14th ITER International School
Aix-Marseille Université
Aix-en-Provence, France
2025-07-02

Integrated Modeling for Heating and Current Drive

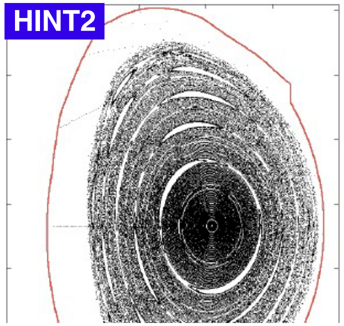
A. Fukuyama

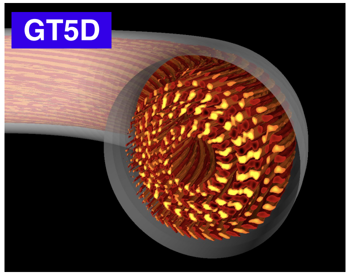
National Institutes for Quantum Science and Technology (QST)
Professor Emeritus, Kyoto University

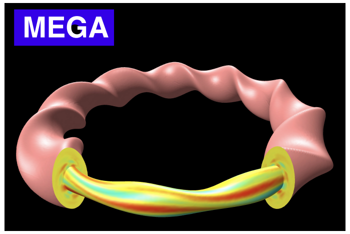
Contents

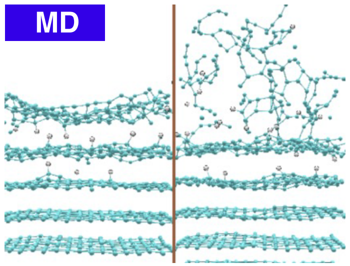
- **Role of heating and current drive**
  - **start-up, sustainment, probabilistic incidents, shut-down**
  - **control of density, temperature, current and rotation**
  - **plasma model: fluid, multi-fluid, kinetic**
- **Modeling of actuators**
  - **Neoclassical resistivity and bootstrap current**
  - **Neutral beam**
  - **Pellet**
  - **Waves**
    - **Frequency ranges**: IC, LH, EC
    - **Models**: ray tracing, beam tracing, full wave, kinetic full wave
- **Kinetic integrated modeling**
- **Summary**

Components of integrated tokamak modeling









3D MHD Equilibrium

Turbulence Simulation

Nonlinear MHD

Turbulence Simulation

Material Simulation

Integrated Modeling
- **Diagnostic model**
- **Control model**
- **Equilibrium:** Rotation, Anisotropy; 2D MHD Equilibria; Island, Braiding
  - Fixed boundary
  - Free boundary
  - Time evolution
- **Core Plasma Transport:** Neoclassical Transport; Neutral Transport; Turbulent Transport; Impurity Transport; Energetic Particles; Radiation Transport
  - Diffusive transport
  - Dynamic transport
  - Kinetic transport
  - Particle orbit
- **Global Stability:** Linear Stability; Plasma Deformation
  - MHD stability
  - Kinetic stability
- **Source:** NBI; Wave: IC, LH, EC; Pellet
  - Ray/beam tracing
  - Full wave
  - Beam/Pellet injection
- **Peripheral Plasma Transport:** Collisional Transport; Neutral Transport; Turbulent Transport; Impurity Transport; Plasma-Wall Interaction; Radiation Transport
  - Fluid model
  - Kinetic model
  - Particle model

Role of heating, current drive and fueling

- **Start-up of fusion reaction**
  - Initial heating for high temperature
  - Initial fueling for high density
- **Sustainment of fusion reaction**
  - Continuous fueling to sustain fuel density
  - Pressure profile control to optimize fusion power output
- **Sustainment of stable plasma**
  - Continuous current drive to keep plasma current
  - Pressure profile control to avoid MHD instability
  - Rotation control to avoid MHD instability

Various role of actuators

Actuator	Heating	Current drive	Fueling	Others
PF coil E_{toroidal}	Joule heating	Inductive current drive		
NBI	NBI heating	NBI current drive	NBI fueling	NBI rotation drive*1
Wave	Wave heating	Wave current drive		
Pellet injection			Fueling	Cooling*1
Gas injection			Fueling*2	Cooling*1

*1 Control of MHD stability, *2 SOL physics

Source modeling

- **Heat and momentum sources**:
  - **Toroidal electric field** and **density gradient**
    - Neoclassical resistivity, bootstrap current
  - **Alpha particle heating**:
    - sensitive to fuel density and momentum distribution
  - **Neutral beam injection**:
    - birth profile, finite size orbit, deposition to bulk plasma
  - **Waves**:
    - IC ($\sim 50\,\mathrm{MHz}$): fuel ion heating, current drive, rotation drive(?)
    - LH ($\sim 10\,\mathrm{GHz}$): current drive
    - EC ($\sim 170\,\mathrm{GHz}$): current drive, pre-ionization
- **Particle source**
  - **Neutral beam injection**:
  - **Pellet injection**: penetration, evaporation, ionization, drift motion
  - **Gas injection**: massive gas injection, wall recycling

Description of heating and current drive

NBI

1. **Beam injection**
2. **Beam penetration**
3. **Beam deposition**
   sensitive to $f_s(\boldsymbol{v})$
4. **Generation of energetic ions**
5. **Slowing down of energetic ions**
6. **Deviation of $f_s(\boldsymbol{v})$ from Maxwellian**
7. Modification of **Beam deposition** $\Rightarrow$ 3

Wave

1. **Wave excitation**
2. **Wave propagation**
3. **Wave absorption**:
   sensitive to $f_s(\boldsymbol{v})$
4. **Velocity diffusion of resonant particles**
5. **Collisional relaxation**
6. **Deviation of $f_s(\boldsymbol{v})$ from Maxwellian**
7. Modification of **Wave absorption** $\Rightarrow$ 3

How to include deviation of $f_s(\boldsymbol{v})$ from Maxwellian

Various model of plasma

- **Fluid model**

1D Diffusive transport equation: $n, u_\phi, T(\rho, t)$	**diffusive transport**
2D Diffusive transport equation: $n, u_\parallel, T(\rho, \chi, t)$	**SOL transport**
1D Fluid-type transport equation: $n, \boldsymbol{u}, T(\rho, t)$	**multi-fluid model**
2D Fluid-type transport equation: $n, \boldsymbol{u}, T(\rho, \chi, t)$	**2D multi-fluid**
3D Gyro fluid equation: $n, \boldsymbol{u}, T(\rho, \chi, \zeta, t)$	**Gyro fluid model**

- **Kinetic model**

Bounce-averaged drift-kinetic equation: $f(p, \theta_{\rm p}, \rho, t)$	**BAFP**
Axisymmetric gyrokinetic equation: $f(p, \theta_{\rm p}, \rho, \chi, t)$	**AGK model**
Gyrokinetic equation: $f(p, \theta_{\rm p}, \rho, \chi, \zeta, t)$	**Gyro kinetic model**
Full kinetic equation: $f(p, \theta_{\rm p}, \phi_{\rm g}, \rho, \chi, \zeta, t)$	**Full kinetic model**

Bounce-averaged Fokker-Planck equation

- **Multi-species momentum distribution functions**:

$$f_s(p_{||}, p_\perp, \rho, t)$$

  - **3 phase space variables**:
    - **parallel and perpendicular momentum, minor radius**
    - toroidal symmetry, gyro-motion average, bounce-motion average

- **Fokker-Planck equation**

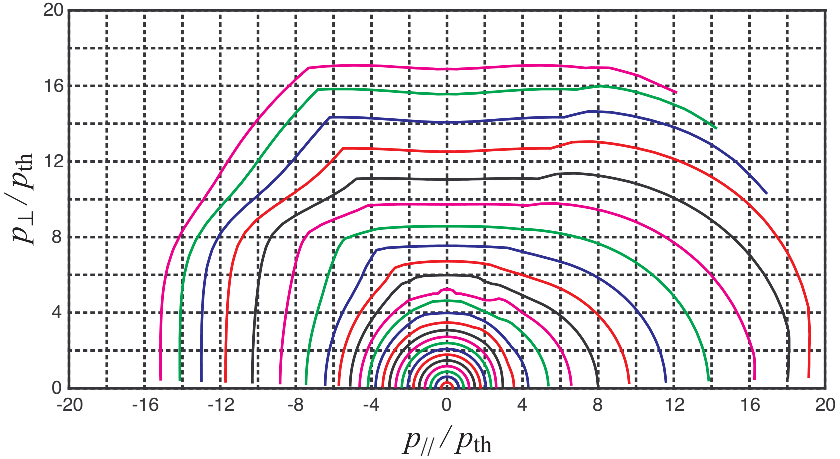
$$\frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + D(f_s) + S_s$$

  - $E(f)$: **Acceleration due to DC electric field**
  - $C(f)$: **Nonlinear Coulomb collision**
  - $Q(f)$: **Quasi-linear diffusion due to wave-particle resonance**
  - $D(f)$: **Spatial diffusion** (model, collisional, QL)
  - S : **Particle source and sink** (NBI, Fusion reaction, Pellet)

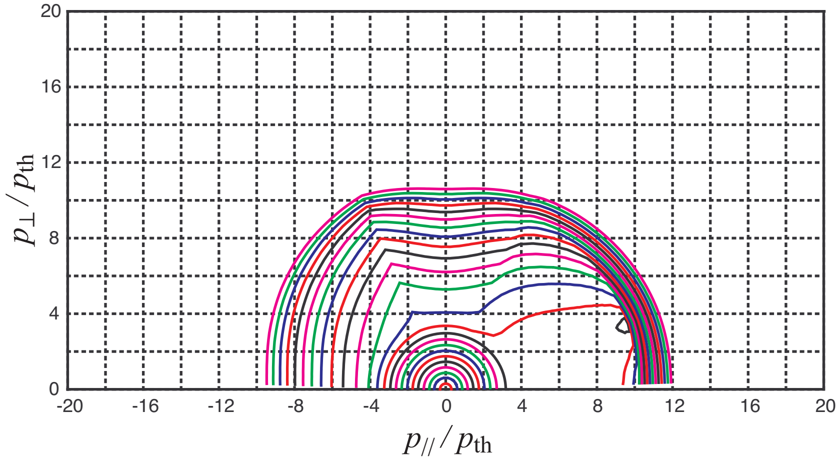
Examples of velocity distribution functions

- **3D multi-species Fokker-Planck analysis** of $f_s(p_{\|}, p_{\perp}, \rho, t)$

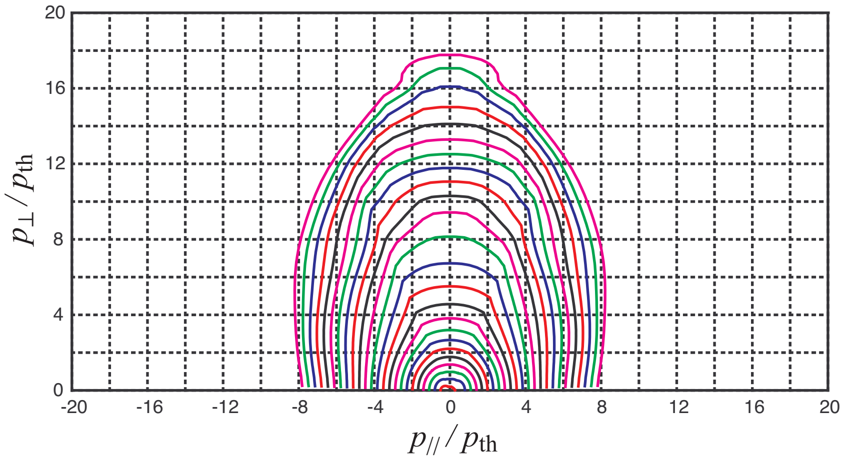
Electron : EC+LH



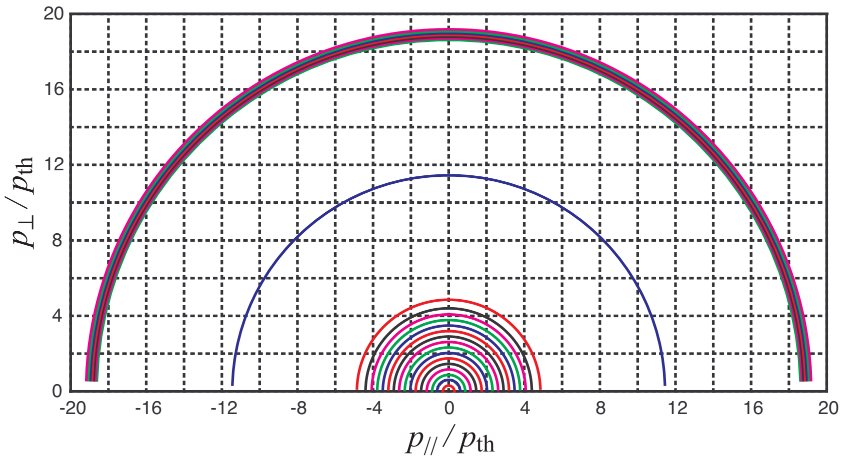
D : NBI



T : ICRF



He : DT reaction



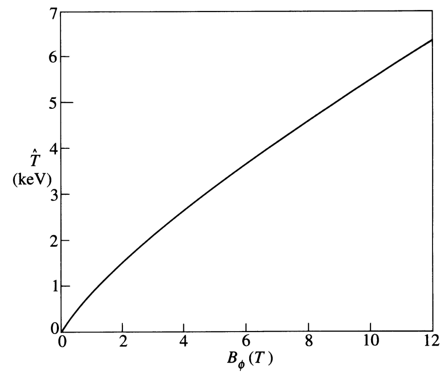
Joule heating

- **Heating by plasma current**
  - Heating power density: $P_\Omega = \eta j^2$ (η: resistivity, j: current density)
- **Upper limit on plasma current**: MHD stability requires

$$q_a = \frac{aB_\phi}{RB_{\theta a}} = \frac{2\pi a^2 B_\phi}{R\mu_0 I_\mathrm{p}} \geq 2$$

  - Upper limit of current density: $\langle j \rangle = \dfrac{I_\mathrm{p}}{\pi a^2} \leq \dfrac{1}{\mu_0}\dfrac{B_\phi}{R}$
- **From the balance of heating power and transport loss**:

$$\eta \langle j^2 \rangle = \frac{3nT}{\tau_\mathrm{E}}$$



  - Resistivity: (T_e: keV): $\eta \sim 8 \times 10^{-8} Z_\mathrm{eff} T_\mathrm{e}^{-3/2}\ \Omega\mathrm{m}$
  - Energy confinement time: $\tau_\mathrm{E} = 0.5(n/10^{20})a^2$
- **Upper limit of temperature**: $T_\mathrm{max} = 1.8\, Z_\mathrm{eff}^{2/5} \left(\dfrac{a}{R} B_\phi\right)^{4/5} \sim 0.87\, B_\phi^{4/5}$

Joule heating: neoclassical resistivity

- **Resistivity in tokamak plasma**
  - **Trapped particles** do not contribute to parallel current.
  - **Collisional transition** between trapped and untrapped
  - Steady-state solution of **bounce-averaged Fokker-Planck equation**
- **Approximate formula**: [Wesson, Tokamaks 3rd]

$$\eta = \eta_s \frac{Z_{\rm eff}}{(1-\phi)(1-C\phi)} \frac{1+0.27(Z_{\rm eff}-1)}{1+0.47(Z_{\rm eff}-1)}$$

where

$$\eta_s = 0.51\frac{m_{\rm e}}{n_{\rm e}e^2\tau_{\rm e}}, \quad C = \frac{0.56}{Z_{\rm eff}}\left(\frac{3.0-Z_{\rm eff}}{3.0+Z_{\rm eff}}\right)$$

$$\phi = \frac{f_{\rm T}}{1+(0.8+0.20Z_{\rm eff})\nu_{*\rm e}}, \quad f_{\rm T} = 1 - \frac{(1-\epsilon)^2}{(1-\epsilon^2)^{1/2}(1+1.46\epsilon^{1/2})}$$

