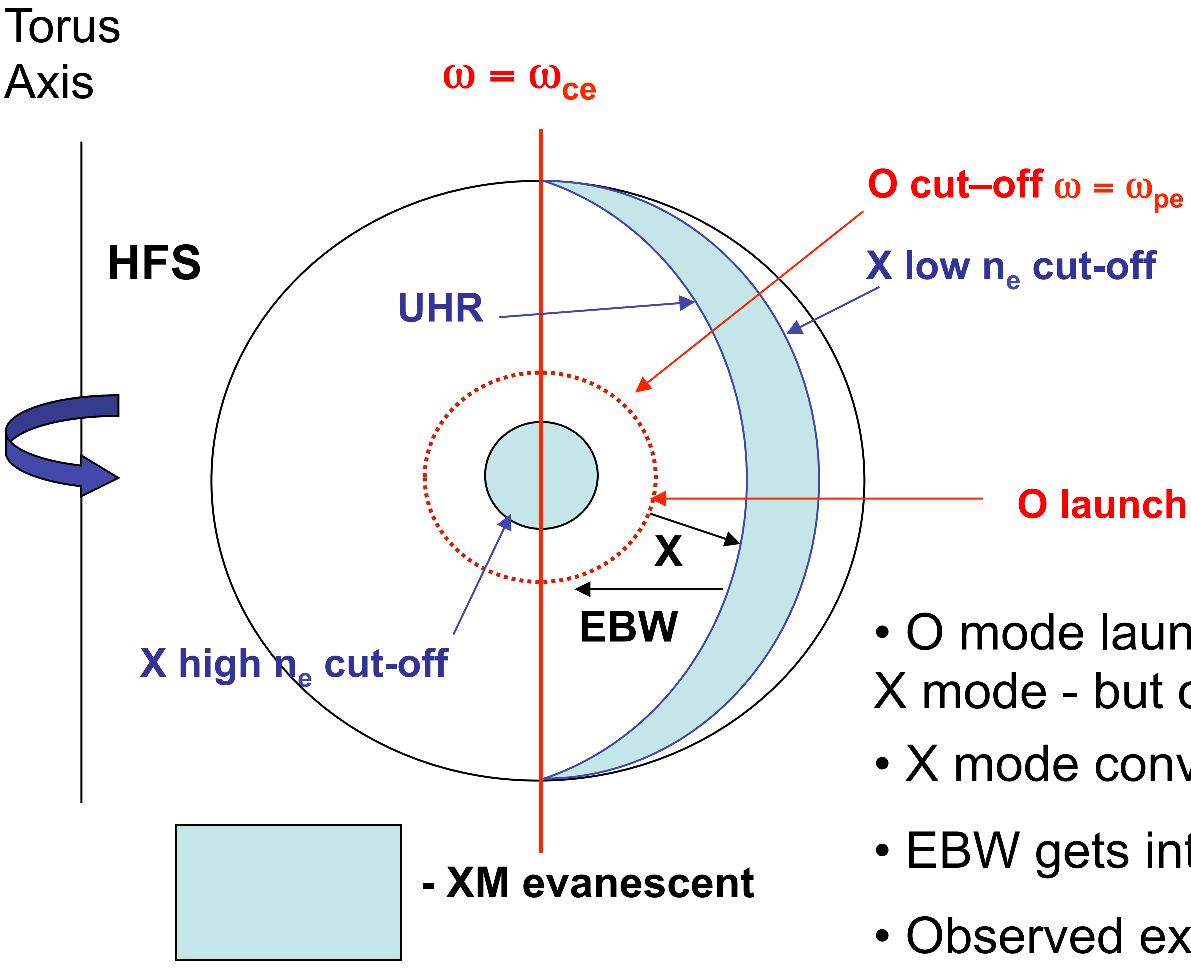
Shevchenko V. et al, PRL 26, 265005 (2002)

- Simulated with VIBETRAC (Ioffe, ray tracing) + BANDIT-3D (CCFE, Fokker-Planck) codes
- X-B EBW CD calculations agreed with experiments. CD peaks at perp launch when X mode damping ~ 0 and EBW is maximised

(symbols = expt at different times)

(lines = codes)