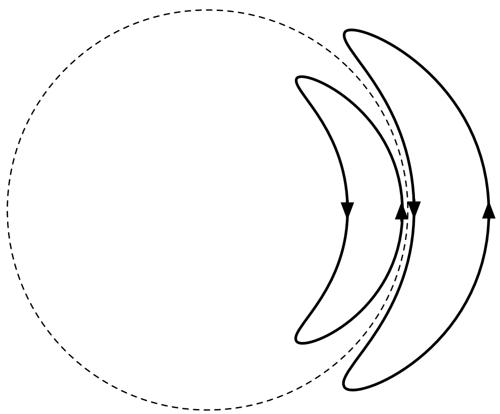
$$\epsilon = \frac{r}{R}, \quad \nu_{*\rm e} = \frac{Rq}{v_{\rm Te}\tau_{\rm e}}, \quad \tau_{\rm e} = 3(2\pi)^{3/2}\frac{\epsilon_0^2 m_{\rm e}^{1/2} T_{\rm e}^{3/2}}{n_{\rm e}e^4 \ln\Lambda}$$

Bootstrap current (1)

- **Current driven by density gradient in tokamak plasma**
  - **trapped particle current**

$$j_{\text{trapped}} \sim -e\,\epsilon^{1/2}(\epsilon^{1/2} v_{\text{T}})\,\frac{\mathrm{d}n_{\text{e}}}{\mathrm{d}r} w_{\text{b}} \sim -q\frac{\epsilon^{1/2}}{B} T \frac{\mathrm{d}n_{\text{e}}}{\mathrm{d}r}$$

  - $\epsilon^{1/2}$: fraction of trapped particles
  - $\epsilon^{1/2} v_{\text{Te}}$: magnitude of parallel velocity of trapped particles
  - $w_{\text{b}} = \epsilon^{-1/2} q\rho$: radial width of trapped particle orbit
  - $\dfrac{\mathrm{d}n_{\text{e}}}{\mathrm{d}r} w_{\text{b}}$: density difference of upward and downward electrons



Bootstrap current (2)

- **Collisional momentum transfer**
  - **from trapped electrons to untrapped electrons, and balance with ions**

$$\nu_{\mathrm{effective}}\, j_{\mathrm{trapped}} \frac{\nu_{\mathrm{ee}}}{\epsilon}\, j_{\mathrm{trapped}} \sim \nu_{\mathrm{ei}}\, j_{\mathrm{BS}}$$

- **Bootstrap current density**

$$j_{\mathrm{BS}} \sim \frac{\nu_{\mathrm{ee}}}{\epsilon \nu_{\mathrm{ei}}} j_{\mathrm{trapped}} \sim -\frac{\nu_{\mathrm{ee}}}{\nu_{\mathrm{ei}}} \frac{q}{\epsilon^{1/2}} \frac{T}{B} \frac{\mathrm{d}n}{\mathrm{d}r} \sim -\frac{q}{\epsilon^{1/2}} \frac{T}{B} \frac{\mathrm{d}n}{\mathrm{d}r}$$

- **Slightly more accurate form**: $q = \epsilon B / B_\theta$ [Wesson, Tokamaks 3rd]

$$j_{\mathrm{BS}} = -\frac{\epsilon^{1/2} n_{\mathrm{e}} T_{\mathrm{e}}}{B_\theta} \left[2.44 \left(1 + \frac{T_{\mathrm{i}}}{T_{\mathrm{e}}} \right) \frac{\partial \ln n_{\mathrm{e}}}{\partial r} + 0.69 \frac{\partial \ln T_{\mathrm{e}}}{\partial r} - 0.42 \frac{T_{\mathrm{i}}}{T_{\mathrm{e}}} \frac{\partial \ln T_{\mathrm{i}}}{\partial r} \right]$$

- **Finite orbit-width Fokker-Planck analysis for higher accuracy**

NBI heating (1)

- **Neutral beam injection**

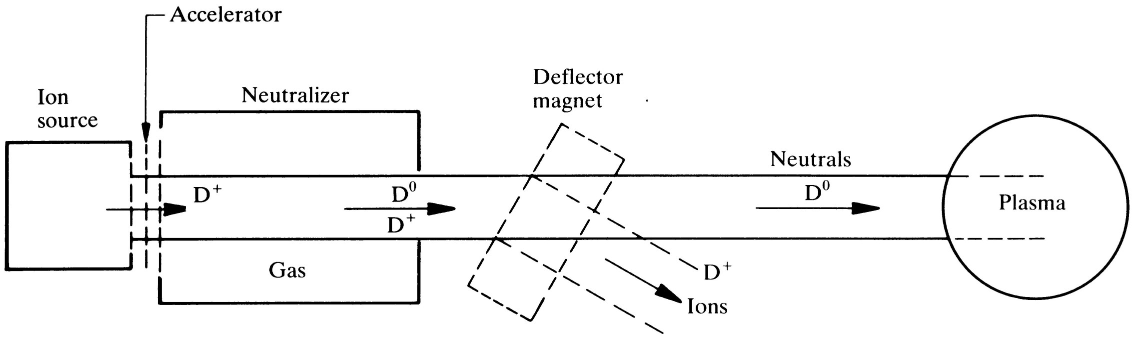
  Ion generation (DC discharge, RF discharge)

  $\Longrightarrow$ Acceleration by electric field (10 keV ~ 1 MeV)

  $\Longrightarrow$ Neutralization (remove ions)

  $\Longrightarrow$ Injection into plasma

  $\Longrightarrow$ Collide with ions and electrons to heat them

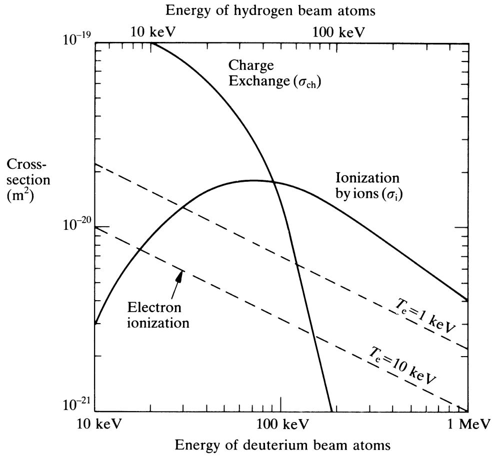


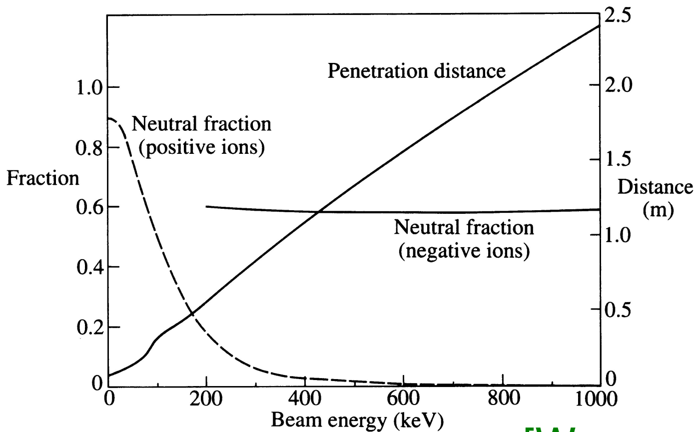
[Wesson, Tokamaks 3rd]

- **Features**
  - Physical mechanism is simple
  - High power heating achieved
  - High current drive efficiency
  - Rotation drive is possible
  - High injection energy for large machine
  - Large aperture area required

NBI heating (2)

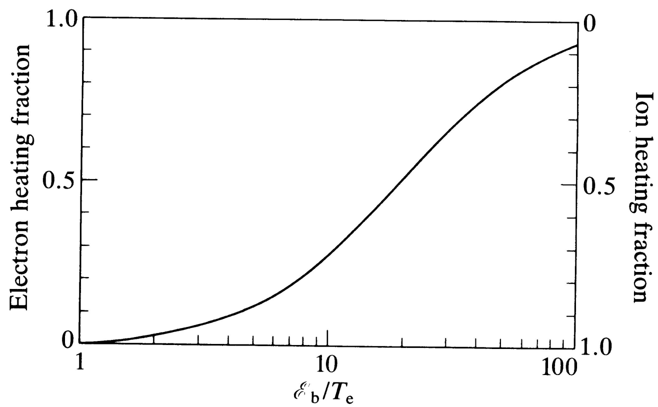
- **Neutralization efficiency**:
  - Charge exchange for less than 80 keV
  - Ionization reduces neutralization for higher than 80 keV.
  - Negative ion is required for high energy





[Wesson, Tokamaks 3rd]

- **Plasma heating**
  - Collisional crosssection $\propto u_{\text{relative}}^{-4}$
  - Collision with ions decreases, Collision with electrons increases, for high beam energy



Modeling of NB heating and current drive (1)

- **Fast neutral** to **fast ion**
  - **Charge exchange**: $\mathrm{A_{fast}} + \mathrm{A}^+ \longrightarrow \mathrm{A} + \mathrm{A}^+_{\mathrm{fast}}$
  - **Collisional ionization (single step model)**

$$\mathrm{A_{fast}} + (e \text{ or } \mathrm{B}^+) \longrightarrow \mathrm{A}^+_{\mathrm{fast}} + (e \text{ or } \mathrm{B}^+) + e$$

  - **Collisional ionization (multi step model)** [Janev et al. NF 1989]

$$\mathrm{A_{fast}} + (e \text{ or } \mathrm{B}^+) \longrightarrow \mathrm{A}^*_{\mathrm{fast}} + (e \text{ or } \mathrm{B}^+) \longrightarrow \mathrm{A}^+_{\mathrm{fast}} + (e \text{ or } \mathrm{B}^+) + e$$

- **Collisional ionization** exceeds charge exchange for Eng $\gtrsim 50\,\mathrm{keV}$
- **Multi-step ionization** exceeds single-step for $Eng \gtrsim 50\,\mathrm{keV}$
- **Rate equation of beam atoms**
  - $I_n(x)$: Beam intensity of excitation level n and distance x

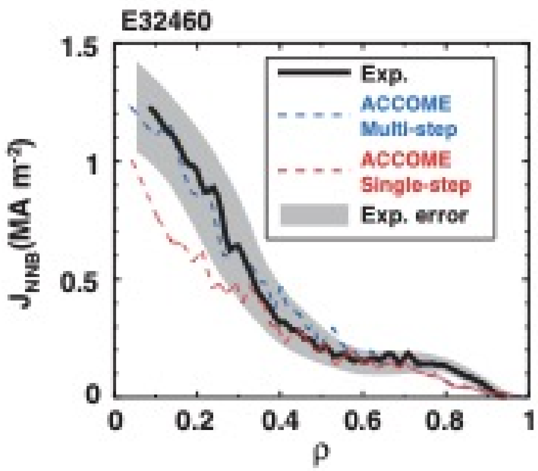
$$v_0 \frac{\mathrm{d}I_n}{\mathrm{d}x} = \sum_{n'=1}^{N} Q_{nn'} I_{n'}$$

Modeling of NB heating and current drive (2)

- **Current drive efficiency**
  - **Simple estimates**

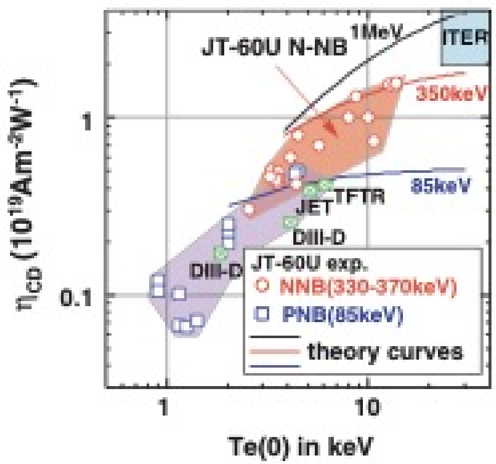
$$j_{\mathrm{NB}} = S_{\mathrm{fastion}}\,\tau_{\mathrm{slowingdown}}\,Z_{\mathrm{fastion}}\,e\,v_{||\mathrm{beam}}$$

  - **Collision with electron** is dominant for high energy beam
  - **Slowing-down distribution** should be taken into account
  - **Trapped particle effect** should be taken account
  - **2D Fokker-Planck** or **Monte Carlo** analysis is required



N-NB driven current in JT-60U

[ITER PB, NF (2007)]



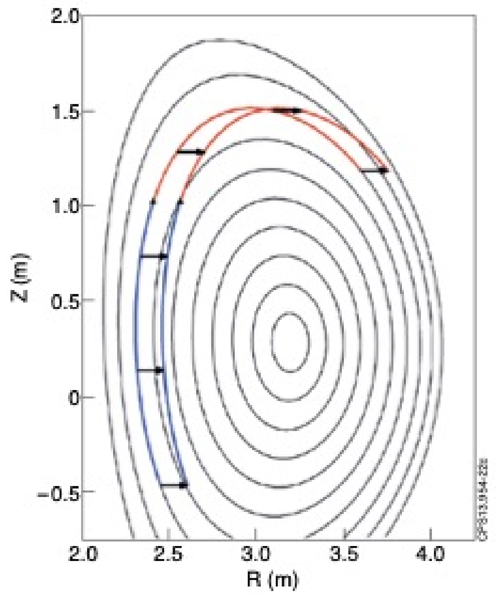
N-NB CD efficiency in JT-60U

[ITER PB, NF (2007)]

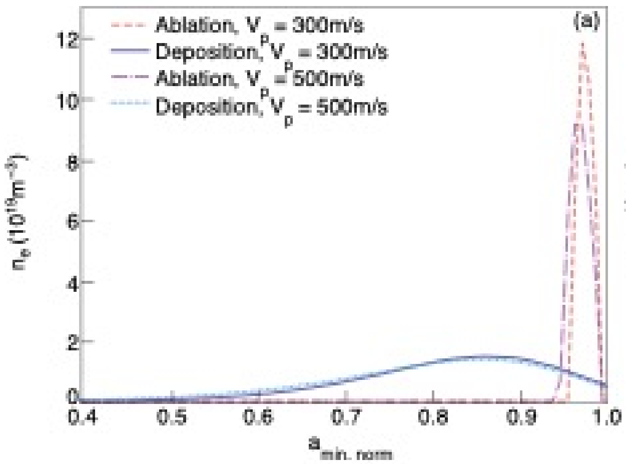
Modeling of pellet injection

- **Physics of ablation and deposition**
  - **Neutral gas shielding**: neutral gas shields pellet
  - **Drift effects**:

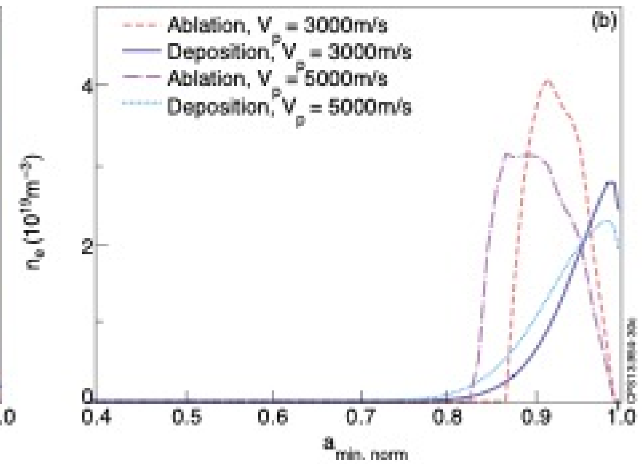
    ∇B drift $\Rightarrow$ Charge separation $\Rightarrow$ Vertical E field $\Rightarrow$ $E \times B$ drift
  - **Deposition**



Outward drift



HFS injection



LFS injection

[Pegourier: EFDA-JET-PR (2014)]

Wave heating (I)

- **Interaction between electromagnetic wave and charged particles**
- **Collisional damping**
  - Accelerated by wave electric field collides with other particles
  - Dominant in low-temperature plasma
  - Very low in high-temperature plasma ($\nu \ll \omega$)
- **Landau damping**
  - Charge particles are accelerated and decelerated by wave field
    - For most particles, time average of acceleration is zero
  - **Resonant particles** $v = \dfrac{\omega}{k}$
    - The particle feels electric field with the same phase
    - Some particles are continuously accelerated
    - If the gradient of velocity distribution function at the wave phase velocity, particles are accelerated in average.