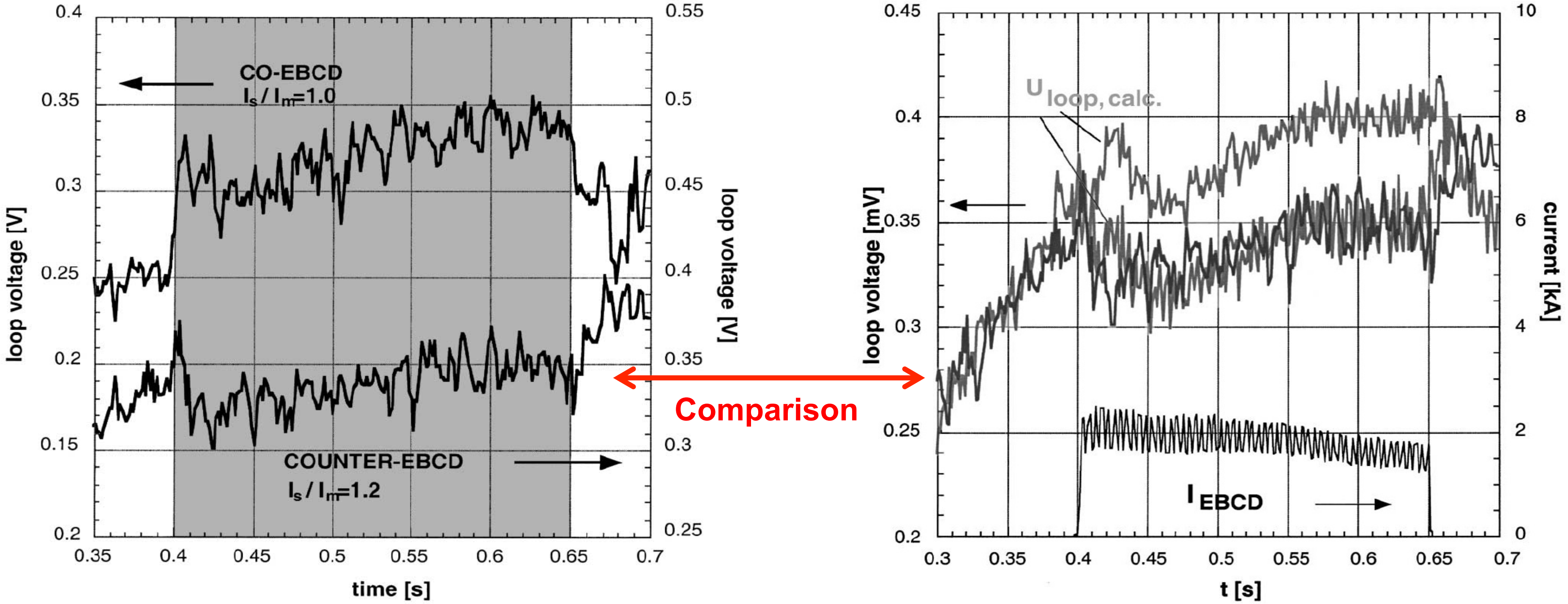
3. EBW from O-X-B mode conversion



Resonances and cut-offs in tokamak

- O mode launched from outside. Converts to X mode - but only at certain angles
- X mode converts to EBW at UHR
- EBW gets into core plasma and damps
- Observed experimentally, e.g. TCV and W7-AS

O-X-B EBW CD detected in W7AS stellarator



Change of loop voltage during co- and counter- EBW CD (0.4–0.65 s). Since magnetic configuration was different for the two cases, the signals refer to different scales.

Calculated loop voltages (grey) for EBW heating only (top trace) and with additional counter current drive (lower trace) in comparison with the measured loop voltage (black curve).

EBCD efficiency was comparable to the COMPASS-D results

3. O-X-B mode conversion

- Outside launch of O then convert to X at O mode cut-off $\omega = \omega_{pe}$
- Conversion only 100% at optimum angle

$$\sin^2\varphi = N^2_{||opt} = \omega_{ce}/(\omega_{ce}+\omega)$$

- Mode conversion window can be estimated by WKB theory. Angular width wider for steeper ∇n_e. Fluctuations and magnetic geometry can affect this – need for further research
- Launch / viewing plane determined by $|B_{tot}|$ and ∇n_e where $\omega = \omega_{pe}$
- Angle between ∇n_e and optimal direction depends only on $|B_{tot}|$
- Magnetic pitch angle can be deduced from EBW emission

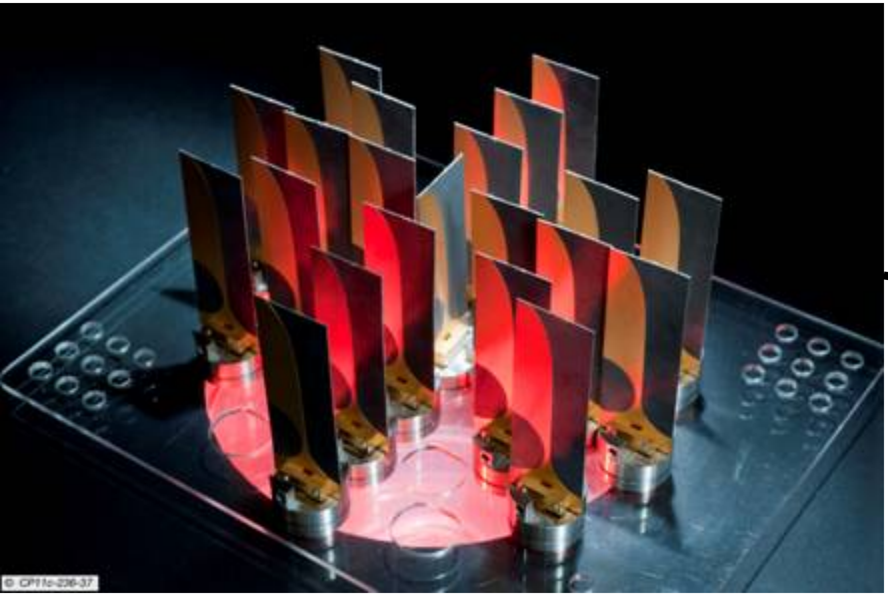
A survey of mode coupling can be found in H. Igami et al, Plas Phys Contr Fus **48** 573 (2006)