Wave heating (II)

- **Cyclotron damping**
  - When wave frequency is close to the cyclotron frequency
  - **Condition of cyclotron resonance**: $v_{||} = \dfrac{\omega - \omega_c}{k_{||}}$
  - Wave phase synchronizes with the cyclotron motion of particles

$$
\begin{aligned}
\exp(\mathrm{i}\,\phi) &= \exp[\,\mathrm{i}\,\phi_0 + \mathrm{i}\,(k_\perp x + k_{||} z - \omega t)] \\
&= \exp[\,\mathrm{i}\,\phi_0 + \mathrm{i}\,k_\perp \rho \sin \omega_c t + \mathrm{i}\,k_{||} v_{||} t - \mathrm{i}\,\omega t] \\
&= \sum_{n=-\infty}^{\infty} J_n(k_\perp \rho) \exp[\,\mathrm{i}\,\phi_0 + \mathrm{i}\,(n\omega_c + k_{||} v_{||} - \omega) t]
\end{aligned}
$$

  - Particle feels constant wave phase: $\Longrightarrow$ resonance condition
  - Particles are accelerated perpendicularly to the magnetic field
  - In general, **cyclotron harmonic resonance** (n: integer)

$$
v_{||} = \frac{\omega - n\omega_c}{k_{||}}
$$

Various Electromagnetic Modes in Tokamak Plasmas

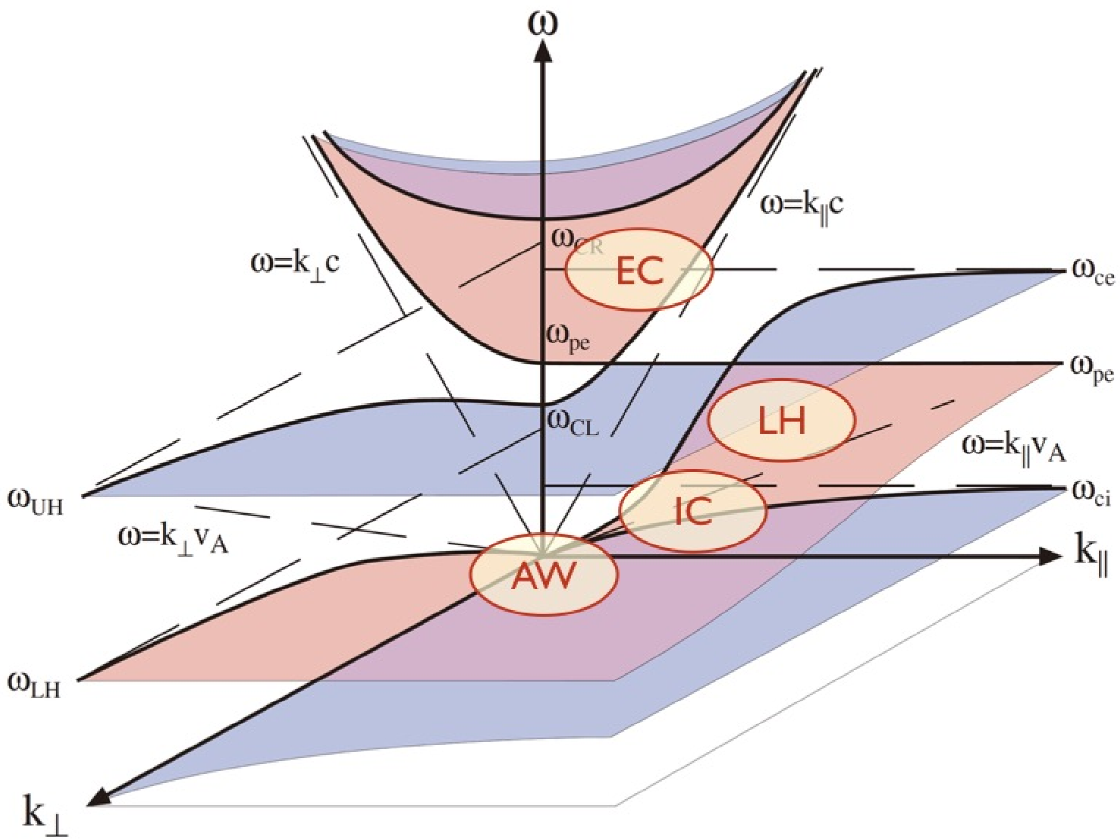
- **Externally-excited modes**: **heating and current drive**
  - Ion cyclotron range: $10\,\mathrm{MHz} \sim 300\,\mathrm{MHz}$
  - Lower hybrid range: $300\,\mathrm{MHz} \sim 10\,\mathrm{GHz}$
  - Electron cyclotron range: $10\,\mathrm{GHz} \sim 300\,\mathrm{GHz}$

- **Internally-excited long-wavelength modes**: **global instability**
  - MHD modes: sawtooth, fishbone, RWM, NTM, ELM
  - Alfvén eigenmodes: TAE, RSAE, HAE, BAE, $\cdots$

- **Internally-excited short-wavelength modes**: **turbulent transport**
  - Drift waves: trapped electron mode, temperature gradient modes
  - Ballooning modes: resistive BM, kinetic BM, current-diffusive BM
  - Ion/Electron cyclotron emission: Induced by energetic particles

Waves Used for Heating and Current Drive

- **Waves with various frequency ranges**
  - **EC** (electron cyclotron): O-mode, X-mode, electron Bernstein wave
  - **LH** (lower hybrid): fast wave (helicon), slow wave (TG and LH)
  - **IC** (ion cyclotron): magnetosonic wave, ion cyclotron wave
  - **AW** (Alfvén waves): compressional Alfvén wave, shear Alfvén wave



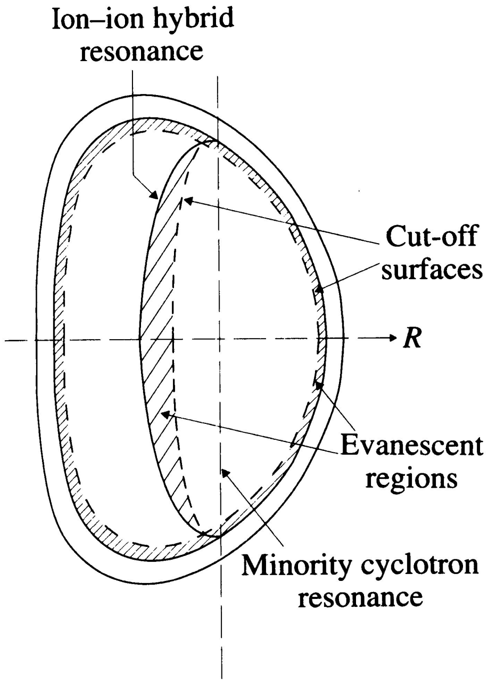
Ion cyclotron wave (I)

- **Fast wave heating**
  - **Ion cyclotron resonance**
    - Fundamental cyclotron heating is weak for one-species of ions
    - Perpendicular acceleration is reduced to zero when $\omega \sim \omega_{\mathrm{c}}$
  - **Two-ion hybrid resonance**
    - Cyclotron resonance of minority ions
    - Electron Landau damping in hot plasmas
  - **Second harmonic heating**: efficient heating even if one ion species
  - **Fast wave current drive**: electrons accelerated with $\omega < \omega_{\mathrm{ci}}$

- **Features**
  - Actual achievements of high power heating (30 MW)
  - Heating near center is easy
  - Large antenna is required
  - Current drive efficiency is slightly lower

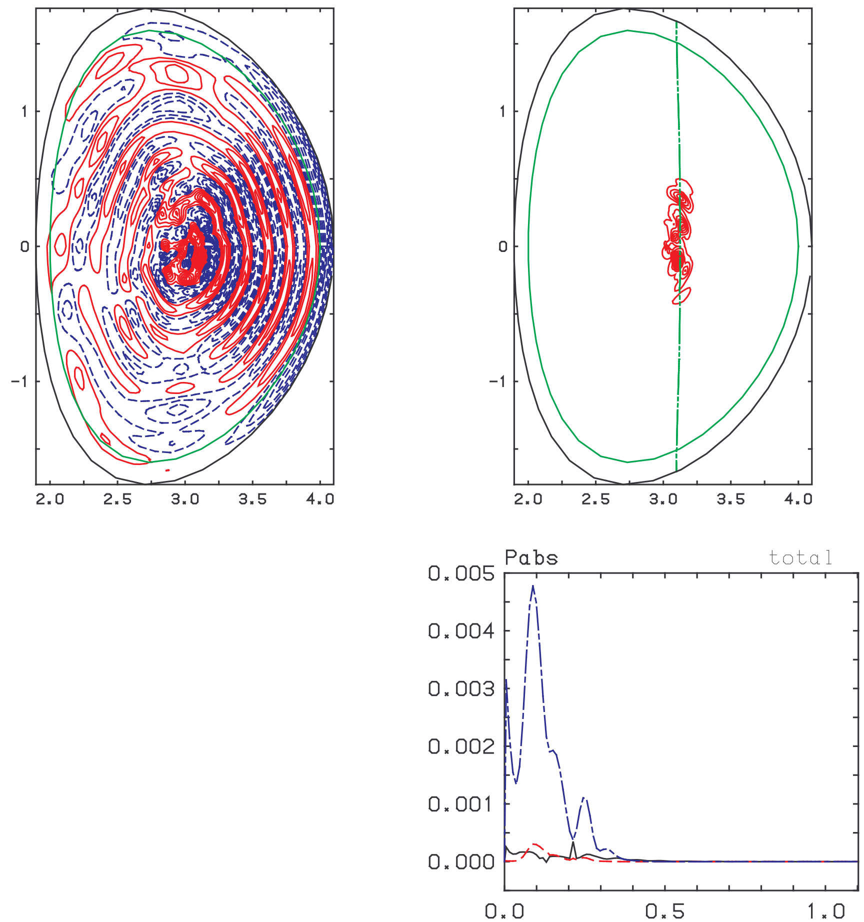
Ion cyclotron wave (II)

- **Two-ion hybrid resonance heating**
  - **10% of hydrogen into deuteron plasmas**
  - **Heating near the cyclotron resonance of hydrogen**



[Wesson, Tokamaks 3rd]

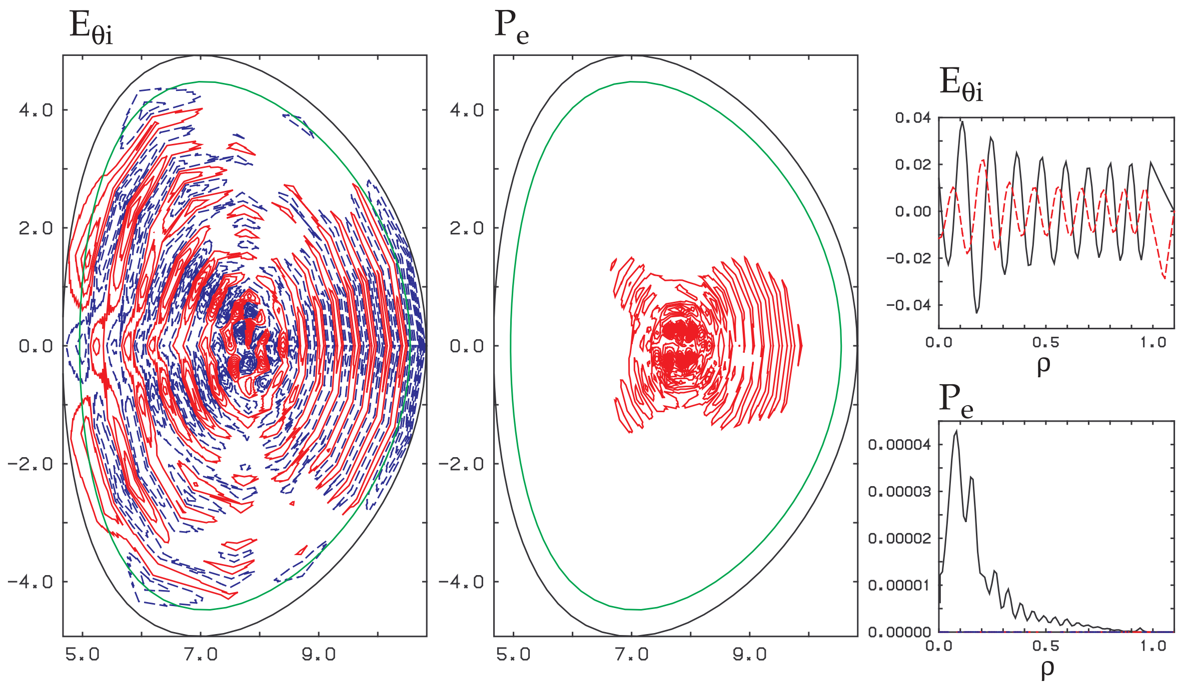
Wave E field Minority ion heating



Ion cyclotron wave (III)

- **Fast wave current drive**
  - **Excite wave which is not absorbed by ions**
  - **Wave absorbed by electrons accelerates in one direction**
  - **Equilibrium balanced by collision with ions**
  - **Asymmetry of electron distribution function drives current**

Wave electric field　　　Electron heating



Lower hybrid wave (I)