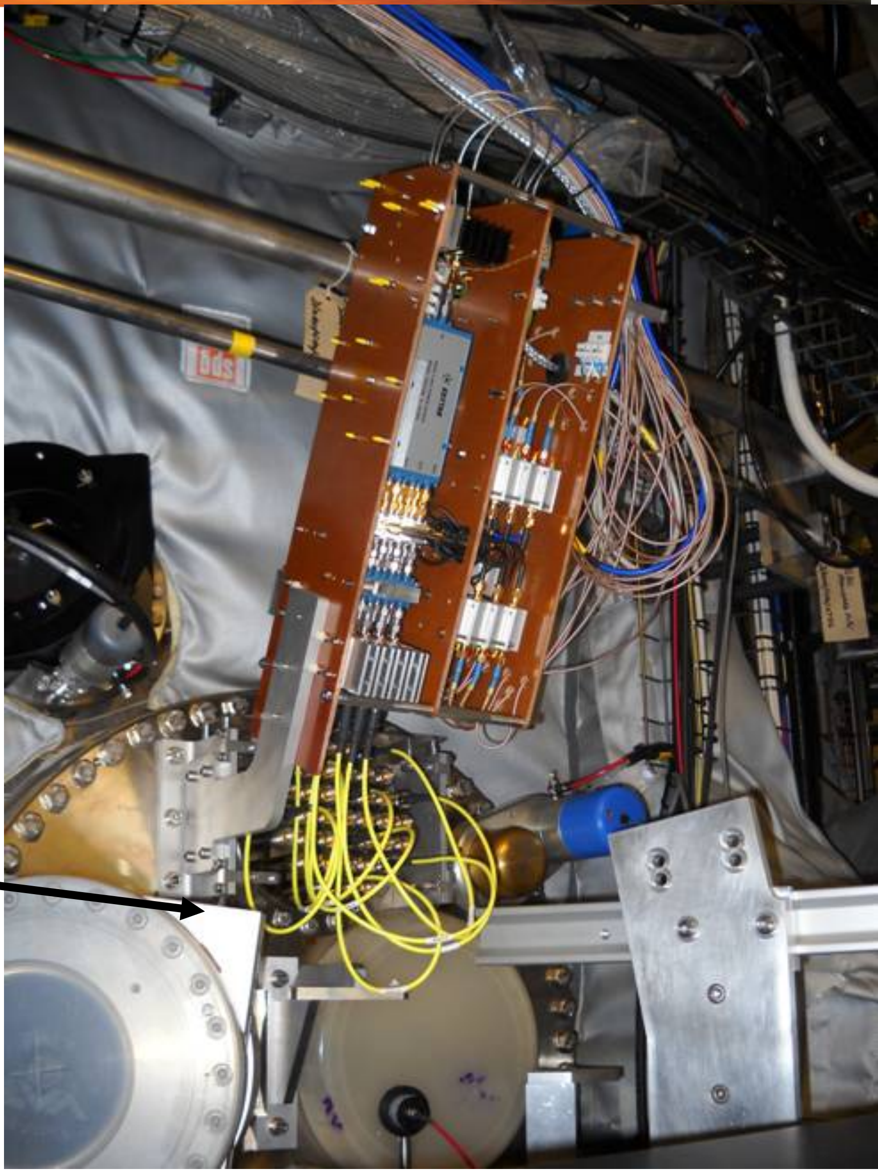
EBW as a Diagnostic

- Reverse process possible.
- B-X-O ⇒ O-mode waves omitted from plasma only at certain angles ⇒ information on local field direction at X-O conversion ⇒ j(r) at edge ⇒ ELM physics
- Used on MAST

Synthetic Aperture Microwave Imaging on MAST

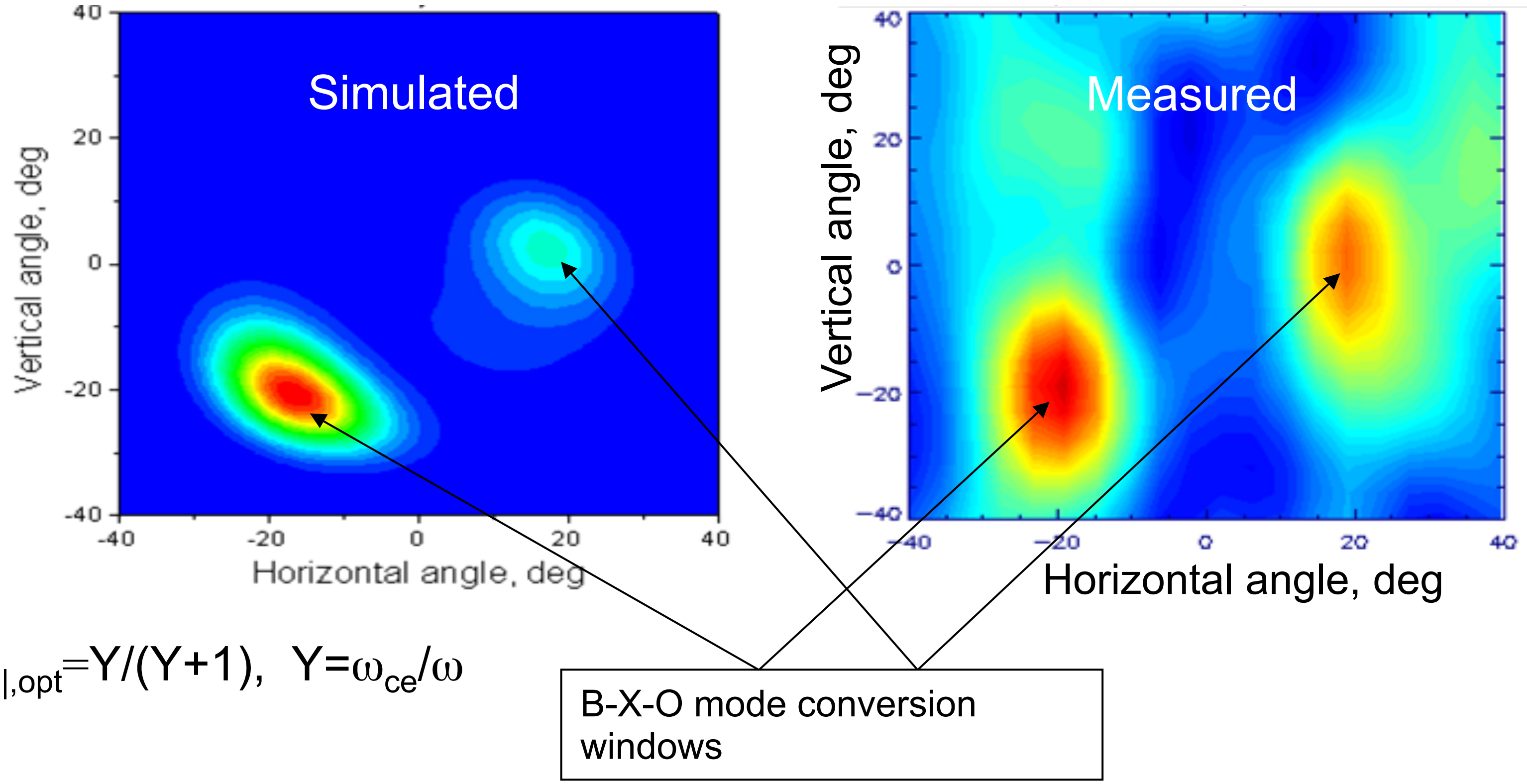
- SAMI - already used in astronomy
- Up to 37 antennas installed
- Imaging time resolution about 10μs
- 4GB of data every shot
- System fully controlled by FPGA
- Upgrade to be installed soon





Assembly of 21 antennas

SAMI results, 15GHz low density L-mode in MAST

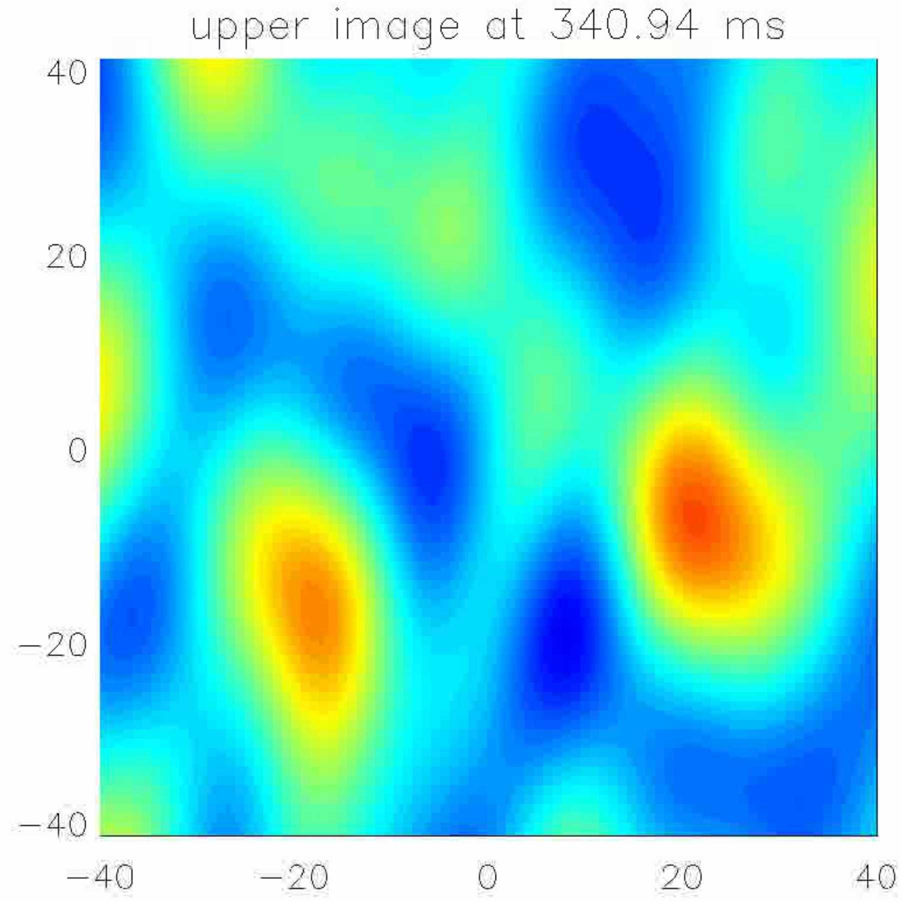


$\sin^2\phi = N^2_{||,opt} = Y/(Y+1)$, $Y = \omega_{ce}/\omega$

- Angular width of mode conversion window wider for steeper ∇n_e
- Launch / viewing plane determined by B_{tot} and ∇n_e in the layer where $\omega = \omega_{pe}$
- Magnetic pitch angle can be deduced from inclination of MC windows
- Active probing also possible $\Rightarrow$ more information e.g. edge turbulence