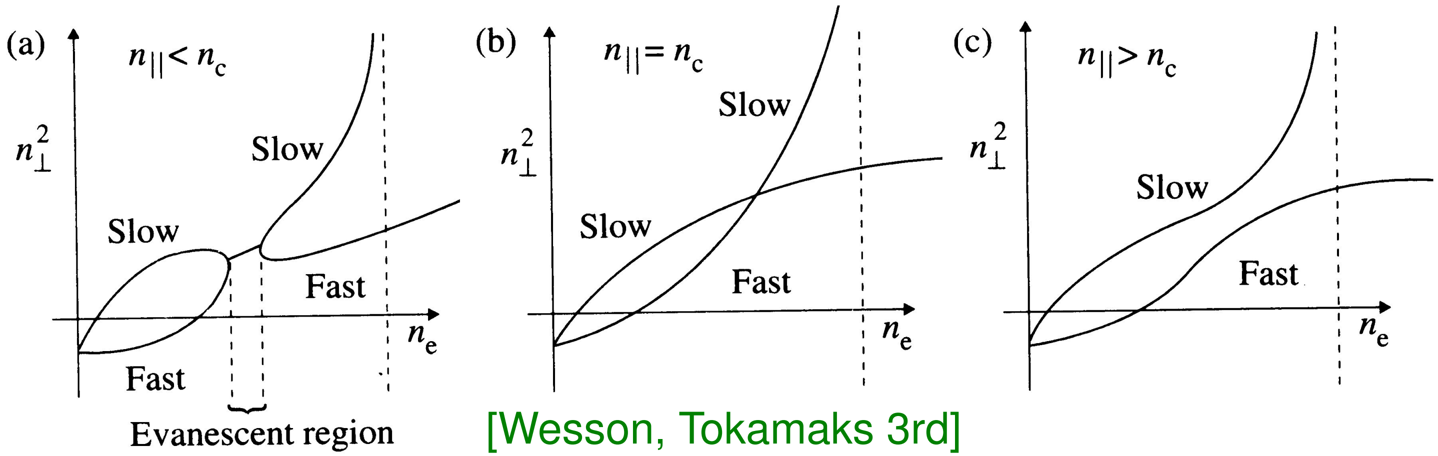
- Frequency range connecting to **lower hybrid resonance** $\omega = \omega_{\mathrm{LH}}$

$$\omega_{\mathrm{LH}} = \frac{\omega_{\mathrm{pi}}}{\sqrt{1 + \omega_{\mathrm{pe}}^2/\omega_{\mathrm{ce}}^2}}$$

- **Features**
  - Electron heating
  - **High current drive efficiency**
  - Central deposition is difficult in high-T_{e} and high-n_{e} plasma
  - Delicate waveguide array antenna is required
  - Long distance between plasma and antenna is difficult
  - Antenna structure with high power injection is complicated

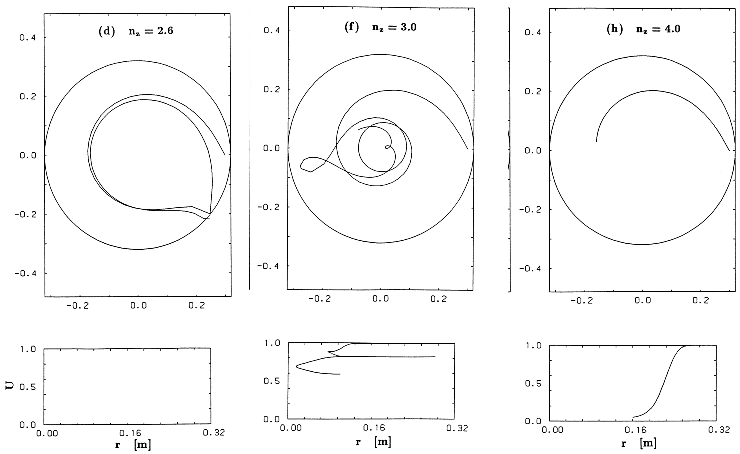
Lower hybrid wave (II)

- **Fast wave and slow wave** (Two branches with different phase vel.)
  - Refractive index along the magnetic field line $n_{||} = k_{||}c/\omega$
  - When $n_{||}$ is small, slow wave is converted to fast wave near
  - When $n_{||}$ is large, slow wave can penetrate into high density region.



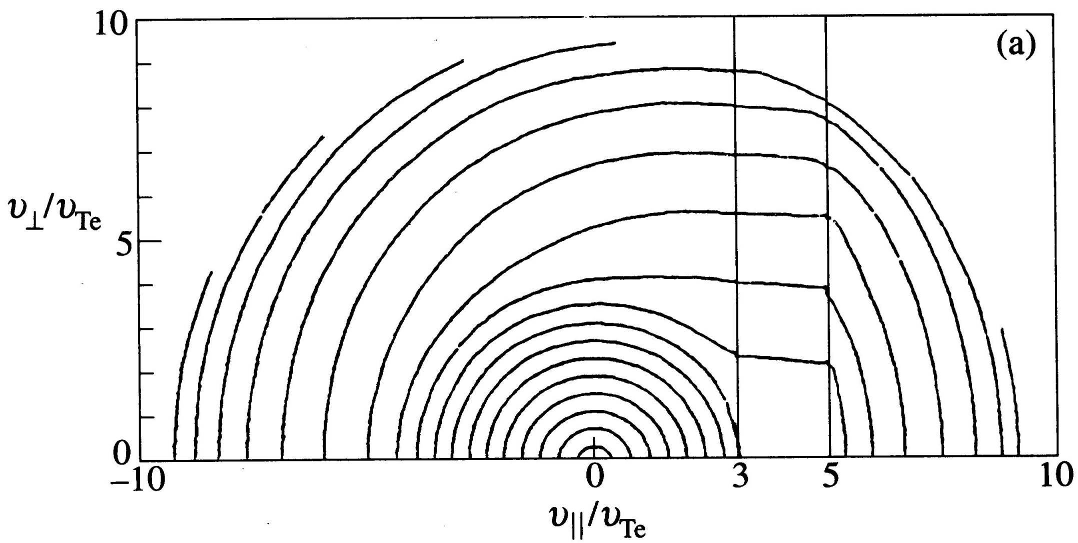
[Wesson, Tokamaks 3rd]

- **Propagation in poloidal plane**: geometrical optics approximation



Lower hybrid wave (III)

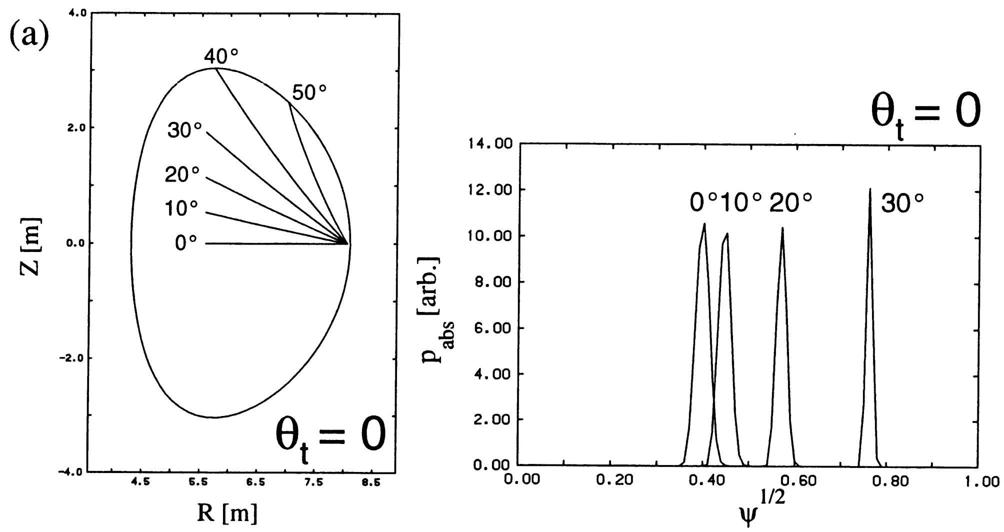
- **Modification of electron velocity distribution function** accompanied with wave-particle resonant interaction
  - **Landau damping**: linear theory, modification of $f_\mathrm{e}(v_{||}, v_\perp)$ is infinitesimal small
  - **Quasi-linear theory**: modification of $f_\mathrm{e}(v_{||}, v_\perp)$ is finite
    - Flattening of $f_\mathrm{e}(v_{||}, v_\perp)$ near the wave phase velocity
    - Isotropic modification due to Coulomb collision
    - Asymmetry of velocity distribution function $\Longrightarrow$ current drive



[Karney & Fisch, PF 1979]

Electron cyclotron wave

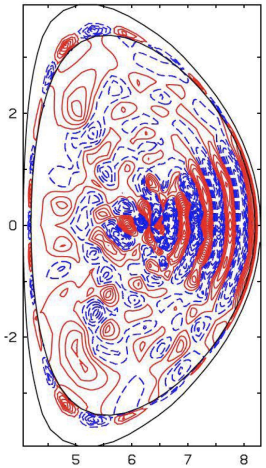
- **Propagation on poloidal plane**: Geometrical optics
  - Wave excited by antenna propagates without divergence
  - Absorbed near the cyclotron resonance



- **Features**
  - Plasma control by spatially localized heating and current drive
  - Heating and current drive in central region
  - Small antenna distant from plasma is possible
  - Slightly lower current drive efficiency
  - Steady-state operation of high-power oscillator, gyrotron.

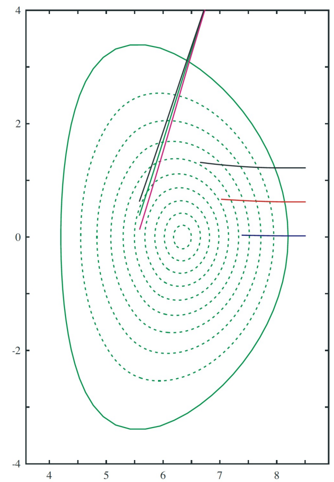
How to describe waves in plasmas (1)

		SPACE	
		Space domain	Wave number domain
TIME	Time domain	**Time-dependent simulation**	**Geometrical optics**
	Frequency domain	**Full wave analysis**	**Dispersion relation**



detailed Time-dependent simulation
Full wave analysis
Geometrical optics
simple Dispersion relation

$\Leftarrow$ Full wave analysis Geometrical optics $\Rightarrow$



How to describe waves in plasmas (2)

- **Time dependent simulation**: $\boldsymbol{E}(\boldsymbol{r}, t)$
  - **Maxwell's equation**

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \qquad \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} - \mu_0 \boldsymbol{J}$$

  - **Fluid simulation**: $\boldsymbol{J} = \sum_s e_s n_s \boldsymbol{u}_s$ (fluid velocity)
  - **Kinetic simulation**: $\boldsymbol{J} = \sum_s \int e_s \boldsymbol{v} f_s(\boldsymbol{r}, \boldsymbol{v}, t)\,\mathrm{d}\boldsymbol{v}$ (distribution function)
  - **Particle simulation**: $\boldsymbol{J} = \sum_s \sum_p e_s \boldsymbol{v}_{sp}$ (particle velocity)
    - **most general**
    - **describe nonlinear phenomena**
    - **requires large computational resources**
    - **not appropriate for parameter survey or optimization**

How to describe waves in plasmas (3)

- **Full wave analysis or stationary wave optics**: $\boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\,\omega t)$

$$\nabla\times\nabla\times\boldsymbol{E} = \frac{\omega^2}{c^2}\overleftrightarrow{\epsilon}\cdot\boldsymbol{E} + \mathrm{i}\,\omega\mu_0\boldsymbol{J}_{\mathrm{ext}}$$

  · **wave tunneling, standing wave, coupling to antenna**

  · **requires less computational resources**

- **Geometrical optics**: $\boldsymbol{E}\exp\left[\mathrm{i}\,\varphi(\boldsymbol{r},t)\right]$, $\boldsymbol{k} = -\mathrm{i}\,\nabla\varphi$, $\omega = \mathrm{i}\,\partial\varphi/\partial t$

  – **ray tracing method**

$$\frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}t} = -\frac{\partial K}{\partial\boldsymbol{r}}\bigg/\frac{\partial K}{\partial\omega}, \qquad \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{v}_{\mathrm{g}} = \frac{\partial K}{\partial\boldsymbol{k}}\bigg/\frac{\partial K}{\partial\omega}$$

  · **no tunneling, no interference, no refraction, point source**

- **Dispersion relation**: $\boldsymbol{E}\exp(\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r} - \mathrm{i}\,\omega t)$

$$K(\boldsymbol{k},\omega;\boldsymbol{r},t) = \det\left[\frac{c^2}{\omega^2}\boldsymbol{k}\times\boldsymbol{k}\times + \overleftrightarrow{\epsilon}(\boldsymbol{k},\omega)\right] = 0$$

Dielectric tensor in cold plasmas

- **Linearized equation of fluid motion**: $\mathrm{d}\boldsymbol{u}_s/\mathrm{d}t = (e_s/m_s)(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}_0)$
- **Cold plasma approximation**: $\boldsymbol{J}_s = n_s e_s \boldsymbol{u}_s = \overleftrightarrow{\sigma}_s \boldsymbol{E}$

$$\overleftrightarrow{\epsilon} = \overleftrightarrow{I} + \sum_s \frac{\mathrm{i}}{\omega\epsilon_0} \overleftrightarrow{\sigma}_s = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

where

$$S = 1 - \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega^2 - \omega_{\mathrm{c}s}^2}, \quad D = \sum_s \frac{\omega_{\mathrm{p}s}^2 \omega_{\mathrm{c}s}}{\omega(\omega^2 - \omega_{\mathrm{c}s}^2)}, \quad P = 1 - \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega^2}$$

$$\omega_{\mathrm{p}s} = \sqrt{n_s e_s^2 / m_s \epsilon_0} \quad \text{and} \quad \omega_{\mathrm{c}s} = e_s B_0 / m_s$$

- **Dispersion relation in magnetized cold plasma**: $N_{x,y,z} = k_{x,y,z} c/\omega$

$$K = \begin{vmatrix} S - N_z^2 & -iD & N_x N_z \\ iD & S - N_x^2 - N_z^2 & 0 \\ N_x N_z & 0 & P - N_x^2 \end{vmatrix} = 0$$

Dielectric tensor in hot uniform plasmas (1)

- Vlasov equation for velocity distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{q}{m} [\boldsymbol{E}(\boldsymbol{r}, t) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{r}, t)] \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

- Linearize,
- Use plane wave $\mathrm{e}^{\,\mathrm{i}\,(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$,
- Follow particle orbit,
- Expand over cyclotron harmonics using Bessel functions,
- Integrate over t,
- we obtain **dielectric tensor in hot plasmas**

Dielectric tensor in hot uniform plasmas (2)

- **Dielectric tensor**

$$\epsilon_{ij} = \delta_{ij} + \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega^2} 2\pi \int_0^\infty v_\perp \mathrm{d}v_\perp \int_{-\infty}^\infty \mathrm{d}v_\parallel$$

$$\times \left[v_\perp \sum_n \Pi_{in}^* \Pi_{jn} L_s^{(n)} + \delta_{zi}\delta_{zj} v_\parallel \left(\frac{\partial f_{s0}}{\partial v_\parallel} - \frac{v_\parallel}{v_\perp} \frac{\partial f_{s0}}{\partial v_\perp} \right) \right]$$

where

$$L_s^{(n)} \equiv \frac{\omega}{\omega - n\omega_{\mathrm{c}s} - k_\parallel v_\parallel} \left[\left(1 - \frac{k_\parallel v_\parallel}{\omega} \right) \frac{\partial f_{s0}}{\partial v_\perp} + \frac{k_\parallel v_\perp}{\omega} \frac{\partial f_{s0}}{\partial v_\parallel} \right]$$

$$\Pi_{xn} \equiv \frac{n}{\zeta} J_n(\zeta), \qquad \Pi_{yn} \equiv \mathrm{i} \frac{\mathrm{d}J_n(\zeta)}{\mathrm{d}\zeta}, \qquad \Pi_{zn} \equiv \frac{v_\parallel}{v_\perp} J_n(\zeta)$$

$*$ implies complex conjugate, δ_{ij} Kronecker delta, s particle species, n cyclotron harmonic number and $\zeta = k_\perp v_\perp / \omega_{\mathrm{c}s}$

Dielectric tensor in thermal equilibrium

- **Maxwellian velocity distribution function** in thermal equilibrium

$$f_{s0}(\boldsymbol{v}) = n_{s0}\left(\frac{m_s}{2\pi k_B}\right)^{3/2} \exp\left[-\frac{1}{k_B T_s}\left(\frac{1}{2}m_s v^2\right)\right]$$

- Dielectric tensor for Maxwellian velocity distribution function

$$\overleftrightarrow{\epsilon}(\boldsymbol{k},\omega) = \overleftrightarrow{I} + \sum_s \frac{\omega_{ps}^2}{\omega^2}\sum_n \times \begin{pmatrix} \frac{n^2\Lambda_n}{\lambda}P_n & \mathrm{i}\,n\frac{\mathrm{d}\Lambda_n}{\mathrm{d}\lambda}Q_n & \frac{k_\parallel}{|k_\parallel|}\frac{n\Lambda_n}{\lambda}Q_n \\ -\mathrm{i}\,n\frac{\mathrm{d}\Lambda_n}{\mathrm{d}\lambda}Q_n & \left(\frac{n^2\Lambda_n}{\lambda} - 2\lambda\frac{\mathrm{d}\Lambda_n}{\mathrm{d}\lambda}\right)P_n & -\mathrm{i}\frac{k_\parallel}{|k_\parallel|}\frac{\mathrm{d}\Lambda_n}{\mathrm{d}\lambda}Q_n \\ \frac{k_\parallel}{|k_\parallel|}\frac{n\Lambda_n}{\lambda}Q_n & \mathrm{i}\frac{k_\parallel}{|k_\parallel|}\frac{\mathrm{d}\Lambda_n}{\mathrm{d}\lambda}Q_n & \Lambda_n R_n \end{pmatrix}$$

where P_n, Q_n and R_n are related to the parallel motion, and Λ_n the perpendicular motion.