EBW emission in MAST H-mode

ELM-free period in H-mode



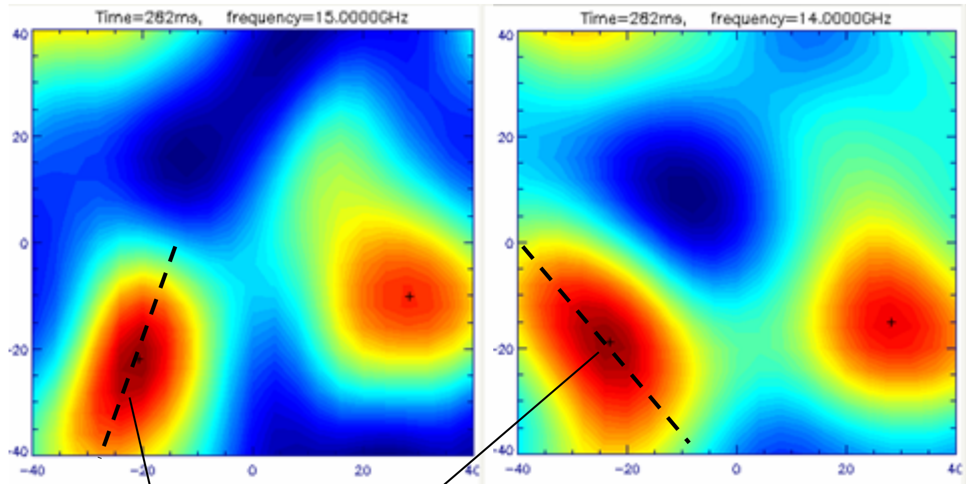
- First image of EBW emission from tokamak plasma
- 2-D space + 1-D frequency

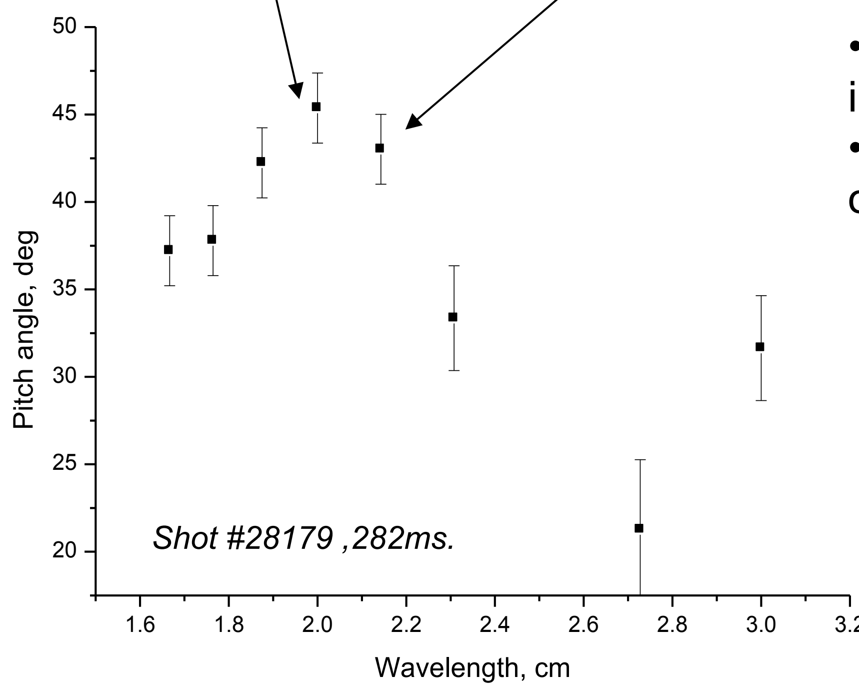
Movie covers 30ms:

- integrated over 10 frames, 160μs between frames
- Emission windows in expected positions