Plasma-wave interaction for Maxwellian plasma

- **Parallel**: **Plasma dispersion function**: $Z(\xi)$

$$Z(\xi) = \frac{1}{\sqrt{\pi}} \int_C \frac{\mathrm{e}^{-u^2}}{u - \xi} \, \mathrm{d}u$$

$$P_n = \frac{\omega}{\omega - n\omega_\mathrm{c}} \xi_n Z(\xi_n), \quad Q_n = \frac{\omega}{\omega_\mathrm{c}} \left[1 + \xi_n Z(\xi_n)\right], \quad R_n = 2\,\xi_0\,\xi_n \left[1 + \xi_n Z(\xi_n)\right]$$

where

$$\xi_n = \frac{\omega - n\omega_\mathrm{c}}{\sqrt{2}|k_{||}|v_{\mathrm{Ts}}}$$

- **Perpendicular**: **Finite Larmor Radius effects**: $\Lambda_n(\lambda)$

$$\Lambda_n(\lambda) = I_n(\lambda)\ \mathrm{e}^{-\lambda}$$

where

$I_n(\lambda)$: the n-th order modified Bessel function of the first kind

$$\lambda = k_\perp^2 \rho^2 = k_\perp^2 \frac{v_{\mathrm{Ts}}^2}{\omega_{\mathrm{c}s}^2} = k_\perp^2 \frac{k_\mathrm{B} T_\mathrm{s}}{m_s \omega_{\mathrm{c}s}^2}$$

Full wave analysis in inhomogensous hot plasma

- **Cold wave number approach**: no kinetic modes
  - Use $\boldsymbol{k}_{\rm cold}$ from the dispersion relation in a cold uniform plasma

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon}(\boldsymbol{r}; \boldsymbol{k}_{\rm cold}) \cdot \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{r})$$

- **Differential operator approach** [1]: difficult for higher order
  - Expand $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{k})$ with respect to $\boldsymbol{k}$ and replace $\boldsymbol{k}$ by $-\mathrm{i}\,\nabla$

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon}(\boldsymbol{r}; -\mathrm{i}\,\nabla) \cdot \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{r})$$

- **Spectral approach** [2]: large dense matrix has to be solved
  - Fourier transform in the direction of inhomogeneity $\boldsymbol{r}$

$$-\boldsymbol{k} \times \boldsymbol{k} \times \boldsymbol{E}(\boldsymbol{k}) - \frac{\omega^2}{c^2} \sum_{\boldsymbol{k}'} \overleftrightarrow{\epsilon}(\boldsymbol{k}, \boldsymbol{k}') \cdot \boldsymbol{E}(\boldsymbol{k}') = \mathrm{i}\,\omega\mu_0 \boldsymbol{J}_{\rm ext}(\boldsymbol{k})$$

- **Inverse Fourier transform of $\overleftrightarrow{\epsilon}(\boldsymbol{r}, \boldsymbol{k})$** [3]: based on uniform $\overleftrightarrow{\epsilon}$

[1] Fukuyama et al., CPR (1986), [2] Jaeger et al. PoP (2000), [3] Sauter et al., NF (1992)

Integral form in uniform plasmas

- **propagation in** z
- **Particle orbit**: $z = z' + v_z(t - t')$
- **Variable transformation** : $v_z = \dfrac{z - z'}{t - t'}$
- **Perturbed distribution function for Maxwellian**: $\tau = t - t'$

$$f(z, \boldsymbol{v}) = \frac{n}{(2\pi T/m)^{3/2}} \frac{q}{T} \int_0^\infty \mathrm{d}\tau\, \boldsymbol{v} \cdot \boldsymbol{E}(z')\, \mathrm{e}^{\,\mathrm{i}\,\omega\tau} \exp\left[-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2T}\right]$$

- **Current density**: variable transformation: $v_z \;\Rightarrow\; z'$

$$\boldsymbol{J}(z) = q \int \mathrm{d}\boldsymbol{v}\, \boldsymbol{v} f(z, \boldsymbol{v}) = \int \mathrm{d}z'\, \overleftrightarrow{\sigma}(z, z') \cdot \boldsymbol{E}(z')$$

- **Electric conductivity tensor**: e.g. zz component: $\tau = t - t'$

$$\sigma_{zz}(z, z') = \frac{nq^2}{\sqrt{2\pi}\, m v_{\mathrm{T}}^3} \int_0^\infty \mathrm{d}\tau \frac{(z - z')^2}{\tau^3} \exp\left[-\frac{1}{2}\frac{(z - z')^2}{v_{\mathrm{T}}^2 \tau^2} + \mathrm{i}\,\omega\tau\right]$$

Ray Tracing

- **Geometrical Optics**
  - Wave length $\lambda \ll$ Characteristic scale length L of the medium
  - Plane wave: Beam size d is sufficiently large
    - **Fresnel condition is well satisfied**: $L \ll d^2/\lambda$

- **Evolution along the ray trajectory τ**
  - Wave packet position $\boldsymbol{r}$, wave number $\boldsymbol{k}$, dielectric tensor D
  - **Ray tracing equation**:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\partial D}{\partial \boldsymbol{k}} \bigg/ \frac{\partial D}{\partial \omega} = \boldsymbol{v}_{\mathrm{g}}$$

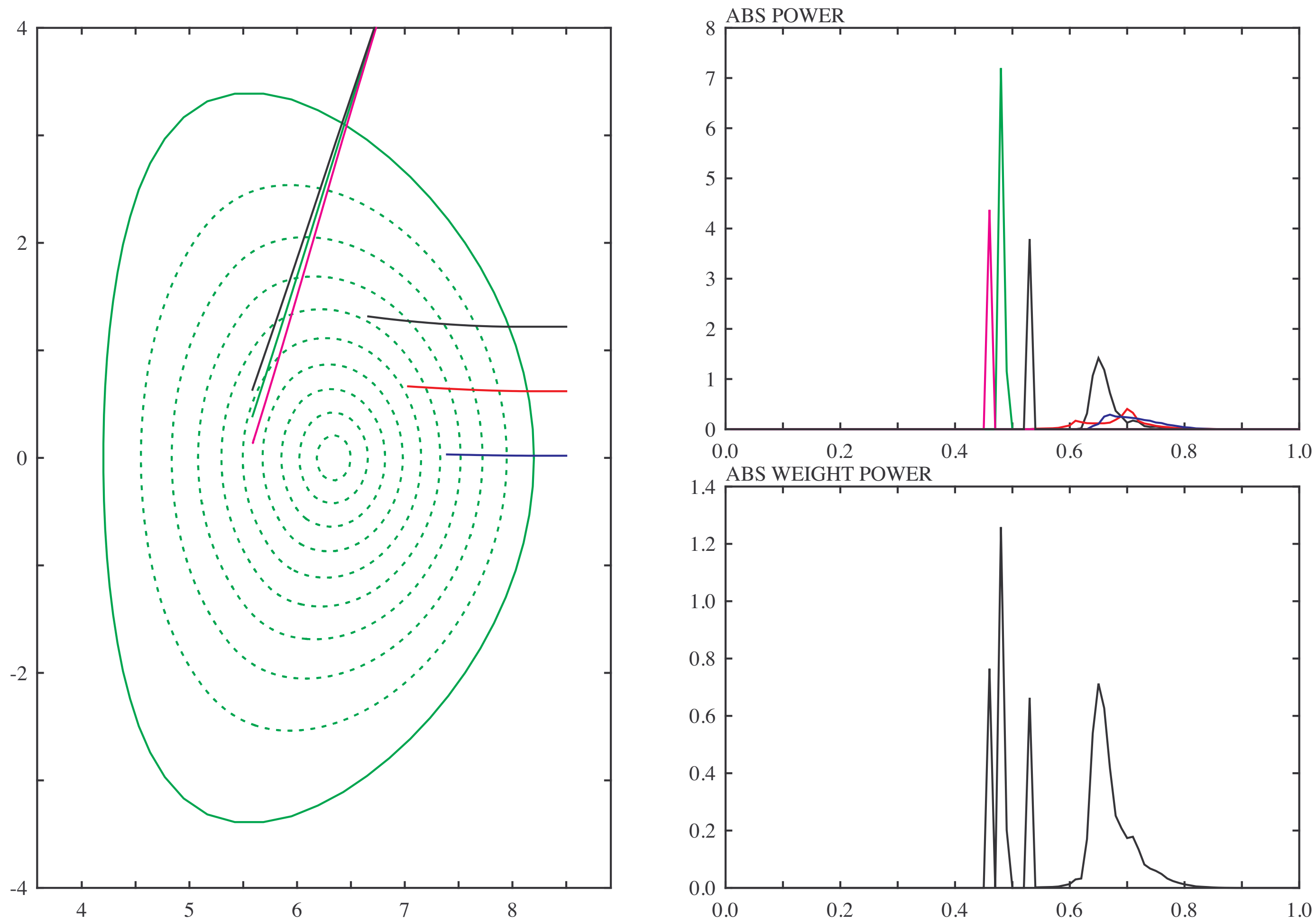
$$\frac{\mathrm{d}\boldsymbol{k}}{\mathrm{d}\tau} = -\frac{\partial D}{\partial \boldsymbol{r}} \bigg/ \frac{\partial D}{\partial \omega}$$

  - **Equation for wave energy** W, group velocity $\boldsymbol{v}_{\mathrm{g}}$, damping rate γ

$$\boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_{\mathrm{g}} W\right) = -2\gamma W$$

Analysis of EC wave propagation in ITER

BB = 5.300	Q0 = 1.000	PA(1) = 0.0005	PTPR(1)= 20.0000	PA(2) = 2.0000	PTPR(2)= 20.0000
RR = 6.200	QA = 3.000	PZ(1) = -1.0000	PTPP(1)= 20.0000	PZ(2) = 1.0000	PTPP(2)= 20.0000
RA = 2.000	RF = 1.700E+05	PN(1) = 0.8000	PTS(1) = 0.0500	PN(2) = 0.4000	PTS(2) = 0.0500
RKAP = 1.700	NPHII= -0.1177	PNS(1) = 0.0100	MODELP1= 25.	PNS(2)= 0.0050	MODELP2= 1.



Beam Tracing: TASK/WR

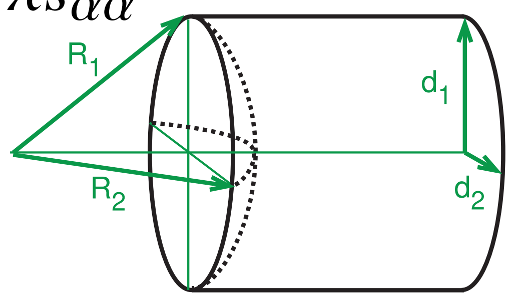
- Beam size perpendicular to the beam direction: first order in ϵ
- **Beam shape** : Hermite polynomial: H_n)

$$\boldsymbol{E}(\boldsymbol{r}) = \mathrm{Re}\left[\sum_{mn} C_{mn}(\epsilon^2\boldsymbol{r})\boldsymbol{e}(\epsilon^2\boldsymbol{r})H_m(\epsilon\xi_1)H_n(\epsilon\xi_2)\,\mathrm{e}^{\,\mathrm{i}\,s(\boldsymbol{r})-\phi(\boldsymbol{r})}\right]$$

  – Amplitude : C_{mn}, Polarization : $\boldsymbol{e}$, Phase : $s(\boldsymbol{r}) + \mathrm{i}\,\phi(\boldsymbol{r})$

$$\begin{aligned} s(\boldsymbol{r}) &= s_0(\tau) + k^0_\alpha(\tau)[r^\alpha - r^\alpha_0(\tau)] + \frac{1}{2}s_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)] \\ \phi(\tau) &= \frac{1}{2}\phi_{\alpha\beta}[r^\alpha - r^\alpha_0(\tau)][r^\beta - r^\beta_0(\tau)] \end{aligned}$$

  – Position of beam axis : $\boldsymbol{r}_0$, Wave number on beam axis: $\boldsymbol{k}^0$
  – **Curvature radius** of equi-phase surface: $R_\alpha = 1/\lambda s_{\alpha\alpha}$
  – **Beam radius**: $d_\alpha = \sqrt{2/\phi_{\alpha\alpha}}$



- Gaussian beam : case with $m = 0$, $n = 0$

Beam Propagation Equation

- **Solvable condition for Maxwell's equation with beam field**

$$\frac{\mathrm{d}r_0^\alpha}{\mathrm{d}\tau} = \frac{\partial K}{\partial k_\alpha}$$

$$\frac{\mathrm{d}k_\alpha^0}{\mathrm{d}\tau} = -\frac{\partial K}{\partial r^\alpha}$$

$$\frac{\mathrm{d}s_{\alpha\beta}}{\mathrm{d}\tau} = -\frac{\partial^2 K}{\partial r^\alpha \partial r^\beta} - \frac{\partial^2 K}{\partial r^\beta \partial k_\gamma} s_{\alpha\gamma} - \frac{\partial^2 K}{\partial r^\alpha \partial k_\gamma} s_{\beta\gamma} - \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} s_{\alpha\gamma} s_{\beta\delta} + \frac{\partial^2 K}{\partial k^\gamma \partial k^\delta} \phi_{\alpha\gamma}\phi_{\beta\delta}$$

$$\frac{\mathrm{d}\phi_{\alpha\beta}}{\mathrm{d}\tau} = -\left(\frac{\partial^2 K}{\partial r^\alpha \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\alpha\delta}\right)\phi_{\beta\gamma} - \left(\frac{\partial^2 K}{\partial r^\beta \partial k^\gamma} + \frac{\partial^2 K}{\partial k^\gamma \partial k_\delta} s_{\beta\delta}\right)\phi_{\alpha\gamma}$$