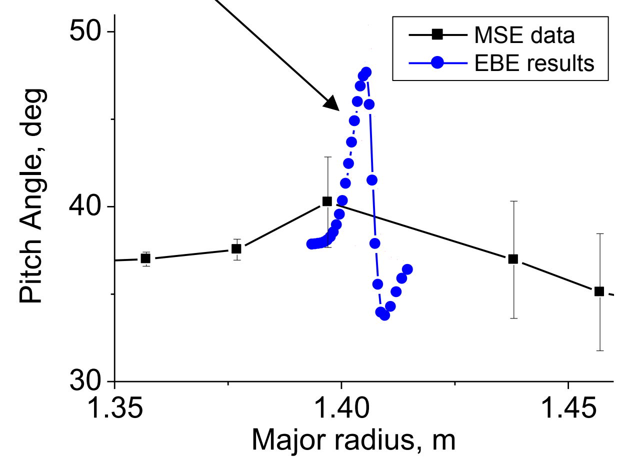
MAST shot #27022

Preliminary estimate of edge current profile on MAST from Mode Conversion ellipse variation with wavelength/frequency

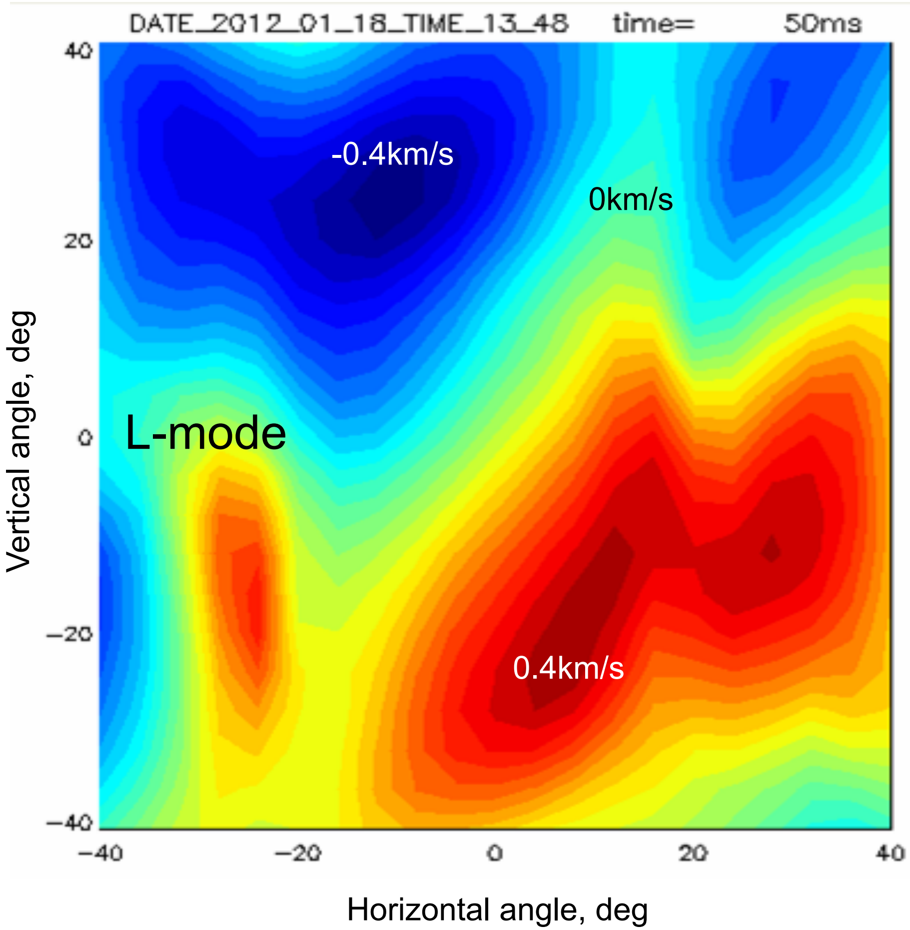


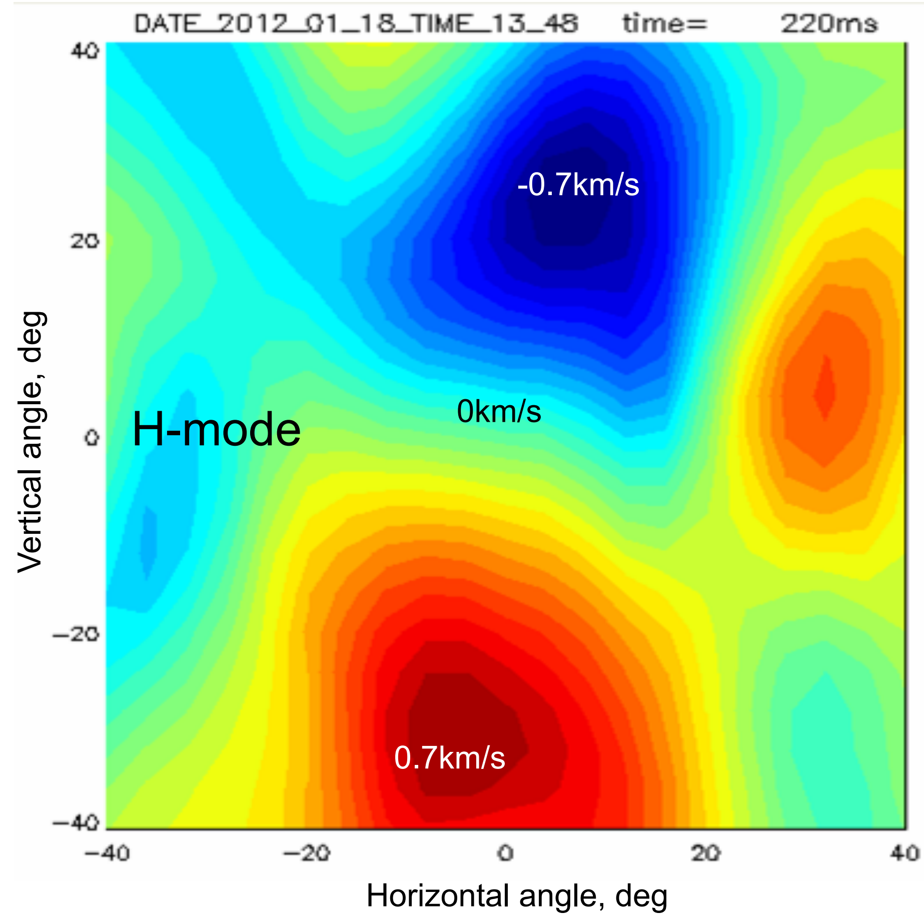


- Change of MC ellipse inclination indicates the change of shear sign
- Pitch angle behaviour is similar to one measured in SM experiments



SAMI at 10 GHz - preliminary turbulence measurements





- Velocity maps of turbulence estimated from Doppler shifts of back-scattered signal.
- Note the difference in flow directions during L-mode and H-mode phases.

Summary - EBWs pros and cons

- ☺ Like ECRH, source and launch technology are straightforward
- ☺ High n_e no problem. Strong damping
- ☺ Current drive: more efficient than ECCD (damps on faster electrons, trapping effects weaker).
- ☺ EBW emission can be a useful diagnostic
- 😐 Experimental base not large, but ~ agrees with codes