- By integrating this set of 18 ordinary differential equations, we obtain trace of the beam axis, wave number on the beam axis, curvature of equi-phase surface, and beam size.
- **Equation for the wave amplitude** C_{mn}

$$\nabla \cdot \left(\boldsymbol{v}_{\mathrm{g}0}|C_{mn}|^2\right) = -2\left(\gamma |C_{mn}|^2\right)$$

Group velocity: $\boldsymbol{v}_{\mathrm{g}0}$, Damping rate: $\gamma \equiv (\boldsymbol{e}^* \cdot \overleftrightarrow{\epsilon}_{\mathrm{A}} \cdot \boldsymbol{e})/(\partial K/\partial\omega)$

Diffraction of wave beam in vacuum

- **Beam radius as a function of path length**: $f = 170\,\text{GHz}, \lambda = 1.76\,\text{mm}$

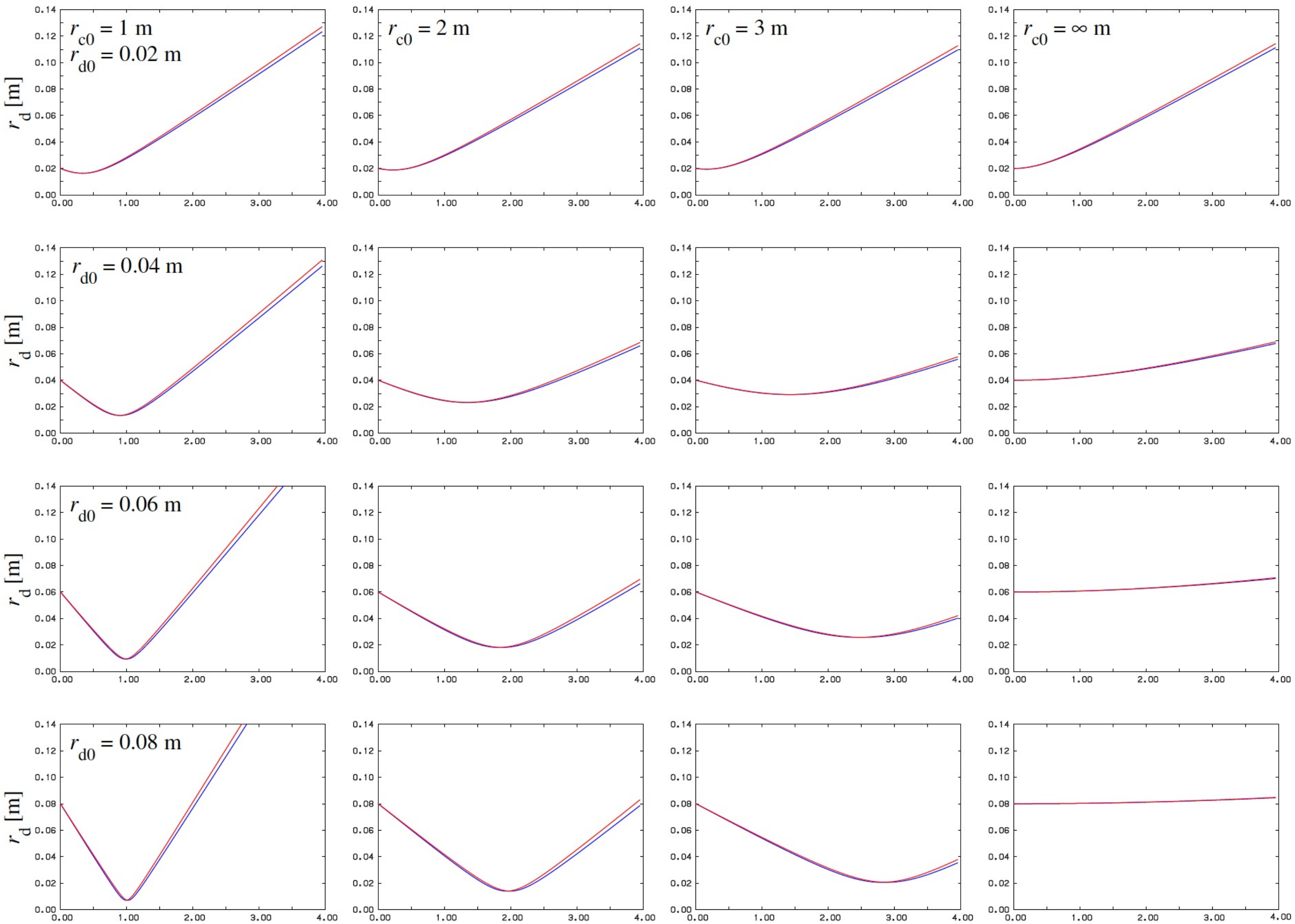
$d_0 \setminus R_0$ 1 m 2 m 3 m ∞ m

20 mm

40 mm

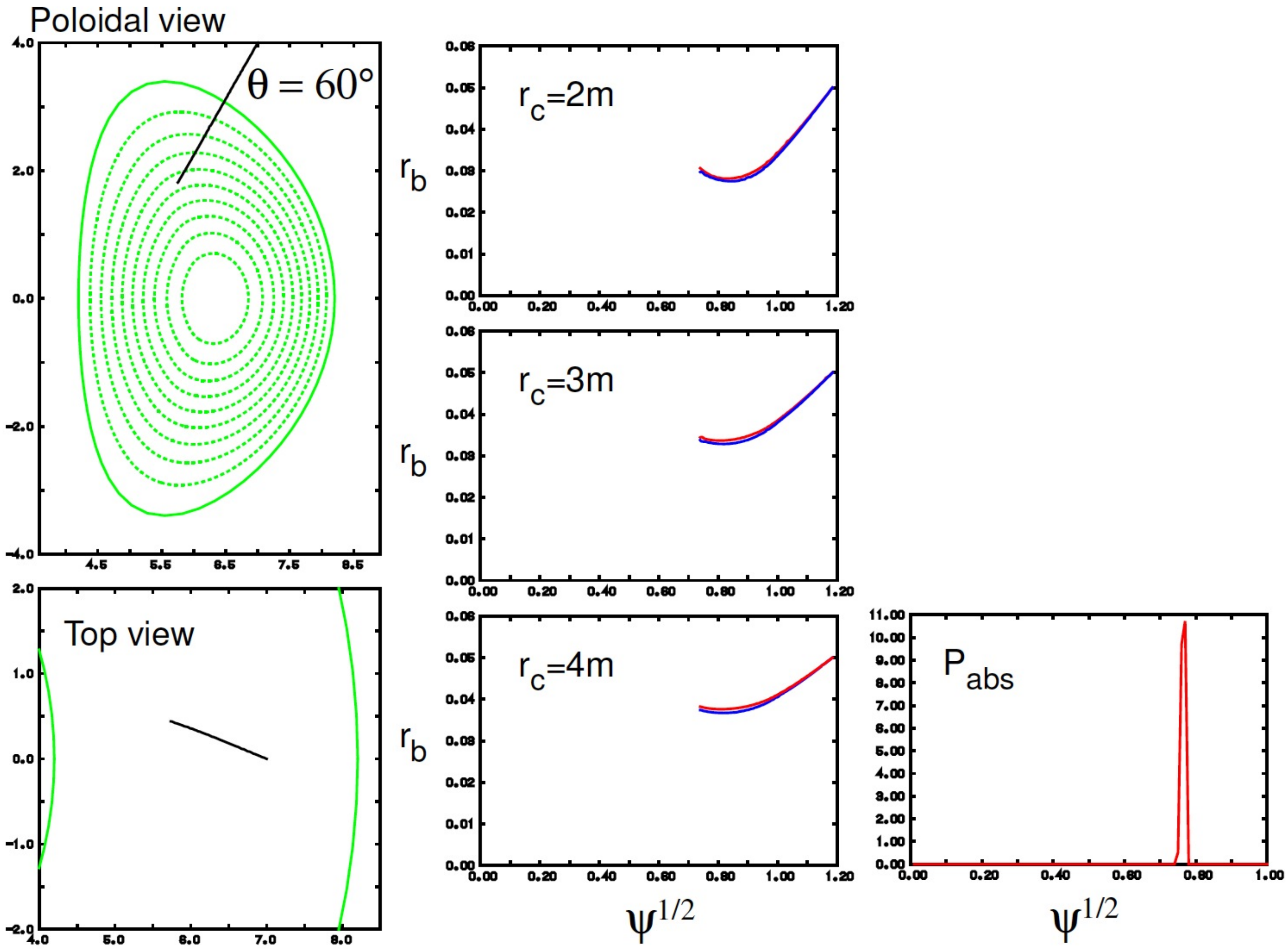
60 mm

80 mm



Application to ITER EC beam

- $f = 170\,\mathrm{GHz}$, toroidal angle $20°$, poloidal angle $60°$
- Initial beam radius: $50\,\mathrm{mm}$



Full wave analysis: TASK/WM

- **magnetic surface coordinate**: (ψ, θ, φ)
- Boundary-value problem of **Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\, \omega \mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- Kinetic **dielectric tensor**: $\overleftrightarrow{\epsilon}$:
  - **kinetic damping**: $Z[(\omega - n\omega_{\mathrm{c}})/k_{||} v_{\mathrm{th}}]$
  - **Drift-kinetic equation for energetic ions to calculate $\overleftrightarrow{\epsilon}$**

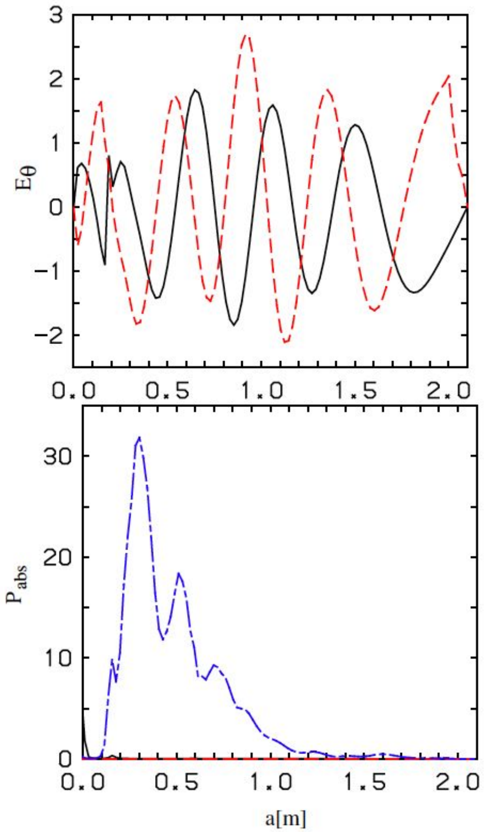
$$\left[\frac{\partial}{\partial t} + v_{||} \nabla_{||} + (\boldsymbol{v}_{\mathrm{d}} + \boldsymbol{v}_{\mathrm{E}}) \cdot \nabla + \frac{e_\alpha}{m_\alpha} (v_{||} E_{||} + \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{E}) \frac{\partial}{\partial \varepsilon} \right] f_\alpha = 0$$

- Poloidal and toroidal **mode expansion**:
  - **Accurate estimation of $k_{||}$**

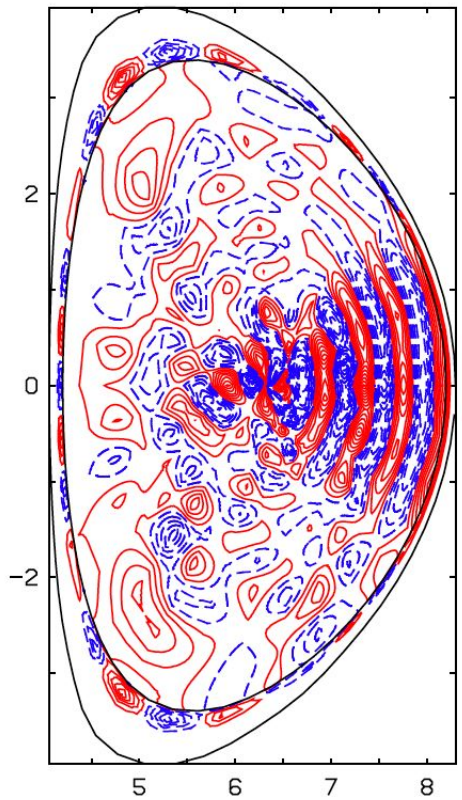
Analysis of IC propagation in ITER

- **Minority heating in D phase**: D + H

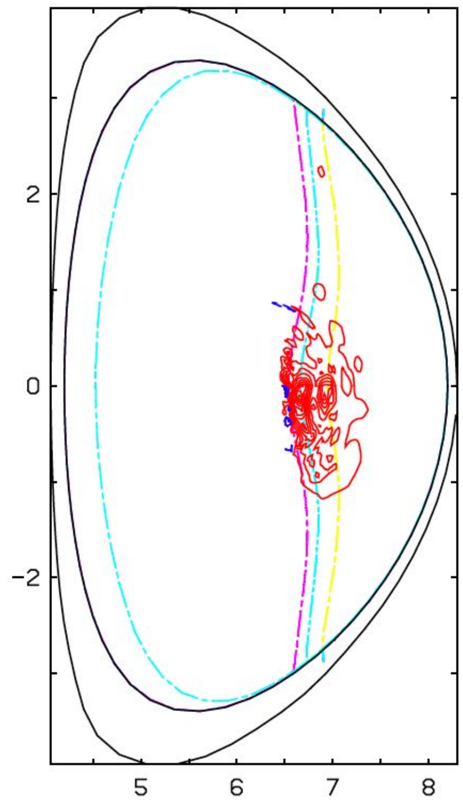
Radial profile



Wave electric field

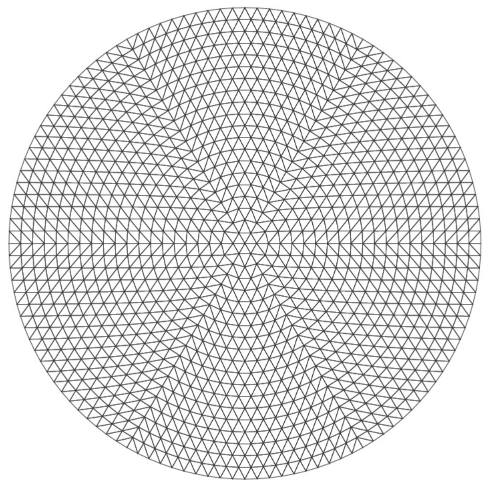


P_{abs} by minority ion



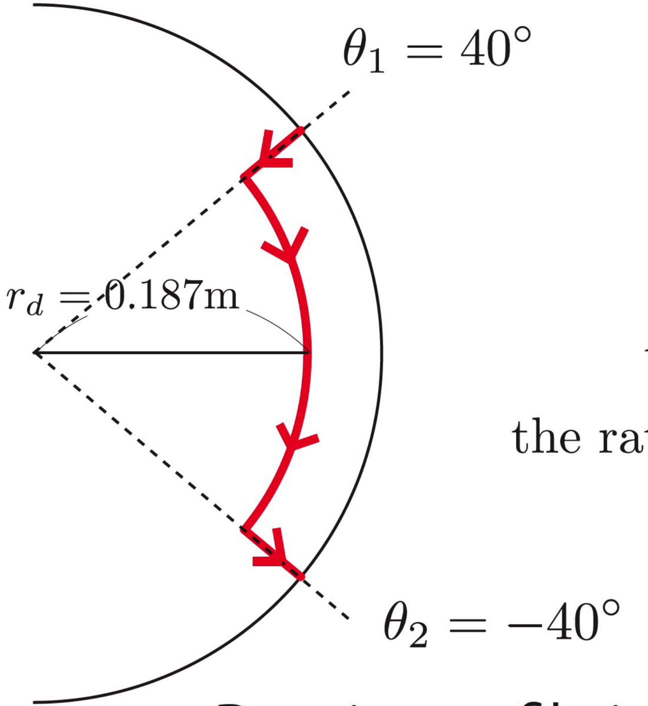
Full wave analysis by FEM: TASK/WF

- **Wave electric field with complex frequency**: $\tilde{\boldsymbol{E}}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\,\mathrm{e}^{-\mathrm{i}\,\omega t}$
- **Maxwell's equation**: $\nabla \times \nabla \times \boldsymbol{E} - \dfrac{\omega^2}{c^2}\overleftrightarrow{\epsilon} \cdot \boldsymbol{E} = \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$
  - $\overleftrightarrow{\epsilon}$: Dielectric tensor
    - Collisional cold plasma model $\Longrightarrow$ **Integral form**
- **Numerical method**: FEM
  - **3D version**
    - Tetrahedron element
    - Electric field along a edge of a tetrahedron
  - **2D version**: axisymmetric cylindrical or plane
    - Triangular element
    - **EB**: Scalar (toroidal) and vector (poloidal) hybrid basis function
    - **Aϕ**: Scalar basis function for vector and scalar potentials



EC waves in a spherical tokamak

- Antenna

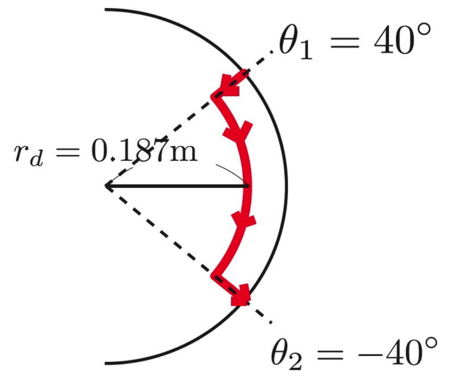


- Used parameters (LATE: Kyoto University)

major radius	R	0.22m
minor radius	a	0.16m
wave frequency	f	5GHz
the number of elements	NEMAX	25350
the ratio of collision frequency to ω	ν/ω	1.0×10^{-3}

- Density profile is parabolic.
- Collisional cold plasma model is used for the dielectric tensor.
  - Mode conversion of EC wave into EB wave does not occur.

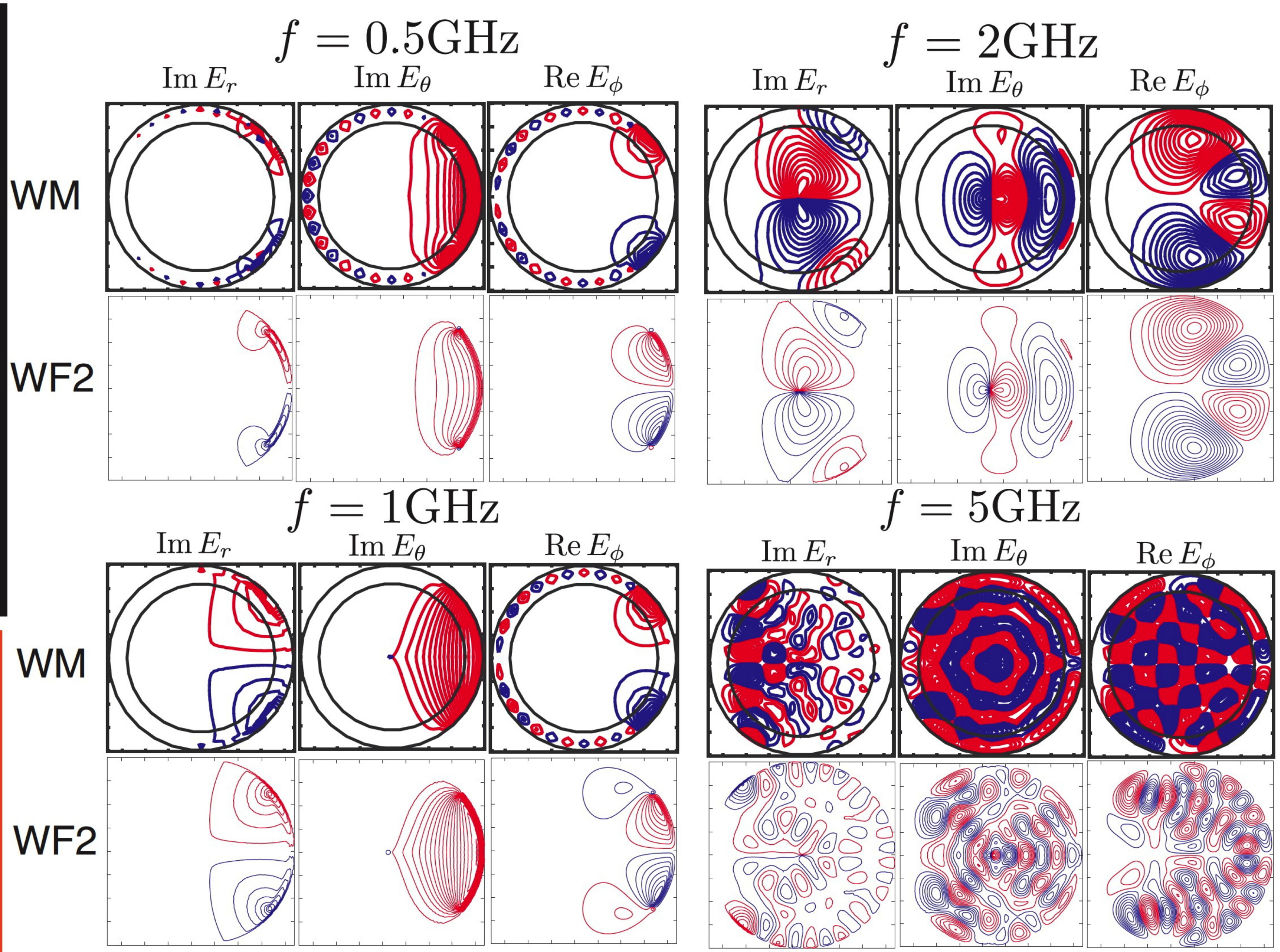
Benchmark test with TASK/WM in Vacuum



major radius	R	0.22m
minor radius	a	0.16m
toroidal mode number	n_ϕ	8

Resolution

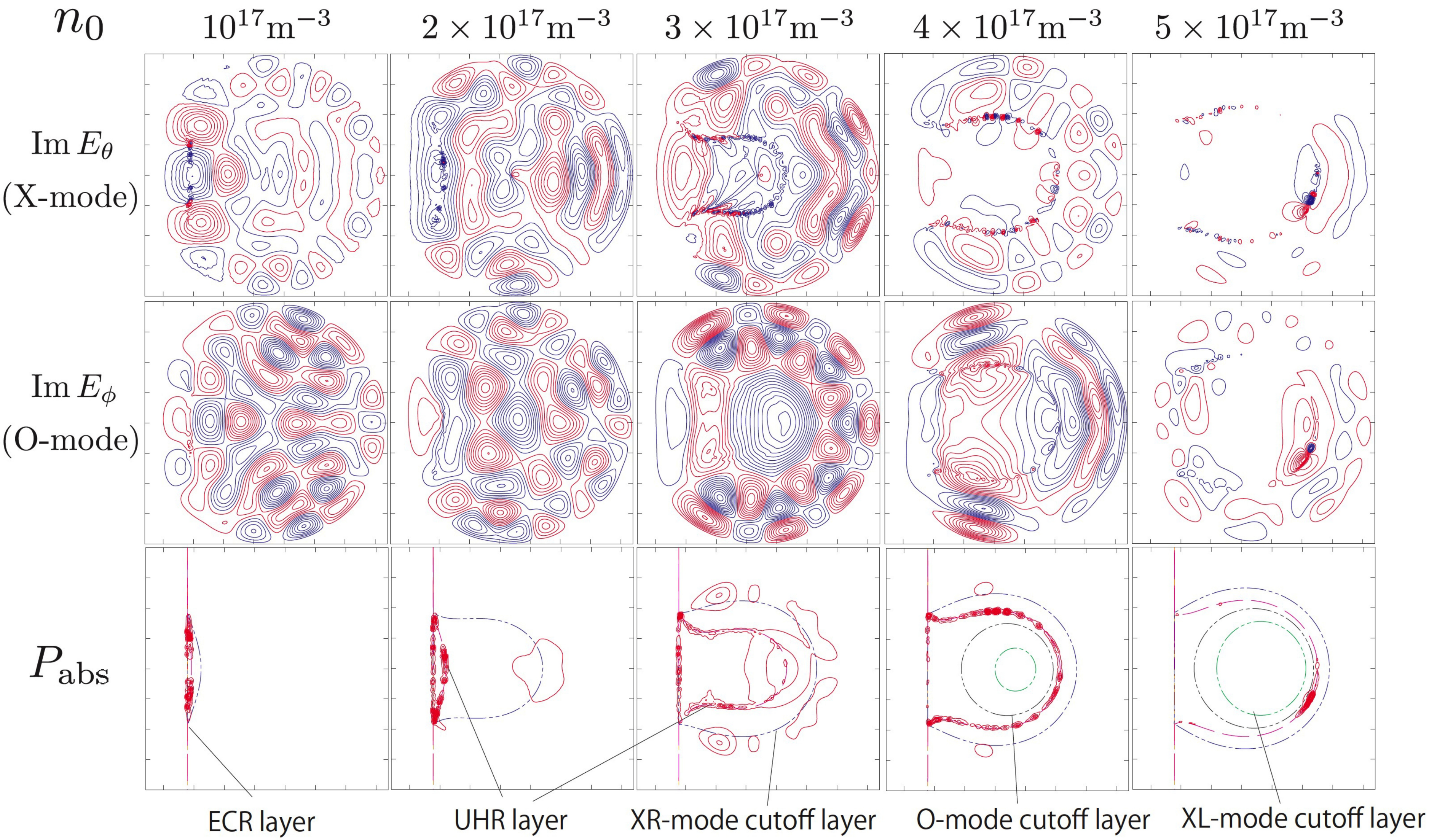
- WF2: 25350 elements
- WM:
  - radial direction: 100 points
  - poroidal direction: 32 modes



- TASK/WM and WF2 have different spacial resolution, so there are some differences in their results.

2D analysis on poloidal plane (Cold plasma)

- $B_0 = 0.072$T



Kinetic Integrated Modeling

Better understanding of burning plasmas

- **Behavior of energetic particles**: production, transport, excitation

Analysis of momentum distribution functions

- **Self-consistent description of energetic particles**
  - No separate description of bulk and energetic components
  - Interaction with electromagnetic field
- **Influence of energetic particles on heating processes**
  - Propagation and absorption of waves
  - Fusion reaction rate
- **Modification of $f(\boldsymbol{v}, \rho, t)$ due to radial transport**
  - Broadening of deposition profile

Modeling based on momentum distribution function is required.

Kinetic Description of Transport

3D Fokker-Planck analysis: TASK/FP

- **Bounce-averaged**:
  Trapped particle effect, zero banana width
- **Relativistic**:
  momentum $\boldsymbol{p}$, relativistic collision operator
- **Non-Maxwellian collision operator**:
  momentum and energy conserved (2nd order Legendre exp.)
- **Multi-species**:
  conservation between species
- **Radial diffusion**:
  Anomalous radial diffusion with momentum dependence
- **Fusion reaction**:
  Contribution of energetic ions (integration in momentum space)

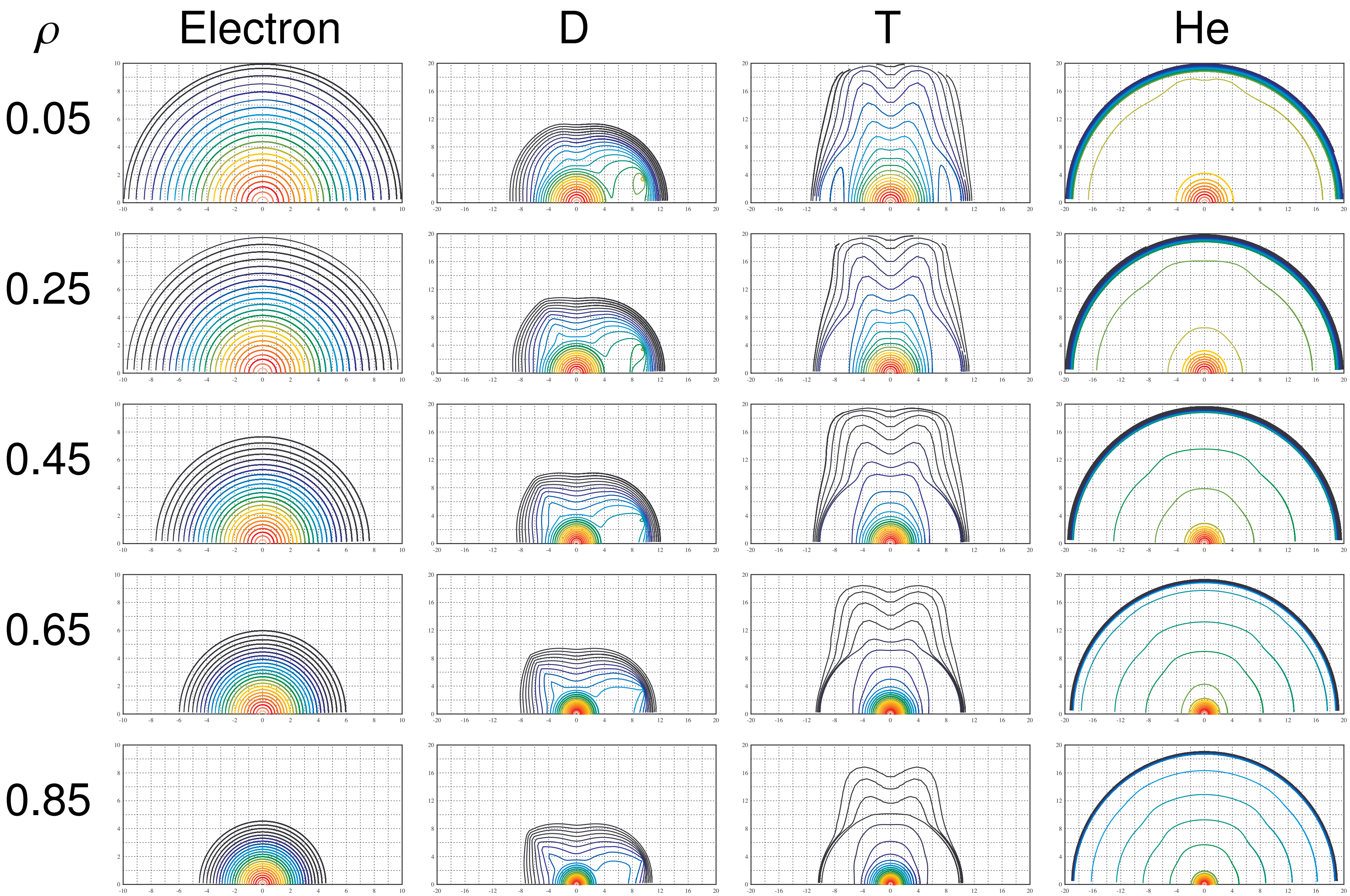
Ref. H.Nuga and A.Fukuyama: Prog. Nuc. Sci. Tech. **2** 76 (2011)