- ☹ Coupling of free space wave to EBW can be difficult
- 😐 Applications mainly of interest for spherical tokamaks and stellarators, i.e. low B^2/n_e machines

Some References

Theory

1. I Bernstein, Phys. Rev. 109, 10 (1958)
2. Waves in Plasmas by T H Stix, 1997
3. S. Puri et al, J. Plasma Phys. 14, 169 (1985).
4. A Ram and S Schultz, Phys Plasmas 7 4084 (2000)
5. R A Cairns and C N Lashmore-Davies, Phys. Plasmas **7**, 4126 (2000);
6. A Ram et al, Phys. Plasmas **9**, 409 (2002)

Experiments

1. V Shevchenko et al., Phs Rev Lett 89 265005 (20002) – COMPASS-D
2. V Shevchenko et al. Fus Science & Technology 52 (2007) - MAST
3. G Taylor et al, Phys. Plasmas 12, 052511 (2005) – NSTX
4. J Urban et al, Nuc Fus 51 083050 (2011) – survey, EBWs for Spherical Tokamaks
5. T Maekawa et al, Phys Rev Lett 86 3783 (2001)
6. S Shiraiwa et al, Phys Rev Lett 96 185003 (2006) – TST-2
7. H P Laqua et al, Phys Rev Lett 90 075003 (2003) - W7AS stellarator