A. Fukuyama et al.: IAEA FEC2012 TH/P6-13

Analysis of Multi-Species Heating in ITER Plasma

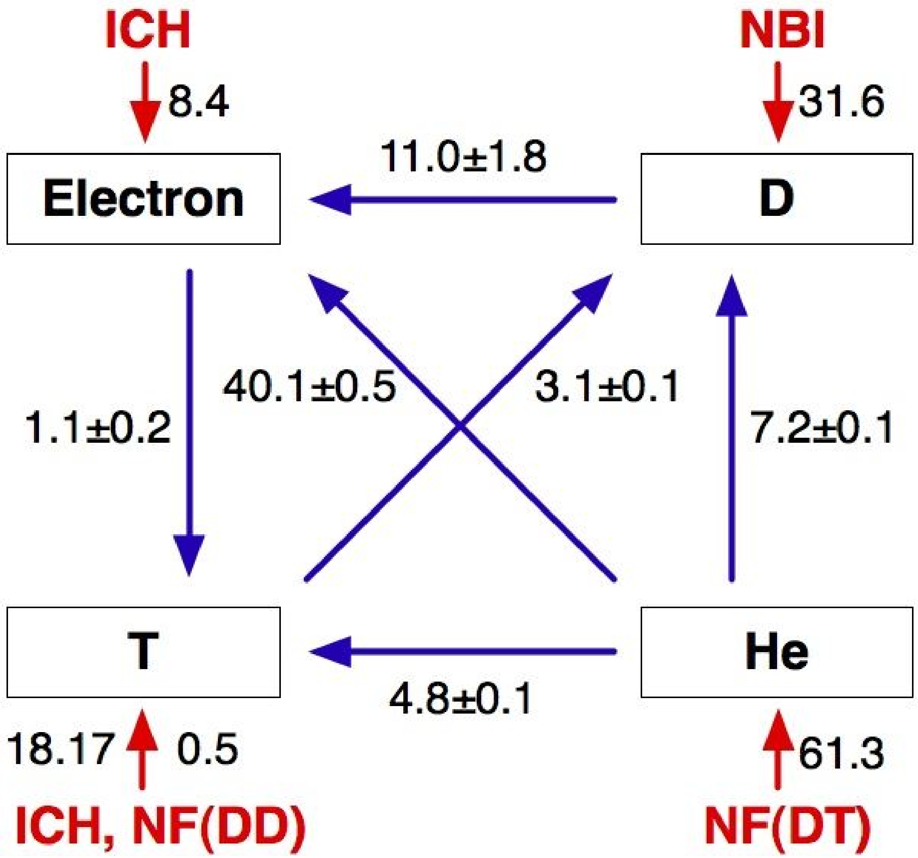
- **2D MHD equilibrium**
  - $R = 6.2\,\mathrm{m}$, $a = 2.0\,\mathrm{m}$, $\kappa = 1.7$, $\delta = 0.33$, $B_0 = 5.3\,\mathrm{T}$, $I_\mathrm{p} = 3\,\mathrm{MA}$
- **Multi species**:
  - Electron, D, T, He
- **Multi scheme heating**:
  - ICH, NBI, NF (DT, DD, TT)
- **Initial density**:
  - $n_\mathrm{e}(0) = 10^{20}\,\mathrm{m}^{-3}$, $n_\mathrm{D}(0) = 5\times 10^{19}\,\mathrm{m}^{-3}$, $n_\mathrm{T}(0) = 5\times 10^{19}\,\mathrm{m}^{-3}$
- **Initial temperature**:
  - $T_\mathrm{e}(0) = T_\mathrm{D}(0) = T_\mathrm{T}(0) = 20\,\mathrm{keV}$
- **Radial diffusion coefficient**: simplest model
  - $D_{rr} = 0.1(1+9\rho^2)\,\mathrm{m/s}$

Momentum Distribution Functions ($t\ =\ 1\,\mathrm{s}$)



Power Transfer between Species

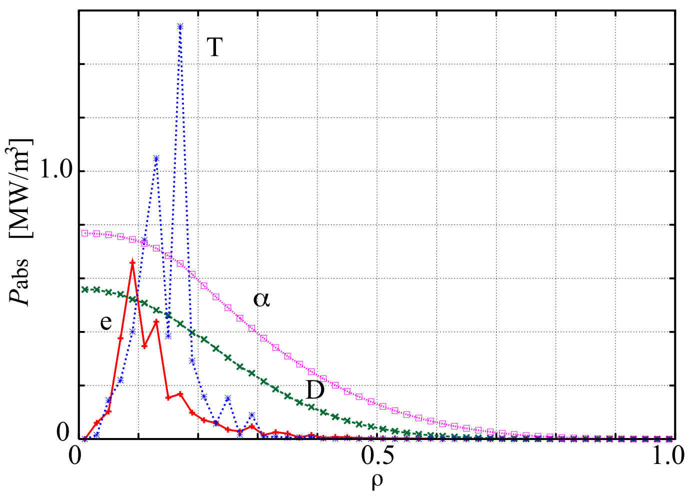
- **Collisional power transfer**



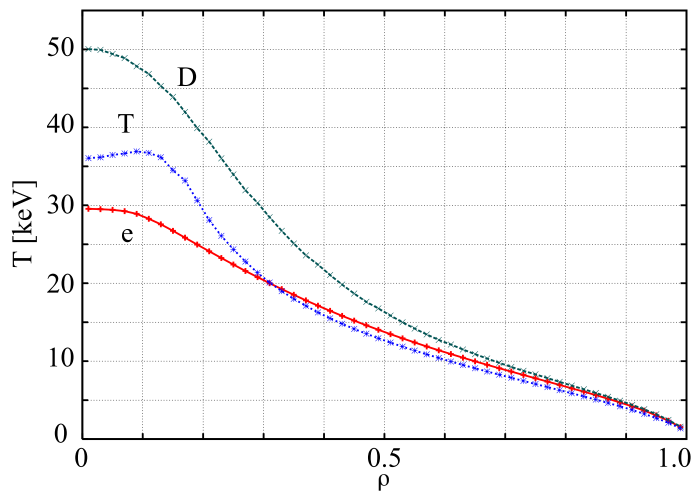
- **Requires more momentum meshes for better accuracy**
  - At present, typically $100 \times 100 \times 50$

Simulation with Radial Transport

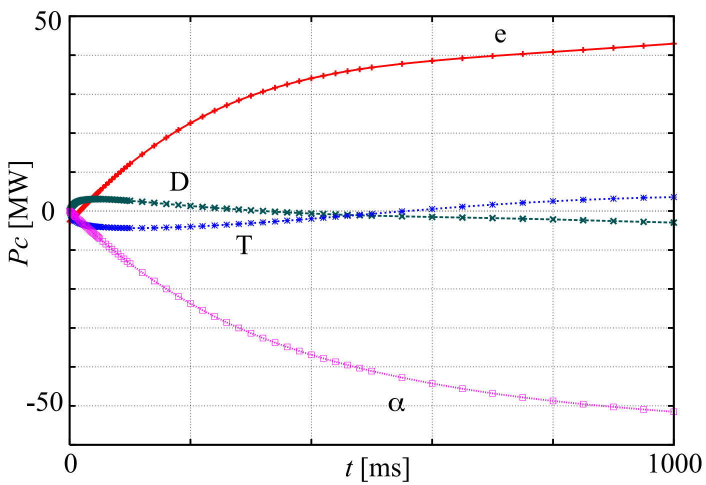
Absorbed power vs ρ



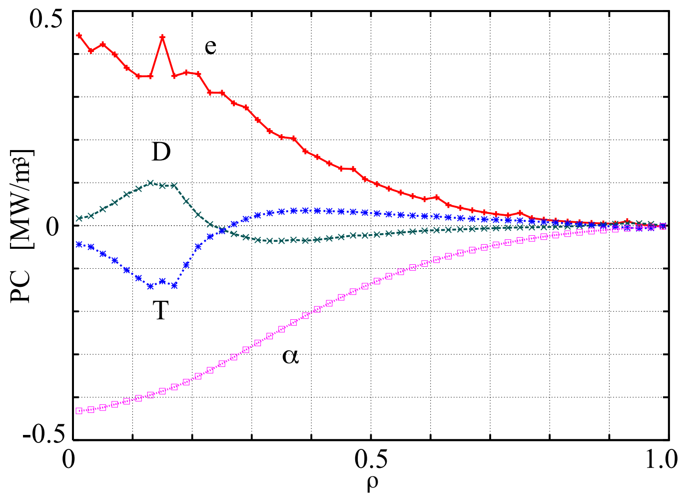
Kinetic energy density vs ρ



Collisional power transfer vs t



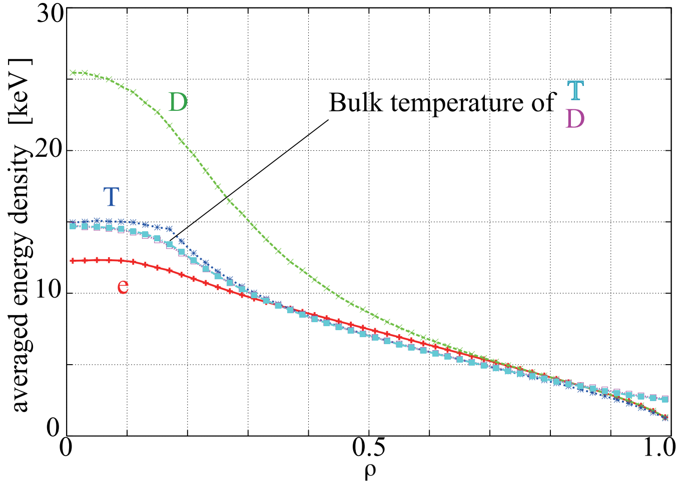
Collisional power transfer vs ρ



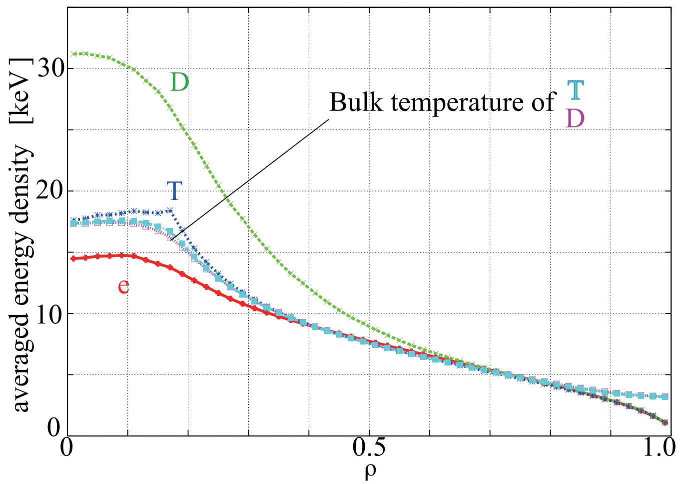
Dependence on Radial Diffusion Model

Radial profile of kinetic energy density: $p' = \sqrt{p^2 + p_{\mathrm{th0}}^2}$

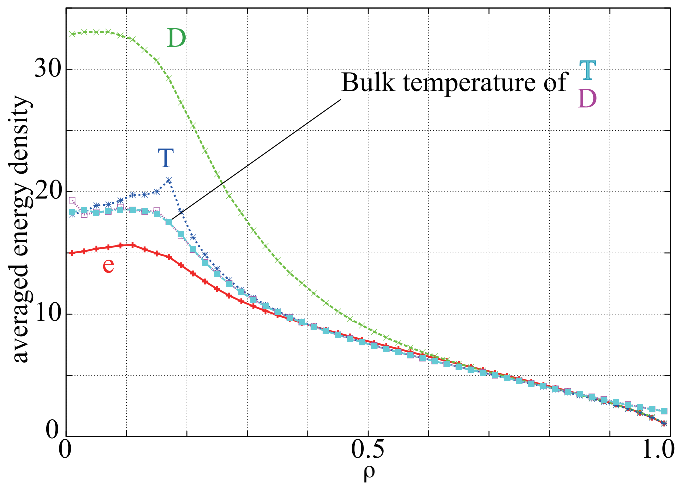
$D_{rr} \propto p'^0$



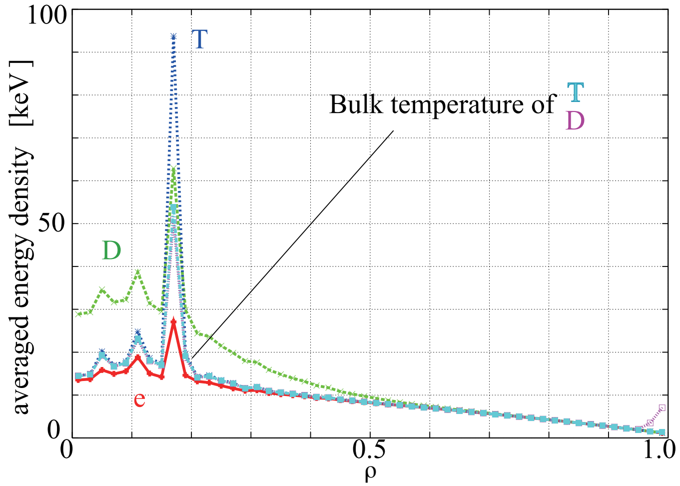
$D_{rr} \propto p'^{-1/2}$



$D_{rr} \propto p'^{-1}$



$D_{rr} = 0$



Dependence on Radial Diffusion Model

		$D_{rr} \propto p'^0$	$D_{rr} \propto p'^{-1/2}$	$D_{rr} \propto p'^{-1}$	$D_{rr} = 0$
E_K [keV]	e	7.13	7.63	7.74	8.18
	D	9.57	10.61	10.82	11.72
	T	7.18	8.00	8.15	9.44
	He	471.70	527.12	558.28	622.75
$T_{bulk,ave}$ [keV]	e	7.16	7.64	7.80	8.18
	D	7.18	8.03	8.06	8.95
	T	7.13	7.98	7.98	8.87
	He	9.88	12.48	12.96	17.88
n_{ave} [10^{16}m^{-3}]	He	6.45	8.36	8.64	9.65
P_{abs} [MW]	IC (e)	7.94	9.24	9.68	10.79
	IC (T)	8.95	10.34	10.87	15.28
	NB (D)	31.68	31.69	31.68	31.69
	NF_{DT}	23.36	30.77	32.70	36.88
I_{CD} [MA]	I (e)	-1.13	-1.36	-1.44	-1.62
	I (D)	2.74	3.01	3.11	3.24
	T (tot)	1.61	1.65	1.67	1.62

Validation with experimental observation is necessary.

Summary

- **Various actuators** are used for **heating, current drive, and fueling** in fusion plasmas.
- Modeling of **Joule heating**, **NB injection** and **pellet injection** are relatively well established.
- Modeling of **wave heating** has various levels of analyses with different accuracy. Appropriate model should be used for its purpose.
- **Self consistent analyses** of **wave propagation and absorption** and **modification of velocity distribution functions** will be important in burning plasmas.